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On the Transitivity of the Parthood Relations

1. The Problem: Are Parthood Relations Always Transitive?

If x is a spatial part of y, and y is a spatial part of z, then necessarily x is a spatial part of z. If x is a temporal part of y, and y is a temporal part of z, then necessarily x is a temporal part of z. Both spatial and temporal parthood are transitive relations. But what about parthood in general? Are the transitivities of spatial and temporal parthood merely special cases of the transitivity of parthood in general? Among philosophers interested in axiomatic mereology, there is an almost complete consensus to the effect that the answer is: 'Yes, all parthood relations are transitive'. But some critical voices have been heard, and I think they are worth re-considering. Below, I have listed a dozen of examples of cases where it has been seen as being problematic whether the conjunction of 'x < y' and 'y < z' really implies 'x < z'.

1. A handle, x, can be part of a door, y, and a door can be part of a house, z, but yet the handle need not be (is not) a part of the house. That is, 'x < y' and 'y < z' but '¬(x < z)'. (Of course, 'part' cannot here and elsewhere in the list be synonymous with 'spatial part'.)
2. A platoon is part of a company, and a company is part of a battalion, but yet a platoon is not part of a battalion.
3. A cell's nucleus is part of a cell, and a cell is part of an organ, but yet the nucleus is not part of an organ.
4. Heart cells are parts of the heart, and the heart is part of the circulatory system, but yet the cells are not parts of the circulatory system.
5. Person P is part (member) of the football club FC, and FC is part (member) of the National Association of Football Clubs, NAFC, but yet P is not a part (member) of NAFC.

6. Simpson's finger is part of Simpson, and Simpson is part of the Philosophy Department, but yet Simpson's finger is not part of the Philosophy Department.
7. Hydrogen is part of water, and water is part of our cooling system, but yet hydrogen is not part of our cooling system.
8. Cellulose is part of trees, and trees are parts of forests, but yet cellulose is not part of forests.
9. A handle is part of a spoon, and a spoon is part of eating soup, but yet a handle is not part of eating soup.
10. This shard was part of a plate, and the plate was part of a dinner service, but yet the shard was not part of the dinner service.
11. This tree is part of the Black forest, and the Black forest is part of Germany, but yet this tree is not part of Germany.
12. These grains of sand are part of the beach, and the beach is part of the island, but yet these grains of sand are not part of the island.[1]

If one finds at least one of these examples convincing, then one has to face the problem I have pointed to, will discuss, and (I think) solve: Are parthood relations always transitive? In the first two sections, two familiar proposed solutions will be presented and rejected – though not without admitting that both of them contain quite a kernel of truth. In ensuing sections, I will put forward my own solution. I will claim that there are both intransitive and non-transitive parthood predicates, but that, when examined more closely, these predicates are at least as complex as so-called relative products of other binary relational predicates or as ternary predicates. Only truly *binary* parthood relations are necessarily transitive.

A ternary predicate is a predicate that has the form Rxyz, but what is a relative product? Complying with Patrick Suppes, I will define it as follows: "If R and S are binary relations, then by the *relative product* of R and S (in symbols R/S) we mean the relation which holds between x and y

[1] The first example comes originally from D. A. Cruse, "On the Transitivity of the Part-Whole Relation," *Journal of Linguistics* 15 (1979), 29-38, and the second and third have their origin in N. Rescher, "Axioms for the Part Relation," *Philosophical Studies* 6 (1955), 8-11. Number four and five are variations of well known themes, and the rest are taken from Morton E. Winston, Roger Chaffin, and Douglas Herrmann, "A Taxonomy of Part-Whole Relations," *Cognitive Science* 11 (1987), 417-444.

if and only if there exists a z such that R holds between x and z, and S holds between z and y. Symbolically, xR/Sy ↔ (∃z)(xRz & zSy)."[2] The formula for relative products contains, just like the form for ternary predicates, three individual variables.

2. Proposed Solutions: (A) Specified parthood need not be transitive

The first three examples in my list have been discussed both by Peter Simons' in his classic book *Parts*, and by Roberto Casati and Achille C. Varzi in their *Parts and Places*.[3] Each claims that these examples trade on an ambiguity between, on the one hand, a basic and broad sense of 'part' that denotes a relation that is necessarily transitive and is the object of mereology and, on the other hand, a narrow sense of 'part' (ϕ-part) that is non-transitive and is not the object of mereology. Casati and Varzi write:

> One can argue that a handle is a functional part of a door, the door is a functional part of the house, and yet the handle is not a functional part of the house. But this involves a departure from the broader notion of parthood that mereology is meant to capture. To put it differently, if the general intended interpretation of 'part' is narrowed by additional conditions (e.g., by requiring that parts make a direct contribution to the functioning of the whole), then obviously transitivity may fail. In general, if x is a ϕ-part of y and y is a ϕ-part of z, it may well be true that x is not a ϕ-part of z: the predicate modifier 'ϕ' may not distribute over parthood. But that shows the non-transitivity of 'ϕ-part' (e.g., of *direct* part, or *functional* part), not of 'part'. And within a sufficiently general framework this can easily be expressed with the help of explicit predicate modifiers.[4]

According to this view, there are ϕs which are such that the conjunction of 'x is a ϕ-part of y' and 'y is a ϕ-part of z' does not imply 'x is a ϕ-part of z'; the conjunction may even imply 'x is *not* a ϕ-part of z'.

