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Fractional Dynamics

This volume is devoted to recent developments in the theory of fractional calculus and its applications. Particular attention is paid to the applicability of this currently popular research field in various branches of pure and applied mathematics. In particular, we have focused on the more recent results in mathematical physics, engineering applications, theoretical and applied physics as quantum mechanics, signal analysis, and in those relevant research fields where nonlinear dynamics occurs and several tools of nonlinear analysis are required. Dynamical processes and dynamical systems of fractional order attract researchers from many areas of sciences and technologies, ranging from mathematics and physics to computer science.

Fractional calculus is an intriguing mathematical theory which dates back to the very foundation of differential calculus when Leibniz (1695) raised some concern about derivatives of fractional order. However, it was Liouville (1832), and nearly contemporarily to him, Riemann (1847) and Fourier (1822), who collected into an independent theory all the basic knowledge about fractional order derivatives and fractional order integrals. Since then and especially in the last century, there has been an increasing interest on fractional calculus and its application in many different fields in Mathematics, Physics, Engineering, Bio-Science, Computer Science, Economics and so on.

In order to explain why fractional calculus is attracting more and more scholars with different scientific and cultural background is due to the following reasons:

1. Although it is almost completely clear what does it mean by fractional derivative and fractional integral and which basic axioms should be taken to define these operators, fractional derivative as operator doesn't fulfill all properties of the ordinary derivative. For this reason, in order to circumvent this problem many additional axioms were assumed, so that, as a consequence, there are different definitions of the same operator, which give rise to different, sometimes competing, fractional derivatives. All definitions are mathematically respectful, but the main problem is that up to now there is not a unique definition of fractional derivative.
2. Since their origins, fractional derivatives were linked to the fractional calculus. There is always a debate on local (for the derivative) and non-local (for derivative) approach to fractional-order operator calculus. For the nonlocal fractional calcu-

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lus, some boundary conditions are needed and this implies global approach to such problems.
3. The fundamental operators of fractional calculus are based on special functions which, in turn, are strongly depending on efficient computational methods and numerical algorithms. Therefore, the modern advancements on computational and numerical methods have pushed the recent development of fractional calculus and the massive interests of the scientific community.

However, as a consequence of these main concerns, we have reached a situation where there are still some unsolved theoretical problems in fractional calculus whereas there are many different fractional models in applications. In fact, any physical, engineering, and mathematical problems that are being investigated by a fractional calculus might have a variety of fractional models, depending on the local-nonlocal approach and/or on the chosen fractional derivative among the many alternative definitions. Moreover, the many recently developed numerical methods add confusion to the multi-facet fractional models. In fact, different choices of orthogonal functions and bases would lead to different fractional models.

Very recently there have also been several successful attempts to extend local fractional calculus to non differentiable, irregular sets like fractals. The possibility to extend the fractional calculus to self-similar unsmooth objects is opening new frontiers in science, such as, for example, in signal analysis, data sets from complex phenomena, image analysis, nuclear medicine, and so on. Nonlinear analysis of data, collected by modern devices, offer still unsolved analytical problems related not only to complex physics and abstract mathematical theories, but also to nonlinear science. From analytical point of view, these kinds of problems are often leading us to deal with the concepts of scales, fractals, fractional operators, and so on.

In this volume, we have collected some of the main contributions to the fields of local and non-local fractional calculus, aiming to explore the most recent advancement in several branches of science and technology.

This volume consists of a total of 22 chapters which are outlined below.

Chapter 1 (by C. Cattani) deals with the fractional calculus applied to Shannon wavelet theory. This chapter introduces a novel definition of local fractional derivative which makes it possible to compute the derivative of any finite energy function, that is, of a localized function which can be represented by wavelet series.

In Chapter 2, V. E. Tarasov studies three-dimensional discrete dynamical systems with long-range properties and gives some applications for different types of dynamical systems.

In Chapter 3, António M. Lopes and J.A. Tenreiro Machado investigate the statistical distributions of earthquakes in Southern California over the time period from the year 1934 up to the year 2013. The reported results reveal relationships and temporal patterns hidden in the data. Also, the investigated methodology and findings can contribute to a comprehensive explanation of these phenomena and to recognize precursory events for earthquake prediction.

Chapter 4 (by Akira Asada) deals with an integral transform arising from fractional calculus, which is used for investigating the solution of some differential equations.

In Chapter 5, Jordan Hristov fully shows how to obtain approximate solutions to time-fractional models by integral balance approach. This long chapter completely describes the theory of integral balance approach for finding approximate solution of fractional differential equations.

Chapter 6 (by Bashir Ahmad, Ahmed Alsaedi and Hana Al-Hutami) presents a study of sequential fractional q-integro-difference equations with perturbed anti-periodic boundary conditions.

In particular, existence results for the given problem are established by applying Krasnoselskii's fixed point theorem, Leray-Schauder nonlinear alternative for single valued maps and Banach's contraction mapping principle. Moreover, correction terms arising due to the perturbation in the anti-periodic boundary data are highlighted.

Several chapters are devoted to surface processes of sorption and reaction processes in electrolysis: In fact, systems For example, in Chapter 7, by M. K. Lenzi, G. Gonçalves, D. P. Leitoles and E. K. Lenzi, give a new approach to surface processes of sorption and reactions processes. The models address a half space with dynamics of the substances governed by a fractional diffusion equation.

