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We have no better way of describing elementary particles than quantum field theory. A quantum field in general is an assembly of an infinite number of interacting harmonic oscillators. Excitations of such oscillators are associated with particles ... All this has the flavor of the 19th century, when people tried to construct mechanical models for all phenomena. I see nothing wrong with it, because any nontrivial idea is in a certain sense correct. The garbage of the past often becomes the treasure of the present (and vice versa). For this reason we shall boldly investigate all possible analogies together with our main problem.

A. M. Polyakov, "Gauge Fields and Strings", 1987 [51]

