Tempered in the Time of the Analemma Harmonics and Architectonics, an Appeal to the "Digital Dignity" of Nature

by Elias Zafiris

In the old tradition of Natural Philosophy, the resolution of an obstacle that is abstracted as a mathematical problem is conceived as a means of tuning into the Cosmos. Since this is an action taking place in the domain of Harmonics, where constellations of wholes abide by the rhythmics of symphony cycles choreographed by the invariance of the obstacle, the mechanics of this invariance paves the path of abducting the metaphysical essence of the obstacle.

Metaphysics in this sense is not a formal logical meta-level lying beyond the physical and addressing the obstacle by means of a tailored ontology but pertains to the concordant circulation around the obstacle enacted metaphorically by the mechanics of its specific invariance. The metaphorical circulation around a fulcrum that hypostasizes the obsta-

cle unveils the invariance underlying its diachronic presence. In this manner, the hypostasis of the obstacle is objectified not by means of an ontology but by means of a natural axiology emancipating from its invariance.

The axiology rests on a canon of scalar values proportionating homologically the points of stasis, that is, the resonance harmonics of tuning into the Cosmos in the presence of the obstacle as it is manifested purely in terms of its invariance. As such, axiology is not hierarchical, but it opens a teleonomic *topos* of communication, whose variable existential bounds are commensurable to the invariance of the obstacle. It is due to the plasticity of the bounds in respecting invariance under reciprocally adjoined encoding/decoding communication bridges that axiology can be always embodied metaphorically in the Cosmos by means of the tuning harmonics of the pertinent canon, persisting through the sequential ordering of time. The roots of Humanism are not intelligible without such axiological canons gauging the invariance of obstacles.

The harmonics of the canon are not absolute truths. Their universal role is the modular infiltration of the consonances of communication through the sieve of the canon, which is expressed in terms of their homologizing ratios with respect to the in-

terference co-bounds of this *topos*. In this manner, consonance or dissonance do not determine classes of binary classification but instantiate categories for the spectral tempering of these ratios by means of rhythmic architectonic forms.

The axiology of consonance implemented by the scale of the canon gives rise to a discrete series of binary digits, a digital string characterizing a *melos*, which modulates any continuous thread through the sieve according to the ubiquitous criterion of symphony expressed by the homologizing ratios. Thus, symphony transcribes the modulated thread to a topologically circular oriented chord, whose universal covering helix can be either ascended or descended palindromically. If the descent is congruent with the directionality of the sequential entropic passage of time, the ascent is congruent with the inverse directionality of in-formation synchronization as diachronic mneme subject to the gnosis of the invariance of the obstacle.

Henceforth, the "melodic" qualification of Humanism according to the proportionate cyclotomy of the invariance in terms of the axiological criterion of symphony, that is, in terms of the canonical scale of harmonic consonance ratios is impossible without the ethos of the pertinent digital string characterizing categorically the *melos*. In this sense,

the current concerns about Digital Humanism pertain to the ethos of these modulating digital sequences, what differentiates a symphonic *melos* eidetically from a series of binary classification digits making a decision product. Digital Humanism cannot be aspired on normative regulations.

The ethos of a digital string as the crucial quality pertaining to the character of a *melos* according to axiology has already served as the major categorial means of cultivating musical in-formation in classical antiquity. The *diataxis* of consonance ratios on the scale of the canon according to their ethos gives rise to three categorical genera; the diatonic, the chromatic, and the enharmonic, where each of them transfuses a distinct character to the *melos*, although all preserve the same invariance. If the pure ratios are stochastically dissolved within well-tempered sections of the canon due to objective variability or indistinguishability in the continuum, twelve distinct categorical genera arise, each characterized by its distinct ethos.

The reduction of a modulating digital string into a formal logical classificatory chain of productivity and decision-making disregarding the categorial ethos associated to a *melos*, according to axiology, annihilates the capability to ascend the helical arc covering the underlying modulated chord in sym-

phony with a categorial genus. What remains is the gravitational descent following dynamically the instantaneous center of curvature according to the directionality of the entropic arrow of time.