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[2] Suppes, *Introduction to Logic*, Van Nostrand: Toronto 1957, p. 226. I will in what follows use Suppes' symbol '/' for this kind of relative product.

[3] See Simons, *Parts. A study in Ontology*, Clarendon: Oxford 1987, pp. 107-108, and Casati and Varzi, *Parts and Places. The Structures of Spatial Representation*, Bradford: London 1999, pp. 33-34.

[4] Casati and Varzi, ibid., p. 34.

In the quotation, Casati and Varzi provide two explicit examples of ϕ-parts, ‘direct part’ and ‘functional part’, but each is unclear. First, ‘functional part’ can mean both *direct* and *indirect* functional part, but the context makes it clear that what is intended is ‘direct functional part’. The predicate ‘indirect functional part’ can lay a much stronger claim on being transitive. Second, ‘direct part’ is an incomplete expression; a direct part has to be direct in a certain respect. Therefore, I will reformulate the first five examples as follows:

1. A handle can be a *direct functional* part of a door, and the door can be a *direct functional* part of a house, but yet the handle need not be (is not) a *direct functional* part of the house.
2. A platoon is a *direct organizational* part of a company, and a company is a *direct organizational* part of a battalion, but yet a platoon is not a *direct organizational* part of a battalion.
3. A cell’s nucleus is a *direct functional* part of a cell, and a cell is a *direct functional* part of an organ, but yet the nucleus is not a *direct functional* part of an organ.
4. Heart cells are *direct functional* parts of the heart,[5] and the heart is a *direct functional* part of the circulatory system, but yet the heart cells are not *direct functional* parts of the circulatory system.
5. I am a *direct organizational* part of the organization X, and X is a *direct organizational* part of the organization Y, but yet I am not a *direct organizational* part of Y.

The instantiations of ‘ϕ-part’ in the above are intransitive, but since for some values of ϕ such as ‘*spatial* part’ and ‘*temporal* part’, it is transitive, too, the general predicate ‘ϕ-part’ is neither transitive nor intransitive but rather non-transitive.[6]

Now what is wrong with this account? The answer is that it gives rise to an extremely curious subsumption relation between the predicates ‘<’ and

[5] In fact, I consider this to be false. There are intermediate functional unities; but the example will fulfil its argumentative function nonetheless.

[6] There seems to be no reason to distinguish between direct and indirect spatial (or temporal) parts. Probably, this fact mirrors the fact that spatial (and temporal) parthood is transitive.

$<^{\phi}$ (‘ϕ-part’) and cannot explain why some specific ϕ-parts are transitive and some are intransitive. According to Simons, Casati, and Varzi, while it is in general true that:

‘x < y’ and ‘y < z’ necessarily implies ‘x < z’,

for some ϕ-parts it is true that:

‘x $<^{\phi}$ y’ and ‘y $<^{\phi}$ z’ and ‘¬(x $<^{\phi}$ z)’.

All the ϕs in question are said to specify (Simons) or modify (Casati and Varzi) a “broader notion of parthood.” Therefore, the relational predicate ‘$<^{\phi}$’ ought to be to the relational predicate ‘<’ what property predicates such as ‘light red’ and ‘quickly running’ are to the more general property predicates ‘red’ and ‘running’, respectively.[7] What is true of ‘red’ is necessarily also true of the ‘light red’ which it subsumes, what is true of ‘running’ is necessarily also true of ‘running quickly’, and what is true of ‘x < y’ ought necessarily be true of ‘x $<^{\phi}$ y’.[8] Since ‘x < y’ is transitive, ‘x $<^{\phi}$ y’ ought to be so as well. But according to the Simons-Casati-Varzi analysis, the latter predicate is non-transitive. I do not think one can make sense of such an odd subsumption relation, and nor have the philosophers mentioned tried to. They seem simply not to have noted the issue that I have raised. However, as will become clear later on, they are quite right in claiming that ‘x ϕ-part y’ is non-transitive, but they give the very false impression that ‘x ϕ-part y’ always denotes a binary relation.

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[7] If, instead, Simons, Casati, and Varzi had intended ‘$<^{\phi}$’ to be to ‘<’ what ‘stuffed animal’ is to ‘animal’, then they ought not to have spoken of “specification” or “modification.” The predicate ‘stuffed animal’ is neither a specification nor a modification of ‘animal’.

