

Contents

| | |
|---|-----------|
| Preface | vii |
| 1 Elliptic curves and equations | 1 |
| 1.1 A general overview | 1 |
| 1.2 Elliptic curves and the Mordell–Weil Theorem | 5 |
| 2 Heights | 9 |
| 2.1 Notations and facts | 9 |
| 2.2 Absolute values in a number field | 11 |
| 2.3 Heights: Absolute and logarithmic | 13 |
| 2.4 A formula for the absolute logarithmic height | 18 |
| 2.5 Heights of points on an elliptic curve | 20 |
| 2.6 The canonical height | 23 |
| 3 Weierstrass equations over \mathbb{C} and \mathbb{R} | 29 |
| 3.1 The Weierstrass \wp function | 29 |
| 3.2 The Weierstrass equation | 31 |
| 3.3 $\psi : E(\mathbb{C}) \mapsto \mathbb{C}/\Lambda$ | 33 |
| 3.4 Weierstrass equations with real coefficients | 36 |
| 3.4.1 $\Delta > 0$ | 38 |
| 3.4.2 $\Delta < 0$ | 40 |
| 3.4.3 Explicit expressions for the periods | 41 |
| 3.4.4 Computing ω_1 and ω_2 in practice | 44 |
| 3.5 $\psi : E(\mathbb{R}) \mapsto \mathbb{C}/\Lambda$ and $\iota : E(\mathbb{R}) \rightarrow \mathbb{R}/\mathbb{Z}\omega_1$ | 47 |
| 4 The elliptic logarithm method | 54 |
| 5 Linear form for the Weierstrass equation | 57 |
| 6 Linear form for the quartic equation | 60 |
| 7 Linear form for simultaneous Pell equations | 69 |

| | | |
|-----------|--|-----|
| 8 | Linear form for the general elliptic equation | 78 |
| 8.1 | A short Weierstrass model | 78 |
| 8.2 | Puiseux series | 80 |
| 8.3 | Large solutions | 84 |
| 8.4 | The elliptic integrals | 86 |
| 8.5 | Computing in practice B_1 of Proposition 8.3.2 | 89 |
| 8.6 | Computing in practice B_2 and c_9 of Proposition 8.4.2 | 91 |
| 8.7 | The linear form $L(P)$ and its upper bound | 94 |
| 9 | Bound for the coefficients of the linear form | 98 |
| 9.1 | Lower bound for linear forms in elliptic logarithms | 98 |
| 9.2 | Computational remarks | 105 |
| 9.3 | Weierstrass equation example | 107 |
| 9.4 | Quartic equation example | 110 |
| 9.5 | Simultaneous Pell equations example | 114 |
| 9.6 | General elliptic equation: A quintic example | 118 |
| 10 | Reducing the bound obtained in Chapter 9 | 121 |
| 10.1 | Reduction using the LLL-algorithm | 122 |
| 10.2 | Examples | 125 |
| 10.2.1 | Weierstrass equation | 125 |
| 10.2.2 | Quartic equation | 127 |
| 10.2.3 | System of simultaneous Pell equations | 131 |
| 10.2.4 | General elliptic equation: A quintic example | 134 |
| 11 | S-integer solutions of Weierstrass equations | 137 |
| 11.1 | The formal group of C and p -adic elliptic logarithms | 137 |
| 11.2 | Points with coordinates in \mathbb{Z}_S | 144 |
| 11.3 | The p -adic reduction | 154 |
| 11.4 | Example | 158 |
| | List of symbols | 165 |
| | Bibliography | 173 |
| | Index | 177 |