The descent tuned unidirectionally towards the centroids of big data utilized for adaptation and classification needs to be turned inside-out to restore the inverse directionality of in-formation synchronization according to a rhythmisizing categorial genus that persists diachronically as an ethical mneme of civilization subject to the gnosis of the invariance of the pertinent obstacle. The interference between these two inverse directionalities from the inside to the outside and from the outside to the inside with respect to the elastic boundary of a *melos* ascribes to it the harmonic equilibration of an architectonic aeon.

The thesis is that the categorial spectral tempering of a *melos* by means of a rhythmic architectonic form takes place by turning inside-out the axiologically modulating digital string characterizing the *melos* according to the ethical genera of harmonics. Equivalently, an architectonic form molded out of the rhythm of a modulor is the enveloping shape that is bounded by the curve of evolutes of the points of stasis of the *melos* according to the *diataxis* of a certain ethical genus. Reciprocally and sym-

metrically, the guiding thread on which the points of stasis lie constitutes the boundary of involutes of the architectonic enveloping shape of the evolutes.

Thus, it is in the axiological terms qualified by the interference between these two inverse directionalities with respect to the binary topological in/out distinction that the static tripod consisting of harmonics, mechanics, and architectonics becomes intelligible. In this stable constellation, mechanics—considered in the original Archimedean sense as a method of tuning through leveraging with respect to a fulcrum—serves as a bidirectional encoding/decoding functorial bridge between the domain of harmonics—where a modulating digital string of homologizing ratios respecting the underlying invariance lies—and the domain of spectral geometry—where the enveloping shape of the corresponding architectonic form lies. The preservation of the same invariance under functorial metamorphosis from one domain to the other through mechanics is attained by the involute/evolute translation code.

The involute/evolute binary code was first conceived by Apollonius of Perga and further developed by Huygens. For any point on a curve, we consider the tangent and normal orthogonal directions at this point as well as the osculating or kiss-

ing circle of the curve at this point. The evolute of this curve is the envelope of all normal directional lines to the curve traced by a moving point. In this way, the original curve is identified with the involute of its evolute. The original normal directional lines become tangential directional lines of the evolute of the curve and reciprocally. The center of the osculating circle at a point, thought of as the center of curvature at this point, is considered in the following way: For a fixed point on the curve, one may construct two normals to the curve, one at and another at a nearby point on the curve. The center of the osculating circle is the limit of the intersection of these two normal lines as approaches . Thus, the evolute of a curve is the enveloping shape of all centers of curvature traced by a moving point along this curve.

The notion of a center of curvature bears the connotation of a center of synchronization in the domain of harmonics, whence the notion of an osculating circle provides the means of tuning a curve at any point with its center of synchronization with respect to this point. The concept of local curvature is associated physically with the idea of the degree of local bending due to gravity, whence the reciprocal of the curvature at a point is the radius of curvature of the tuning osculating

circle. In other words, the bending of a curve, expressed through its curvature at any of its points, is the tuning of the curve to the invariance of the obstacle of gravity taking place through rolling an osculating circle along it, whose radius is variable from point to point being equal to the reciprocal of the curvature.

Let us wonder about the nature of the tuning offered by the osculating circle at any of the points of the bending curve due to gravity. According to Euclid, a point on the curve is something without parts, but a moving point along the curve may be thought of as a fluxion, according to Newton. Physically, it is conceptualized as a moving corpuscle due to gravity, such that the bending curve is its continuous thread subject to this obstacle altering its uniform straight-line motion that would sustain in the absence of the obstacle. The bending around a point of this thread is the local curvature that localizes the invariance of gravity at this point. As a local measure, it expresses the average rotation per unit area with respect to the center of the osculating circle at this point.

Therefore, local curvature is a tempering measure of the degree of bending around a point that is expressed by the average rotation per unit area around the center of the osculating circle at this

point. Since the radius of the osculating circle at each point is the reciprocal of curvature, the product of the curvature with the radius of the osculating circle is invariant at each point of the bending thread. This means equivalently that the continuous variation along the thread gives rise to a rectangular hyperbola on a screen where the radii of the osculating circles extend along the vertical direction and the corresponding curvatures extend along the horizontal direction due to the reciprocal relation between them.