[8] This view follows from the nature of subsumption. It is, by the way, an integral part of so-called description logic in computer science: “when a concept is more specific than some other concept, it inherits the properties of the more general one.” The quotation is from F. Baader, *et al*. (eds.), *The Description Logic Handbook*, Cambridge University Press: Cambridge 2003, p. 5.

3. Proposed Solutions: (B) Seeming parthood non-transitivities are due to equivocations

In their paper "A Taxonomy of Part-Whole Relations," Winston, Chaffin, and Herrmann claim that the "apparent failures of transitivity [of parthood] occur when different types of meronymy occur in the two premises of a syllogism."[9] They claim that all seeming violations of the mereological inference from 'x < y' and 'y < z' to 'x < z' are due to equivocations between six different kinds of meronymic relations (in the terminology here introduced: six kinds of ϕ-parts).[10] According to these authors, to be a part can mean six different things:

(i) to be a component of an integral object;
(ii) to be a member of a collection;
(iii) to be a portion of a mass;
(iv) to be a stuff of an object;
(v) to be a feature of an activity;
(vi) to be a place within an area.

When the conjunction of 'x < y' and 'y < z' does not seem to imply 'y < z', this is due, they say, to the fact that the two premises really have the form 'x ϕ_1-part y' and 'y ϕ_2-part z', respectively.

In my opinion, the authors give their second sense of 'part', "being a member of a collection," too wide a sense. Contrary to their claim,[11] the sense in which a tree is part of a forest (collection) is generically distinct from the sense in which a juror is part of a jury (social unit). A jury is not a collection. I will therefore add a seventh sense of 'to be a part':

(vii) to be a direct organizational part (or: to be a subunit of a group or an organization).

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[9] Winston, Chaffin, and Herrmann, "A Taxonomy of Part-Whole Relations," *Cognitive Science* 11 (1987), p. 438.

[10] They are talking about equivocations between meronymic and non-meronymic relations, too. But I will leave that out of account.

[11] Winston, Chaffin, and Herrmann, "A Taxonomy of Part-Whole Relations," *Cognitive Science* 11 (1987), p. 423.

The term ‘organization’ should here be understood as relating to all social units (human groups and collectivities) that are regulated by formal or informal rules, and the term ‘subunit’ should be understood in such a broad sense that it subsumes both what is normally termed ‘member of an organization’ and ‘part of an organization’, respectively. With this amendment, which will be explained in more detail in section six, the essence of the examples 6 to 12 can be distilled in the following true statements:

6. The fact that Simpson’s finger is a *component-part-of-the-integral-object* Simpson and that Simpson is a *direct-organizational-part-of-the-organization* the Philosophy Department, does not imply that Simpson’s finger is in any of these senses part of the Philosophy Department.
7. The fact that hydrogen is a *stuff-part-of-object* water and that water is a *component-part-of-the-integral-object* our cooling system, does not imply that hydrogen is in any of these senses part of our cooling system.
8. The fact that cellulose is a *stuff-part-of-object* trees and that trees are *member-parts-of-the-collections* forests, does not imply that cellulose is in any of these senses part of forests.
9. The fact that a handle is a *component-part-of-the-integral-object* spoon and a spoon is a *feature-part-of-the-activity* eating soup, does not imply that a handle is in any of these senses part of eating soup.
10. The fact that this shard was a *portion-part-of-the-mass* the plate and that the plate was a *component-part-of-the-collection* a dinner service, does not imply that the shard was in any of these senses part of the dinner service.
11. The fact that this tree is a *member-part-of-the-collection* the Black forest and that the Black forest is a *place-part-of-the-area* Germany, does not imply that this tree is in any of these senses part of Germany.
12. The fact that these grains of sand are *portion-parts-of-the-mass* the beach and that the beach is *place-part-of-the-area* the island, does not imply that these grains of sand are in any of these senses parts of the island.

More generally: The conjunction of 'x ϕ_1-part y' and 'y ϕ_2-part z' implies neither 'x ϕ_1-part z' nor 'x ϕ_2-part z'. However, and as is not explicitly noted by the authors, the very same conjunction does imply 'x < z' if '<' is a determinable that subsumes 'ϕ_1-part' and 'ϕ_2-part'.[12] For instance, the fact that Simpson's finger is a *component-part-of-the-integral-object* Simpson and that Simpson is a *direct-organizational-part-of-the-organization* the Philosophy Department, does really imply that Simpson's finger is, in the *determinable* sense of 'part', part of the Philosophy Department. Another such example: If 'x is a spatial part of y' and 'y is a temporal part of z', then necessarily 'x is a part of z'.

