ED ZALTA'S VERSION OF NEO-LOGICISM – A FRIENDLY LETTER OF COMPLAINT

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Subject: Ed Zalta's version of Neo-Logicism – a friendly letter of complaint

Dear Ed,

It was great to meet you again, especially in such a beautiful setting as Kirchberg, Austria. We had a great time at the conference and in particular at the neo-logicism workshop, and we thoroughly enjoyed our many very stimulating and engaging discussions on just this topic. We are grateful for the patience you showed in explaining and clarifying your views. We are, however, not yet (or rather still not) convinced of your version of neo-logicism, and we would like to take this opportunity to outline our main points of disagreement regarding the philosophical foundations of mathematics.

Rather than indulging in smaller details our aim here is to outline three rather general areas of concern. First, we will discuss broader epistemological issues and your explanation of mathematical knowledge. Second, we will draw attention to some "unusual" consequences of your theo-

ry. Last, but not least, we will take issue with your claim that your account of mathematics follows in the footsteps of Frege's logicism and is thus deservedly called a 'neo-logicist' or even a 'logicist' account of mathematics.

In a recent paper, co-written with Bernard Linsky, you write: "Our version of neologicism constitutes an epistemic foundation, in the sense that it shows how we can have knowledge of mathematical claims."[1] The solution that you offer, is not one that explains how we can have knowledge of *some specific* mathematical theory, such as Peano arithmetic (PA) or even Zermelo-Fraenkel set theory (ZF), rather you offer a very liberal account of which mathematical theory we can properly know. It is so liberal that it accounts for all written and even all yet to be written mathematical theories; including mathematical theories that seem to be mutually exclusive. You write: "This allows us to have knowledge of all the axioms and theorems of mathematical theories, including for example the truths of ZF and those of alternatives to ZF such as Aczel's non-wellfounded set theory. The fact that these latter two theories are inconsistent with one another doesn't mean that we can't have knowledge of their claims." (Linsky and Zalta 2006, p.41). You advertise the fact that this account is all-encompassing to this extent as a unique "feature" rather than a flaw.

Yet, it comes at a price – a price that we are not willing to pay. We fail to see how your conception can account for *genuine* categorical mathematical knowledge. Your epistemic foundation of any mathematical theory is based on a two-step approach. First your show how we can re-interpret any mathematical theory within your theory – third-order Object Theory. Then, in a second step, you show how we can have knowledge of just those re-interpreted statements. That is, your epistemic account only applies to the imported statements of Object Theory. Our claim is that knowledge of these re-interpreted statements is not categorical. To see this more clearly, let us first outline how it is so much as possible to have knowledge of "inconsistent" theories and then explain how you can account for our knowledge of *any* mathematical theory. This will help to see why the resulting knowledge is non-categorical.

The reason why we can have knowledge of "inconsistent" theories is that theories imported in the manner you envisage concern different types of objects and so have their own theory-dependent domain. Hence, they are strictly speaking *not* inconsistent since they concern a different subject matter.

[1] See p.61 of Linsky, B., and Zalta, E.N. "What is Neologicism", *The Bulletin of Symbolic Logic* 12/1 (2006): pp.60–99.

The reason why we can have knowledge of any mathematical theory is because each axiom, imported in the appropriate way, will be true since it is reformulated into what we call a *hedged* statement that is *trivially* true. So, for example, Peano's axiom that zero is a number will result, once imported into third-order Object Theory, in a hedged statement of the form: 'In Peano Arithmetic, $N_{PA}0_{PA}$'.[2] In the same way, any other theory (including merely possible or yet to be written theories) can be, in principle, reinterpreted and given a *true* reading in your theory. Since the account is truly liberal we can even explain our knowledge of some inconsistent theory in this way.