Jocelyn Sabatier in Chapter 8 shows how fractional-order models can be used to capture the dynamical behavior of electrochemical devices such as batteries or super-capacitors, and are also used to build state of charge or state of health estimators. Moreover, in Chapter 9, M. K. Lenzi, G. Gonçalves, F. R. G. B. Silva, R. S. Zola, H. V. Ribeiro, R. Rossato and E. K. Lenzi present a new approach to surface processes of sorption and reactions processes in electrolytic systems. In fact, the electric impedances in electrolytic systems are an old area where fractional derivatives of half-time order are widely applicable. The models address a half-space with dynamics of the substances governed by a fractional diffusion equation to model non-linear phenomena. The processes on the surface are assumed to be of the first order, that is, the kinetic equations are linear and with memory effects which may be connected to an unusual relaxation.

Applications of fractional calculus to epidemiology are given in Chapter 10 by Abdon Atangana by considering the fractional dynamical system which is a generalization of a SIR model in epidemiology. Then, by using some theorems given by the authors, together with the Sumudu transform and the variation iteration model, some significant numerical results for a concrete problem are discussed.

In the category of fractional differential equations there are several chapters. A new numerical method based on Laguerre polynomials and operational matrix for the approximate solution of fractional differential equations on a semi-infinite interval is given in Chapter 11 by A.H. Bhrawy, T.M. Taha, M.A. Abdelkawy and R.M. Hafez.

Chapter 12 (by Hong-Yan Liu and Ji-Huan He) describes some thermal conduction on the wall of a typical Mongolian construction. This problem is summarized by a set of fractional differential equations which are analytically solved and many interesting results are presented. They also find an optimal thickness of the fractal hierarchy of the felt cover.

In Chapter 13, Mohamad Rafi, Segi Rahmat, Dumitru Baleanu and Xiao-Jun Yang present and employ the Cantor-type spherical-coordinate method to solve a problem involving a class of local fractional differential operators on Cantor sets. This method converts differential equations on Cantor sets from Cantorian-coordinate system to Cantor-type spherical-coordinate systems. The capability of the proposed method is confirmed through several examples for some important physical problems namely, the damped wave, Helmholtz and heat conduction equations.

In Chapter 14 on the approximate methods for local fractional differential equations, H. M. Srivastava, J. A. Tenreiro Machado and Xiao-Jun Yang present some analytical techniques for solving a class of local fractional differential equations. They also focus upon the local fractional variational iteration, Adomian decomposition and series expansion methods. In particular, some classical problems such as the Boussinesq and wave equations are considered in the local fractional calculus sense and solved on Cantor sets.

Numerical Solutions for ODEs with local fractional derivative are investigated in Chapter 15 by Xiao-Jun Yang, Dumitru Baleanu and J. A. Tenreiro Machado. In particular, these authors give an efficient numerical algorithm for solving local fractional ODEs by applying the extended differential transform method which employs the generalized local fractional Taylor theorems.

A particular equation arising in fractal heat transfer is studied in Chapter 16 in which Xiao-Jun Yang, Carlo Cattani and Gong-Nan Xie address the homogeneous and non-homogeneous heat conduction, the Poisson and the Laplace equations of fractal heat transfer. The 2D partial differential equations of fractal heat transfer in Cantor-type circle coordinate system are also discussed by these authors.

Fractional partial diferential equations are discussed in Chapter 17 by Hossein Jafari, Hassan Kamil Jassim and Syed Tauseef Mohyud-Din by using the local fractional Laplace decomposition method in order to find the solution of some local fractional differential equations.

Chapter 18 (by Alireza Khalili Golmankhaneh and Dumitru Baleanu) discusses some fundamental principles of fractional calculus and presents a framework for fractional calculus and a calculus on fractals. These authors also present some relations and formulas for calculus on fractal subset of the real-line, F-limit and F- continuity and Taylor series on fractal sets. Some applications on F-calculus in physics and dynamics have also been given.

The solutions of nonlinear fractional differential systems through an implementation of the variational iteration method are given in Chapter 19 by Haci Mehmet Baskonus, Fethi Bin Muhammad Belgacem and Hasan Bulut.

In Chapter 20 (by Tofik Mekkaoui, Zakia Hammouch, Fethi B. M. Belgacem and Ahmed El-Abbassi), the chaotic dynamics of an electrical circuit is analyzed and the corresponding fractional order nonlinear dynamical system is studied.

Chapter 21 (by E. Guariglia) deals with the fractional analysis of the Riemann zeta function by giving some fractional series expansions of the Riemann zeta function and by studying their convergence properties.

A treatment of generalized fractional differential equations: Sumudu transform series expansion solutions and their applications, is presented in Chapter 22 by Fethi Bin Muhammad Belgacem, Vartika Gulati, Pranay Goswami and Abdullah Aljoujiee.

All of the above 22 chapters deal essentially with current problems in the topics within the aim and scope of the present volume. However, they are not an exhaustive representation of the area of fractional calculus and its widespread applications, but they are meant to represent some of the more modern and fascinating fields of research on the subject of this volume. In all of these 22 chapters, the authors have focused on the main aspects of the theory and, although they have proposed some solutions and models, most problems remain still open, thus giving the opportunity to readers for further research and discussions in this field. Thanks to the authors for their excellent contributions addressing all of the key aspects raised in this volume.

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