So far so good. In all probability, the equivocations spotted have sometimes fooled some people. But Winston *et al*. also claim that "meronymy is transitive when the same kind of meronymic relation occurs in both premises of a syllogism."[13] In other words, they claim that the conjunction of 'x ϕ_1-part y' and 'y ϕ_1-part z' necessarily implies 'x ϕ_1-part z'. This view contradicts not only my own view but also that of Simons, Casati, and Varzi. If Winston *et al*. were right, then the term 'direct functional part' would be used in two different senses in examples one, three, and four above. Similarly, 'direct organizational part' would have to mean different things in examples two and five. This seems not to be the case.

The concept of "component-part," as introduced by Winston *et al*., suffers from the same ambiguity which I have pointed out in relation to 'functional part'. It can mean either direct component-part or indirect component-part. Here, it ought to mean direct component-part. Examples one to five can now be rewritten as follows:

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[12] One may also, as A. Artale, E. Franconi, N. Guarino, and L. Pazzi do, say that "the WCH approach seems to exclude the existence of a single, very general *part-of* relation assumed to be transitive;" see p. 350 of their paper "Part-whole relations in object-centered systems: An overview," *Data & Knowledge Engineering* 20 (1996), 347-383.

[13] Winston, Chaffin, and Herrmann, "A Taxonomy of Part-Whole Relations," *Cognitive Science* 11 (1987), p. 438.

1. A handle can be a *direct-component-part-of-the-integral-object* a door, and the door can be a *direct-component-part-of-the-integral-object* a house, but yet the handle need not be (is not) a *direct-component-part-of-the-integral-object* a house.
2. A platoon is a *direct-organizational-part-of-the-organization* a company, and a company is a *direct-organizational-part-of-the-organization* a battalion, but yet a platoon is not a *direct-organizational-part-of-the-organization* a battalion.
3. A nucleus is a *direct-component-part-of-the-integral-object* a cell, and a cell is a *direct-component-part-of-the-integral-object* an organ, but yet the nucleus is not a *direct-component-part-of-the-integral-object* an organ.
4. The heart cells are *direct-component-parts-of-the-integral-object* the heart, and the heart is a *direct-component-part-of-the-integral-object* the circulatory system, but yet the heart cells are not *direct-component-parts-of-the-integral-object* the circulatory system.
5. I am a *direct-organizational-part-of-the-organization* X, and X is a *direct-organizational-part-of-the-organization* Y, but yet I am not a *direct-organizational-part-of-the-organization* Y.

In this list, the non-transitivity cannot be due to different senses of ‘part’. Winston *et al*. have greatly over-generalized their very useful insight. However, my strongest reasons for the view that ‘ϕ-part’ need not always denote a binary transitive relation are presented in the next two sections.

4. The Solution: (C) Intransitive parthood predicates are not binary predicates

Let us now look at two new examples of ϕ-parts; one where the predicate in question is non-transitive (13), and one where it is intransitive (14):

13. x can be a large spatial part of y and y can be a large spatial part of z, but yet x need not necessarily be a large spatial part of z.

14. If the part x is a spatial 60%-part of y and y is a spatial 60%-part of z, then x cannot possibly be a spatial 60%-part of z (x is necessarily a spatial 36%-part of z).

Obviously, in these two examples the relational predicates (‘large spatial part of’ and ‘spatial 60%-part of’) have exactly the same sense in all their occurrences. Therefore contra Winston *et al*., there are surely some ϕ-part-predicates that are non-transitive and some that are intransitive. How to explain this fact without, like Simons, Casati, and Varzi, doing violence to the ordinary logic of subsumption? The answer, to be worked out and explained in this and the next two sections, is that, for many values of ϕ, ‘ϕ-part’ is not a binary relational predicate subsumable under ‘<’. Instead it is either a relative product of two binary relations ‘ϕ’ and ‘<’ (so that it ought to be written ‘ϕ/< ’) or it is an implicitly ternary relation (and so ought to be written ‘Rxyz’). In both cases, although in different ways, there are at least three relata involved; not just two, as in the parthood relation of mereology.[14] And both relative products and ternary relations may well be non-transitive or intransitive.

The predicate ‘is an aunt of’ is a relative product. If ‘a is the aunt of b’ (aAb), then necessarily there is a w such that ‘a is the sibling of w’ (aSw) and ‘w is the parent of b’ (wPb). We can write: ‘A = S/P’ as shorthand for:

$$xAy \leftrightarrow (\exists w)(xSw \,\&\, wPy).$$

Similarly, if ‘a is a large spatial part of b’, then necessarily there is at least one object of size comparison (Cw) such that ‘a is larger than w’ (aLw). The relational predicate ‘is a large spatial part of’ contains, apart from its reference to some comparison object(s), the relative product of the binary relations ‘L’ and ‘a is a spatial part of b’ ($a <^S b$), i.e., it should be symbolized ‘L/<’, not ‘$<^L$ ’ as if it (like the ‘$<^S$ ’ for spatial part) were subsumable under ‘<’. In other words:

$$x \text{ is a large spatial part of } y \leftrightarrow (\exists w)(Cw \,\&\, xLw \,\&\, (x <^S y)).$$

[14] The view that the predicates in the examples (13) and (14) might not denote binary relations was first suggested to me by Kevin Mulligan.