Our claim is that this is only possible by regarding our mathematical knowledge as hypothetical. Hedged statements, statements that are prefaced by the 'in theory T'-operator are clearly not categorical statements and so our resulting mathematical knowledge is non-categorical. Now, even if you can drop the *explicit* occurrence of the 'in theory T'-operator, the content of the imported statements is still hedged, albeit now in a clandestine way. The imported statement is about properties and objects that are always theory-bound – and in this sense our knowledge of these statements is non-categorical.[3]

Note also that our mathematical knowledge is easy to come by; and we think: a little too easy. On your account there is, as we have seen, no epistemically relevant difference between mathematical knowledge of a consistent theory and "mathematical knowledge" of an inconsistent theory. Also, if we follow you, the mark of mathematical knowledge, in contrast to, say, fictional knowledge, is not a matter of substance. Knowledge is mathematical if the underlying theory (story) is generally regarded mathematical, it is fictional, if, the underlying theory (story) is generally regarded to be that of a fiction.[4]

All this, of course, is not to say that your explanation of our knowledge of mathematics is internally flawed, but rather that the resulting conception of non-categorical mathematical knowledge is not the right type of explanation, at least to our taste.

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[2] Where 'N_{PA}' and '0_{PA}' are the obvious arithmetical constants, indexed to the theory in question.

[3] This point is further developed and discussed in Ebert, P.A. & Rossberg, M. "What Neo-Logicism Could Not Be" (forthcoming).

[4] See p.260 of Zalta, E.N. "Neo-Logicism? An Ontological Reduction of Mathematics to Metaphysics", *Erkenntnis*, 53/1–2 (2000): pp.219–265. Our criticism is further developed in Ebert, P.A. and Rossberg, M. "What is the Purpose of Neo-Logicism?" *Traveaux de Logique* 18 (2007): pp.33–61.

Leaving aside these epistemic issues, let us briefly note what we regard as unhappy metaphysical consequences of your neo-logicist account of mathematics. We noted earlier that ZF, re-interpreted in Object Theory, has its private domain of sets, while Aczel's non-wellfounded set theory also has its own distinct domain of sets. The same applies to other theories of sets. As a result, none of the candidate set theories is, or can be, *the* theory of all set-like objects, but there are many distinct domains of sets. From this it follows for example that there are many, presumably infinitely many, *empty sets*. One for each mathematical theory that states the existence of an empty set. This sounds very unappealing to us.

A further interesting consequence for your conception is that there are strictly speaking no genuine disputes about the truth of a mathematical theory. Since every axiom is, once re-interpreted in Object Theory, (trivially) a logical consequence of its respective theory, there can only be mathematical disagreement whether a certain statement does in fact follow from a set of axioms, and thus merely the question who of the disagreeing parties made a mistake in their proof. There is simply no point in arguing about the possible falsity or the truth of a given theory of some mathematical subject matter. Yet, classical mathematicians and constructivists do not simply talk past each other, it seems to us; neither are disputes about the existence of large cardinals merely verbal. Well, some (at least one person) might consider this result a welcome *feature*, while some others (at least two others) regard it as a *flaw*.

Lastly, in your presentation in Kirchberg you labeled your conception as a true version of logicism and not just neo-logicism. We thought we just note here briefly our reservation about calling your view a neo-logicist, or even logicist, conception. First to note is that your very liberal view of acceptable mathematical theories is surely not one that Frege, the founder of logicism, would have welcomed. Yet, more importantly, Frege's aims in reducing mathematics to logic were at least twofold: first, his mathematical motive was to prove theorems within logic and so reduce the enterprise of mathematics to logic. This is not, however, what happens in your theory since you take what mathematicians have proved and only then import it into Object Theory. Second, Frege also had an epistemological motive. Namely, by reducing mathematics to logic he hoped to establish the philosophical status of arithmetical knowledge, showing, ultimately, how it flows from pure logic. Again, we don't think you offer an explanation of our mathematical knowledge as based upon our logical knowledge. Hence we are hesitant in regarding your view a

logicist conception of mathematics if, by this, you mean to follow in the footsteps of Frege's logicist project.

Yours truly,

Philip Ebert Marcus Rossberg