

Contents

List of Figures — VII

List of Tables — IX

Preface — XI

1 Introduction — 1

2 Review of orthogonal polynomials — 3

2.1 Introduction — 3

2.2 Generalities — 4

2.3 Orthogonal polynomials via a three-level recurrence — 8

2.4 Darboux–Christoffel rule — 11

2.5 Continued fractions — 13

2.6 Orthogonal polynomials via Rodrigues rule — 16

2.7 Orthogonal polynomials via differential equations — 17

2.8 Some classical orthogonal polynomials — 18

2.8.1 Legendre polynomials — 19

2.8.2 Laguerre polynomials — 22

2.8.3 Hermite polynomials — 23

2.8.4 Chebyshev polynomials — 25

2.8.5 Gegenbauer polynomials — 28

2.9 Conclusion — 30

3 Homogenous polynomials and spherical harmonics — 31

3.1 Introduction — 31

3.2 Spherical Laplace operator — 32

3.3 Some direct computations on S^2 — 34

3.4 Homogenous polynomials — 37

3.5 Spherical harmonics — 40

3.6 Fourier transform of spherical harmonics — 44

3.7 Zonal functions — 47

3.8 Conclusion — 50

4 Review of special functions — 51

4.1 Introduction — 51

4.2 Classical special functions — 51

4.2.1 Euler's Γ function — 51

4.2.2 Euler's beta function — 59

4.2.3	Theta function —	64
4.2.4	Riemann zeta function —	66
4.2.5	Hypergeometric function —	72
4.2.6	Legendre function —	77
4.2.7	Bessel function —	81
4.2.8	Hankel function —	89
4.2.9	Mathieu function —	94
4.2.10	Airy function —	97
4.3	Hankel–Bessel transform —	103
5	Spheroidal-type wavelets —	105
5.1	Introduction —	105
5.2	Wavelets on the real line —	105
5.3	Chebyshev wavelets —	109
5.4	Gegenbauer wavelets —	110
5.5	Hermite wavelets —	111
5.6	Laguerre wavelets —	112
5.7	Bessel wavelets —	112
5.8	Cauchy wavelets —	115
5.9	Spherical wavelets —	116
6	Some applications —	125
6.1	Introduction —	125
6.2	Wavelets for numerical solutions of PDEs —	125
6.3	Wavelets for integrodifferential equations —	129
6.4	Wavelets in image and signal processing —	131
6.5	Wavelets for time-series processing —	134
	Bibliography —	137