Necessarily, the predicate ‘is a large spatial part of’ involves three individual variables. The seeming monadic predicate ‘is spatially large’ does not, like the monadic predicate ‘is round’, denote only a monadic property. Shapes like roundness inhere in things, and, of course, so do sizes. But the predicate ‘is large’ does not just denote a size. It also denotes a relation between the thing to which it is primarily attributed and certain other, smaller things. This fact seldom creates a problem in everyday communication since the context implicitly affords us the necessary (but vaguely delimited) contrasting sizes. However, when discussing parthood in relation to mereology, it is important to make this implicit relationship explicit.

It should, though, be noted that the predicate ‘L/<’ is not a relative product in exactly the same sense as this concept is defined by Suppes, and according to which ‘is an aunt’ is a relative product of S and P in our example above. It is more complex. The explicit structure of ‘x L/< y’ contains three conjuncts whereas the explicit structure of ‘x S/P y’ contains only two. We have:

$$x\,S/Py \leftrightarrow (\exists w)(xSw \,\&\, wPy), \text{ and}$$

$$x\,L/< y \leftrightarrow (\exists w)(Cw \,\&\, xLw \,\&\, (x <^{S} y)), \text{ respectively.}$$

This difference does not make the term ‘relative product’ inapplicable to a case like L/<; but ‘qualified relative product’ would be more to the point. Both ‘xL/< y’ and ‘xS/Py’ share the feature that whereas only two relata are explicitly mentioned there is nonetheless a hidden and indefinite reference to a third relatum, w, which appears explicitly in the definiens. Clearly, mereological axioms for binary parthood cannot be applied to xL/< y.

I guess and hope that no further arguments are now needed to show that, just like the predicate ‘large spatial part of’, the predicate ‘spatial 60%-part of’ designates a relative product to which mereological axioms cannot be applied. In this case, it is even more obvious that there is an indefinite reference to one or several comparison objects. It is the specific numerical relationship mentioned in ‘spatial 60%-part of’ that makes this predicate

intransitive in contradistinction to the merely non-transitive predicate ‘large spatial part of’.

At the beginning of this section I claimed that the (seemingly two-place) predicate ‘large spatial part of’ is non-transitive. I have now claimed that the very same predicate is in fact a relative product and a kind of three-term relation. Are these claims consistent with each other? The answer is: Yes, they are, once we have isolated a natural definition of transitivity for relative products. The definitional statement

L/< is transitive if and only if necessarily:
if xL/< y & yL/< z
then xL/< z

can be explicated more fully as

L/< is transitive if and only if necessarily:
if [(∃w)(Cw & xLw & (x $<^S$ y)) & (∃v)(Cv & yLv & (y $<^S$ z))]
then (∃u)(u=v & Cu & xLu & (x $<^S$ y)).

If there are no restrictions on w and v, then the consequent need not be true. If, however, one introduces a constraint to the effect that w is larger than or equal to v, then the consequent becomes true. In short, for some values of the variables there is transitivity and for some others there is not. The general predicate and relative product ‘large spatial part of’ is non-transitive. Q.E.D.

5. The solution (C) applied to functional parthood

Let us next look at the seemingly binary predicate ‘is a direct functional part of’, or ‘is a *direct-component-part-of-the-integral-object*’. I regard these expressions as more or less synonymous. Consider, first, artefactual-functional parthood. What to be said, in light of section four, about the sentence: ‘This handle is a *direct functional* part of this door, and this door is a *direct functional* part of this house, but yet the handle is not a *direct functional* part of the house’?

If a handle is a functional part of a door, then the handle has to be a spatial part of the door, and the door has to be a functional unity. However, there is a third requirement as well. The handle has to be able to act on something else that is of relevance for its function in relation to the door;

and in order to have this ability it has to be in spatial contact with this other thing. Of course, this thing is the panel of the door. The function of the handle, in relation to the door, is to make it easier to move the panel. Leaving as an open question whether the handle is mono- or multifunctional, and at the same introducing variables, we can write:

- In the artefactual-functional unit (A) of a door (y),
- one function of the handle (x) is
- to make it easy to move (M) the panel (w).

Next, if a door is a functional part of a house, then the house has to be a functional unity, the door has to be a spatial part of the house, and the door has to be able to act on (and therefore be in contact with) something else that is of relevance for its function in relation to the house. Such a thing is the wall in which it is placed. The function of a door is to make it easy to have a part of a wall sometimes contain a hole and sometimes not.

- In the artefactual-functional unit (A) of a house (y),
- one function of the door (x) is
- to make it easy to open and close (M) a hole in the wall (w).