The unit area is the area of the square on this screen lying under the rectangular hyperbola whose orthogonal sides are the unit radius and the unit curvature correspondingly. The unit area is invariant. Any increase along the radii on the vertical is compensated by a reciprocal decrease along the curvatures on the horizontal, such that the corresponding area bounded by the hyperbola and the two orthogonal directions of the screen remains invariant.

The above gives rise to a gravitational monochord. The variation of local curvatures corresponds to a continuous spectrum of frequencies which is modulated by the directly invisible discrete harmonics of gravitation. Local curvature at a point expresses the tempering frequency around

this point since it is the average rotation per unit area. According to Archimedes and Kepler, the invariance of gravity with respect to a center of curvature is subsumed in enclosing equally tempered areas at equal times on our screen where the rectangular hyperbola lies. Therefore, local curvature is indeed a tempering frequency on a continuous spectrum expressing locally the average rotation per unit time.

The discrete gravitational harmonics of this monochord are abducted through the unit area invariance on the screen, which may be identified with the complex plane. The homologizing ratios of integer radii on the real horizontal axis determine corresponding co-homologizing ratios of integer frequencies on the imaginary vertical axis subject to the invariance of the unit area. The tempered intervals of this gravitational monochord, that is, the gravitationally consonant intervals, are precisely the areas bound by the hyperbola and the two orthogonal axes of the complex plane screen, which preserve the unit area under synchronized reciprocal extension/contraction of the real radii/imaginary frequencies on the screen.

The invariant synchronization condition pertaining to the enclosure of equal areas at equal times gives rise to the equally tempered scale of the

gravitational monochord. The tempered intervals of this monochord are abducted, in turn, through the homologizing ratios, whose sequential melodic strings modulate the continuous threads of corpuscles under gravity. A homologizing ratio literally means "homologous logos-arithmetic," which is abridged by the name-symbol "logarithm." The latter may be universally expressed through the Euler basis cipher , which modulates continuously the unit area on our screen under synchronized reciprocal extension/contraction of the real radii/imaginary frequencies, such that:

$$\log_e(e) = 1, \text{subject to} \log_e(1) = 0 \text{ and inversely } e^0 = 1.$$

In this manner, the unit area on the screen is synchronically preserved and tempers equally the intervals of the gravitational monochord upon their abduction through the logarithms. If the imaginary continuous frequencies are periodically ordered according to the harmonic progression, and the reciprocal real radii are sequentially ordered according to the geometric progression, then the logarithmic equally tempered intervals are synchronically ordered according to the arithmetic progression.

Since these intervals correspond to areas under the hyperbola and the axes, the categorization of

these areas is chromatically abridged in terms of the quality of color. In this sense, the equally tempered scale is qualified as a chromatic scale that is enunciated spectrally on the screen, that is by means of light rays. This abstraction is intelligible by wrapping the screen around the Riemann sphere by means of the stereographic projection and considering the light rays as penetrating the sphere after radiation or absorption through a fixed point identified with one of the poles. In this way, the projection imprinted via light rays on the screen if the radiation point is the North pole is topologically turned outside-in upon translation of the fixed point to the South pole bearing the role of an absorption point reciprocally. The spectral ichnography of light on the screen of the complex plane is unveiled through the gesture of the analemma.

In consonance with Kepler, gravity presents an obstacle in the domain of harmonics. The rolling osculating circle encodes the invariance of this obstacle through its variable radius, and thus, is tuning the bending of the curve by the reciprocal of the osculating circle radius at any of its points. The *topos* of all centers of curvatures, that is, the *topos* of all centers of all osculating circles, gives rise to the evolute of the bending curve by means of turning inside-out the invariance of the obstacle. The evo-

lute is the enveloping spectral architectonic form of all centers of curvature of the bending curve due to gravity. According to Apollonius, the evolute form is identical with the envelope of all normal directional lines to the bending curve due to gravity, such that the latter constitutes the harmonic involute of the spectral geometric architectonic form that unfolds it in an area-preserving manner.

