

Contents

Preface — v

0 Introduction — 1

1 Lévy processes and Itô calculus — 5

- 1.1 Poisson random measure and Lévy processes — 5
 - 1.1.1 Lévy processes — 5
 - 1.1.2 Examples of Lévy processes — 8
 - 1.1.3 Stochastic integral for a finite variation process — 11
- 1.2 Basic materials to SDEs with jumps — 13
 - 1.2.1 Martingales and semimartingales — 13
 - 1.2.2 Stochastic integral with respect to semimartingales — 15
 - 1.2.3 Doléans' exponential and Girsanov transformation — 21
- 1.3 Itô processes with jumps — 24

2 Perturbations and properties of the probability law — 33

- 2.1 Integration-by-parts on Poisson space — 33
 - 2.1.1 Bismut's method — 35
 - 2.1.2 Picard's method — 45
 - 2.1.3 Some previous methods — 51
- 2.2 Methods of finding the asymptotic bounds (I) — 59
 - 2.2.1 Markov chain approximation — 59
 - 2.2.2 Proof of Theorem 2.3 — 63
 - 2.2.3 Proof of lemmas — 70
- 2.3 Methods of finding the asymptotic bounds (II) — 77
 - 2.3.1 Polygonal geometry — 77
 - 2.3.2 Proof of Theorem 2.4 — 78
 - 2.3.3 Example of Theorem 2.4 – easy cases — 88
- 2.4 Summary of short time asymptotic bounds — 96
 - 2.4.1 Case that $\mu(dz)$ is absolutely continuous with respect to the m -dimensional Lebesgue measure dz — 96
 - 2.4.2 Case that $\mu(dz)$ is singular with respect to dz — 97
- 2.5 Auxiliary topics — 99
 - 2.5.1 Marcus' canonical processes — 99
 - 2.5.2 Absolute continuity of the infinitely divisible laws — 102
 - 2.5.3 Chain movement approximation — 107
 - 2.5.4 Support theorem for canonical processes — 109

3	Analysis of Wiener–Poisson functionals — 114
3.1	Calculus of functionals on the Wiener space — 114
3.1.1	Definition of the Malliavin–Shigekawa derivative D_t — 116
3.1.2	Adjoint operator $\delta = D^*$ — 120
3.2	Calculus of functionals on the Poisson space — 122
3.2.1	One-dimensional case — 122
3.2.2	Multidimensional case — 125
3.2.3	Characterisation of the Poisson space — 128
3.3	Sobolev space for functionals over the Wiener–Poisson space — 132
3.3.1	The Wiener space — 132
3.3.2	The Poisson Space — 134
3.3.3	The Wiener–Poisson space — 139
3.4	Relation with the Malliavin operator — 148
3.5	Composition on the Wiener–Poisson space (I) – general theory — 150
3.5.1	Composition with an element in S' — 150
3.5.2	Sufficient condition for the composition — 156
3.6	Smoothness of the density for Itô processes — 160
3.6.1	Preliminaries — 160
3.6.2	Big perturbations — 163
3.6.3	Concatenation (I) — 167
3.6.4	Concatenation (II) – the case that (D) may fail — 170
3.7	Composition on the Wiener–Poisson space (II) – Itô processes — 178
4	Applications — 181
4.1	Asymptotic expansion of the SDE — 182
4.1.1	Analysis on the stochastic model — 184
4.1.2	Asymptotic expansion of the density — 205
4.1.3	Examples of asymptotic expansions — 210
4.2	Optimal consumption problem — 216
4.2.1	Setting of the optimal consumption — 216
4.2.2	Viscosity solutions — 220
4.2.3	Regularity of solutions — 239
4.2.4	Optimal consumption — 243
4.2.5	Historical sketch — 246
	Appendix — 249
	Bibliography — 253
	List of symbols — 263
	Index — 264