Something x is a functional part of something else y (xFy) if and only if, y is a functional unity or integral object of some kind (Ay), and there is a w such that x makes something happen (M) to w that is relevant for Ay. If, in this sentence, the clause ‘that is relevant for Ay’ is left out of account, the formal structure of the right hand side can be written ‘$(\exists w)(Ay \,\&\, xMw \,\&\, (x <^{S} y))$’. Since it is a relative product, it can be symbolized ‘$xM/< y$’. Formally, therefore, we get:

‘$xFy \rightarrow xM/< y$’ and ‘$xM/< y \leftrightarrow (\exists w)(Ay \,\&\, xMw \,\&\, (x <^{S} y))$’.

Note that some clause like ‘Ay’ is necessary in the formula. If it were absent, one could let the value of ‘x’ be the handle, the value of ‘w’ be the panel, and the value of ‘y’ be not the door, but our solar system, and so get the odd result that the handle has the relation M/< to the solar system.

When we claim that a handle (x) is a functional part of a door (y), we seem to be using a binary relational predicate. In fact, however, we are using a predicate that contains a relative product and that, therefore, involves at least three relata (x, y, and w). And the same kind of reasoning

applies to the door-to-house case, too. Since the mereological axioms for binary parthood cannot be applied to ‘xM/< y’, neither can they be applied to ‘xFy’. The sentences ‘The handle is a functional part of the door’ and ‘The door is a functional part of the house’ fall outside mereology as the theory of the binary parthood relation.

In spite of this result, however, we can still of course ask whether the three-term relative product ‘xM/< y’ is transitive or not. Using the definition put forward in section four we get:

> x M/< y is transitive if and only if necessarily:
> *if* [(∃w)(Ay & xMw & (x $<^S$ y)) & (∃v)(Av & yMv & (y $<^S$ z))]
> *then* (∃u)(u=v & Au & xMu & (x $<^S$ y)).

If, here, we let the value of x be the handle, that of w the panel, of y the door, of v the wall, and of z be the house, then it is easily seen that the only expression in the consequent whose truth might be questioned is ‘xMu’. This says “the handle makes it directly easy to open and close a hole in a wall,” and it is false. Why? Answer: since the handle is not directly connected to the wall it cannot directly act on it. Conclusion: the general relative product predicate ‘direct artefactual-functional parthood’ cannot be transitive.

What, then, about biological-functional parthood?[15] Examples three and four on our list can be brought out as follows:

[15] In the philosophy of biology, some authors have explicitly made claims like “relationships between phenomena at different levels will in general be taken to be nontransitive,” “while we may take the gene to be part of a cell, it is *not* part of the organism of which that cell is a part,” and “nontransitivity is not really a separately imposed constraint but an implication of the triadic system itself;” quotations from Stanley N. Salthe, *Evolving Hierarchical Systems*, Columbia University Press: New York 1985, p. 118.

(a)

- In the biological-functional unit (B) of a cell (y),
- one function of the cell nucleus (x) is
- to store information (M) about cellular proteins (w);

(b)

- In the biological-functional unit (B) of the heart (y),
- one function of the heart cells (x) is
- to make possible the contractions and expansions (M) of the heart tissue (w);

(c)

- In the biological-functional unit (B) the circulatory system (y),
- one function of the heart (x) is
- to pump (M) blood (w).

This means, that instead of the two-term relation ‘the nucleus is a direct functional part of the cell’, we have something that involves the three relata ‘nucleus-proteins-cell’; instead of the two term relation ‘the heart cells are direct functional parts of the heart’, we have something that involves the three relata ‘cell-tissue-heart’; and instead of the two-term relation ‘the heart is a direct functional part of the circulatory system’ we have something that involves the three relata ‘heart-blood-circulatory system’. Logically speaking, these biological-functional parthood predicates contain relative products in the same way as artefactual-functional parthood predicates do. Therefore, even biological-functional parthood predicates fall outside mereology. Formally, we now have as before (but with ‘By’ instead of ‘Ay’):

‘$xFy \rightarrow xM/< y$’ and ‘$xM/< y \leftrightarrow (\exists w)(By \,\&\, xMw \,\&\, (x <^{S} y))$’.

In order to investigate whether the relative product predicate ‘biological-functional parthood’ is transitive or not, we can proceed exactly as in the case of artefacts. If, in the definition of transitivity for relative products, we insert the values that (a) and (b) afford us, then the problematic consequent-sentence becomes: ‘Cell nuclei make possible the contractions and expansions of the heart tissue’. If, instead, we insert values from (b)

and (c), then the questionable sentence becomes: ‘Heart cells pump blood’. Both these sentences are false, and this being so, the expression ‘direct biological-functional part of’ cannot be a transitive predicate.