If we consider the tangent and normal orthogonal directions at any point of a bending curve due to the invariance of the obstacle of gravity, they give rise to the complex plane, where the real axis is directed along the normal, and the imaginary axis is directed along the tangent at this point. The turning inside/out of the *topos* of all centers of the tuning osculating circles from the harmonic to the geometric domain, which bounds the enveloping shape of the evolute architectonic form, is expressed as an imaginary rotation in the complex plane at every point, which is as a rotation conducted by the imaginary unit. The reason is that the normal to the harmonic involute, which is the direction of the real axis, is imaginarily rotated by a right angle—under the action of the imaginary unit—after turning inside-out from the harmonic to the spectral geometric domain. This is the case because the normal to the harmonic involute at a

point becomes the tangent of the evolute architectonic form, and thus, bears the directionality of the imaginary axis after unfolding the bending curve to its evolute.

Essential care is needed for interpreting these imaginary rotations properly. From the standpoint of Archimedes an imaginary rotation should be always considered with respect to a fulcrum that essentially establishes the equilibration condition. In the case of the harmonic involute the fulcrum is identified with the point of the bending curve where the real and imaginary axes of the complex plane are erected by means of the directionality of the normal and tangential lines respectively. In the case of the architectonic evolute the fulcrum is not the same. Precisely, it is identified with the center of curvature of the harmonic involute at the considered point.

Hence, it is the center of the tuning osculating circle (based at the harmonic fulcrum) that the architectonic fulcrum is located under turning inside-out. Therefore, the imaginary rotation applies under translation of the harmonic fulcrum to the architectonic fulcrum, which is a translation from a point of the bending curve to the center of its tuning osculating circle. The translation from the harmonic fulcrum to the architectonic ful-

crum preserves the rate of enclosing area through imaginary rotation in symphony with the Pythagorean theorem.

Concisely put, the action of the imaginary unit on the normal at each point of the harmonic involute is mechanically rotating it to the tangent of its architectonic evolute under translation of this point to the center of its osculating circle along the radius. In this manner, the involute/evolute binary code of turning inside-out from the harmonic to the architectonic, and reciprocally outside-in, is mechanically encoded/decoded by means of an imaginary right-angle rotation on the complex plane under translation of the fulcrum in an area-preserving way. Algebraically, since such a rotation is expressed via the action of the imaginary unit *i*, which constitutes a fourth root of unity together (with its complex conjugate *-i,* 1, and -1); the universal equation of ciphers arises, known as the Euler identity , via the natural logarithm basis which modulates the unit area.

It is worth pondering on the nature of the evolute spectral architectonic form arising from the binary code of turning inside-out the corresponding involute bending curve. Since the form is imprinted spectrally on the screen of the complex plane, and the normal directional lines of the harmonic

involute are imaginarily rotated to tangential directional lines of the architectonic evolute, the latter bear the status of light rays whose ichnography is the contour (peri-gramma) of the evolute form. In particular, these light rays being tangential to the evolute concentrate light at its contour, which becomes brightly lit in this way, thus, unveiling the form spectrally. Therefore, the evolute architectonic form arises in a natural way as the optical spectrum of the envelope of its tangential caustics.

We conclude that the involute/evolute binary code of turning inside-out from the harmonic to the architectonic, and reciprocally outside-in, is not a design principle like the one devised to engineer the material self-stability of an architectonic form, for instance the catenary arch by means of inverting the shape of a hanging chain due to gravity with respect to the horizontal axis. Rather it pertains to the "digital dignity" of Nature in relation to the "static tripod" of Harmonics/Mechanics/Architectonics enunciated in axiological terms. It is the preservation of the interwoven distributed area between the radii and curvatures of the tuning osculating circles on the complex screen of the gravitational monochord - under functorial metamorphosis from the harmonic domain to the spectral geometric domain and inversely by the lev-

eraging bridges of mechanics—that constitutes the invariance and objectivity attained by the involute/evolute translation code.

December 2021, Budapest