6. The solution (C) applied to organizational parthood

The two remaining examples, (2) and (5), describe parthood relations between social units: ‘platoon-company-battalion’ and ‘person(P)-organization(FC)-organization(NAFC)’, respectively. In everyday language, a platoon is part of a company, a company is part of a battalion, a person can be part of a club, and a club can be part of an association. In my terminology, all these four parthood cases contain a relation of direct organizational parthood. There are, though, differences. Whereas P can stop being a member of FC and still exist, and FC can leave NAFC without ceasing to exist, a platoon cannot leave its company (and a company cannot leave its battalion) without losing its identity.

In the sense that I am here using the term ‘organization’, there can be no organizational parthood relations without consciousness and language. But this might be possible with respect to functional parthood (see the concluding section). This is one reason for keeping these parthood relations separate. Another indication of their generic difference is the fact that, whereas x cannot be a functional part of y without also being a spatial part of y, x can very well be an organizational part of y without being a spatial part of y. Many organizations such as clubs, associations, platoons, companies, and battalions simply lack a definite spatial delimitation.

When there is an organization, there are both persons and rules. First, even though all the persons of an organization may be exchanged and nonetheless the organization remain the same, there must at any specific moment at which the organization exists be some existing persons that can perform functions related to the organization. Normally, such persons are members, but they need not necessarily be; some kinds of organizations can survive a total death of members. Second, if a unit of some kind is a direct subunit of an organization, then necessarily it is regulated by rules, be they formally stated or merely informally imposed. Mostly, such rules are constraining in certain respects and enabling in others. As a member of

FC, P has both rights and duties; and as a member of NAFC, FC, too, has both rights and duties. Even military units have both rights and duties in relation to their direct superordinated units. With respect to some organizations, such rules cannot only be changed, they can be completely exchanged for other rules without affecting the identity of the organization.

Every organization necessarily combines one concrete aspect (the persons involved) and one abstract aspect (the rules involved). Let us now look at the direct organizational parthood relations involved in 'P-FC-NAFC'.

It is beyond doubt that 'P is a member of FC' and 'FC is a member of NAFC' need not imply 'P is a member of NAFC'. Why? Both FC and NAFC have explicit rules for membership, and these rules can very well (but need not necessarily) be such that, although P is a member of FC, he cannot possibly become a member of NAFC. In the regulations of many clubs and associations, the second paragraph reads something like this:

"§2. A person is a member of X if he or she supports the purpose of X as stated in §1, and if this person pays the annual membership fee."

Let us assume that the membership rules of FC and NAFC contain such a paragraph. That is, we have:

"§2. A *person* is a member of FC if he or she supports the purpose of the club as stated in §1, and if this person regularly pays the membership fee," and

"§2. A *football club* is a member of NAFC if it supports the purpose of the association as stated in §1, and if the club fulfils its economic and representative duties as stated in §§ ..."

In these regulations it is explicitly stated what kind of entities are allowed as members, i.e., persons and football clubs, respectively; and since persons cannot be members of NAFC, P cannot possibly be a member and direct organizational part of this organization. And similar remarks can be made in relation to platoon-(member/part of)-company-(member/part of)-battalion. Because of this, 'direct organizational part' cannot be a transitive predicate.

Regulations like §2 above have two specific features. They are themselves a kind of part of the organization in question, and they connect the organization to its members. At first, it might be tempting to claim that 'x is a direct organizational part of y' contains a relative product, 'O/< ',

because if this predicate is applicable then this implies “There is a rule z such that x has an organization relation O to z (xOz) and z is part of y (z < y).” We would then have the structure: xO/< y ↔ (∃z)(xOz & (z < y)).

If this were true, I would have no qualms. To the contrary. I could then say: “Fine, direct organizational parthood has essentially the same formal structure as direct functional parthood.” However, I do not think that it is true. For in a relative product, it is taken for granted that the two connected binary relations are logically independent of each other. That is, in the case at hand, ‘xOz’ should be able to be true when both ‘z < y ’ and ‘xO/< y’ are false. What is denoted by ‘x’ should be able to have the relation O to z without thereby becoming part of the organization y. But this is impossible, since the rule z (§2) explicitly mentions both possible members and the organization y. The unit x cannot conform to z without being part of y. To be a direct organizational part is to be one relatum in a relation that is at least ternary and holds between members (x) and an organization (y) because of some membership rules (z); in symbols, Oxyz. Just like the ternary relation ‘x is more similar to y than to z’, the ternary relation ‘x is an organizational part of y by means of z’ cannot possibly be reduced to a conjunction or a combination of binary relations.

The expression ‘x is a direct organizational part of y’, which contains two individual variables, has to be regarded as shorthand for an expression that contains at least three such variables. As in the case of predicates for functional parthood, but in another way, even predicates for organizational parthood contain a hidden third relatum. And the conclusion is the same: binary mereology cannot be applied.

I have already informally explained why ‘direct organizational part’ cannot be a transitive predicate; but it may be worthwhile to take a more formal look at this truth, too. First, we need a definition of transitivity for ternary predicates. In my opinion, it has to take the form of two complementary definitions that I will call left-transitivity and right-transitivity, respectively:

‘Rxyz’ is left-transitive if and only if,
necessarily: *if* Rxyz & Ryzw *then* Rxzw;

‘Rxyz’ is right-transitive if and only if,
necessarily: *if* Rxyz & Rwxy *then* Rwyz.

The ternary relation ‘x lies on a line between y and z’ is transitive in both senses, but whether or not the two definitions are always extensionally equivalent is of no concern for our purposes here. Rather it is another aspect that is of interest. Both the definitions have an implicit requirement built into them, namely that all the variables have to be variables for the same kind of entities. Why? Because in left-transitivity the y-variable figures both as the second and as the first relatum, and the z-variable figures both as the third and as the second; and in right-transitivity the x-variable figures both as the first and as the second relatum, and the y-variable figures both as the second and as the third. This requirement of categorial homogeneity of the variables cannot be fulfilled in the case of Oxyz. The values of its first variable have to be either persons or organizations, the values of its second should be organizations, and the values of its third variable are sets of rules. That is, the first and the second relata are always categorially distinct from the third relatum. Strictly speaking, therefore, transitivity is not defined for Oxyz (meaning ‘x is a direct organizational part of y by means of z’). Loosely speaking, however, one might still say that Oxyz cannot be a transitive relation.

I have now argued that direct functional and direct organizational parthood lack transitivity for quite other reasons than those put forward by Winston, Chaffin, and Herrmann. But I will end this section by stressing that an equivocation between functional parthood (in the first premises) and organizational parthood (in the second premises) is involved in the following two fallacies:

- (6) Simpson’s finger is part of Simpson and Simpson is part of the Philosophy Department, therefore Simpson’s finger is part of the Philosophy Department.
- (15) The arm is part of the musician and the musician is part of the orchestra, therefore the arm is part of the orchestra.

7. Three conclusions

The first conclusion of this paper is simple and not in any way astonishing: All binary parthood relations are transitive.[16] The second conclusion is, as far as I know, quite new: Seemingly intransitive and non-transitive binary parthood predicates, both in everyday and in scientific language, are in every case hiding a reference to a third relatum, which explains their lack of transitivity. In appearance these predicates are binary predicates, in reality they are at least as complex as either relative-product-predicates or as ternary predicates. Together, these two conclusions imply a third, which can be phrased as a warning: be careful if you try to apply the transitivity axiom of binary mereology to parthood predicates found in areas outside mereology proper. Such predicates might very well be intransitive, non-transitive or fall outside the scope of any natural definition of transitivity.

Coda on constituent functions

What has been said in this paper about functional parthood is worth exploring a bit further. The two schemas used for artefactual-functional and biological-functional parts, respectively, have a common form:

- In the (artefactual or biological) functional unit of y,
- one function of the spatial part x is
- to M in relation to w.

In both cases the functionality of x has as one of its presuppositions the functionality of y; x is a derived kind of functionality. The function of x is a *part-(to-)whole* or *constituent* function relative to y.

Such a kind of functionality does of course contain an infinite regress problem: If x can have a constituent function, F, only if it is itself part of a larger functional whole, y, what about the function of y? Where should we end our constituent function talk? In the case of artefactual functionality, one might with good reasons say that we end in a functional unit whose

[16] If there were intransitive binary parthood relations, it ought, in analogy with non-Euclidean (non-Classical) geometry, to be possible to construe an axiomatic "non-Classical mereology."

function is a purpose merely ascribed to it by human beings. With respect to biological functionality things are not so simple. There are here two conflicting intuitions. On the one hand is the common sense view that there really are units that are intrinsically functional, so that functionality inheres in the unities in the way a monadic property like mass is assumed to inhere in Newtonian corpuscles or the way human intentions are assumed to inhere in individual persons. On the other hand, there is the post-Darwinian view of science that to think in such terms of biological function is to anthropomorphize nature. I will not try to resolve this issue here. Rather, I will content myself with making the following two claims, the first of which has already been explained:

1. Where there is a constituent function, xFy, there is also necessarily either (a) an infinite regress of constituent functions, or (b) an intrinsic function, or (c) a merely man-made and conventionally ascribed function/purpose.
2. Independently of whether (a), (b), or (c) is the case, the constituent function predicate 'xFy' can describe objectively existing features of the world.

With respect to the second claim, the most controversial part is case (c). However, think briefly of the following.[17] If, counterfactually, one regards a certain house as lacking functionality and being just a material structure, then the doors seem to lose their functionality, too. But if the house as a whole has its normal house-function, then it is an empirical question whether or not the doors have a function.

The fact that there can be objectively existing constituent functions even where the function of the whole is merely an ascribed purpose is, at bottom, no more curious than the fact that there can be an objective means-end rationality even in relation to completely irrational ends.[18]

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[17] For a much fuller argumentation see my "Functions, Function Concepts, and Scales," *The Monist* 87 (2004), 96-115.

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