# TEMPORAL BONDS: TRIPODAL SPECTRAL ARCHITECTONIC SYNTAXIS OF TIME

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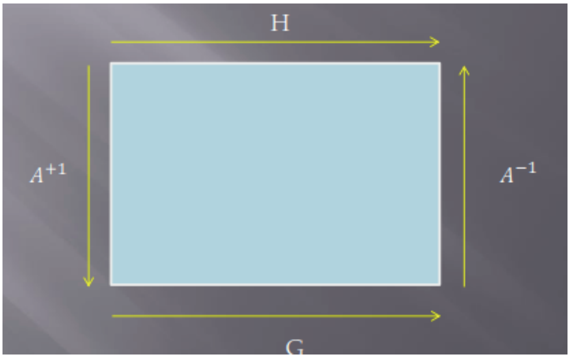
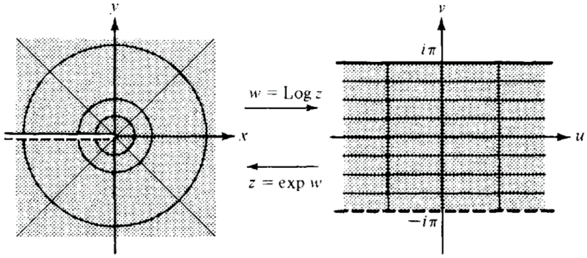
## 3.1 HARMONICS TO GEOMETRY: ENCODING LOGARITHM-DECODING ALGORITHM

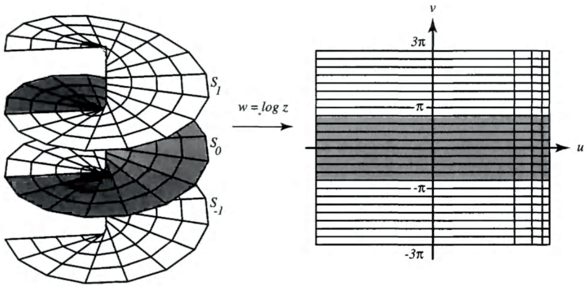
It is elucidating to consider in more detail the temporal aspects of the metaphora from harmonics to geometry along the lines already established. The temporal helical axis of unfolding perpendicular to the epiphaneia bears the meaning of a temporal diastasis. Any appropriate finite bounded portion of this diastasis can be transfigured conformally, via a branch of the complex logarithm on the imaginary axis of the complex plane equipped with the rectangular grid, as a spatialized spectral interval. In the context of each present, the harmonics appear as powers through their corresponding action on the complex roots of unity, in accord with the cyclotomy of the circle, or equivalently, the imaginary ring in the complex analytic setting. The diachronic persistence of the harmonics, elevates them to topological invariants that manifest as points of stasis geometrically. In this sense, a point of stasis bears temporal depth, since harmonic resonance is qualified in terms of such points diachronically. The crucial issue is that a spatialized spectral interval is not merely a geometric interval, but carries topological and harmonic information that manifests in discrete or quantum terms with respect to the temporal unfolding diastasis. More precisely, the implicated geometry is the one arising out of a cohomological spectrum, congredient with the sheaf localization structure. We qualify this geometry simply as spectral geometry to avoid further technicalities at this stage. The objective is to foreground the metaphora from harmonics to geometry in relation to the temporal aspects of this metaphora, not to focus exclusively on the terms of the complex analytic function-theoretic setting of our previous discussion.

The motivation for this reflection follows from the idea that the complex analytic function-theoretic setting is based on and presupposes for its consistency a certain notion of generalized number domain; the domain of complex numbers, which is an algebraically closed field. In a well-defined sense, the metaphora from harmonics to geometry cannot be consistently implemented without reference to the arithmetic cosmos of the complex number field. We recall that we employ two distinct types of metaphora in order to invert exponentiation. The first, referring to the powers with respect to a base necessitates the extension of the field of the rationals to the field of the reals, so that logarithmization becomes a total operation in the domain of real numbers. The second, referring to the roots, calls for the extension of the rationals to the complex numbers, to accommodate the roots of negative numbers. Both of these inversions are unified in the field domain of the complex numbers under the notion of the complex logarithm. But, in turn, the complex logarithm inverts complex exponentiation only locally, namely by considering a branch of this multi-valued function corresponding to the interval between two consecutive harmonics. As a result, the inversion of the logarithmic encoding bridge from harmonics to geometry, i.e. the decoding bridge from geometry back to harmonics, requires the algorithmic instantiation of the harmonics as powers acting on the roots of unity. Therefore, the notion of *arithmos* pertains to the metaphora from harmonics to geometry in terms of the *logarithm* notion in the encoding direction, and in terms of the *algorithm* notion in the decoding direction.



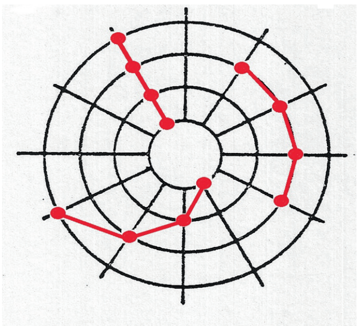

In the above setting, arithmos via its dual connotation as a logarithm and algorithm correspondingly, in relation to the encoding of harmonics to geometry and, inversely to the decoding from geometry back to harmonics, provides the bridges of the metaphora from harmonics to geometry. We may assert that the domain of harmonics directs the global choreography, the domain of arithmetics in its double role, as previously, directs the scenography and the orthography respectively, while the domain of spectral geometry directs the ichnography. The latter interprets a point of stasis as the trace of its temporal depth in each present. Taking into account the concomitant weaving pattern on the geometry depicted by the rectangular grid of the complex plane via the branched action of the logarithm, we realize that it descends from the epiphaneia conceived in its polar Euler representation bounded by the imaginary ring.




3.2 **HARMONICS-ARITHMETIC-GEOMETRY: THREEFOLD *METAPHORA* AS A *STATIC TRIPOD***

According to our argument, the spectral geometry on the epiphaneia manifests itself like the spider's web, called *"arachne"* in ancient Greek. It is telling that in nature the spider ascends and expands its web by means of the logarithmic spiral, i.e. following the *geometric progression*, whereas it descends back to the center and stabilizes its web along a radius, i.e. following the *arithmetic progression*.



The ancient Greek naming is very interesting since all three of the involved notions -armonia, arithmos, and arachne- bearing the correspondence with harmonics, arithmetic, and spectral geometry respectively, emanate from the same linguistic root. We argue that this phenomenon is not accidental, but it is intimately correlated with the personification of Time in mythology. This provides a very elucidating conceptual insight on all the preceding. Before going to this matter, it is significant to point out a fundamental symmetry property pertaining to the metaphora from harmonics to geometry through arithmetics. This symmetry property, which is of a topological origin, is that the round trip at the level of harmonics via the level of geometry under the arithmetic bridges can equivalently and isomorphically be conceived as a round trip via the level of arithmetic under geometric bridges.

The latter is feasible if the level of arithmetic is identified simply as a spectrum characterized by the integer frequencies, whereas the encoding and decoding geometric bridges are identified on the epiphaneia in terms of the ascending logarithmic spiral and the descending Archimedean spiral respectively. In the latter setting, the domain of harmonics still directs the global choreography, the domain of the geometry in its double role, as above, directs the scenography and the orthography respectively, and the domain of arithmetic directs the ichnography.

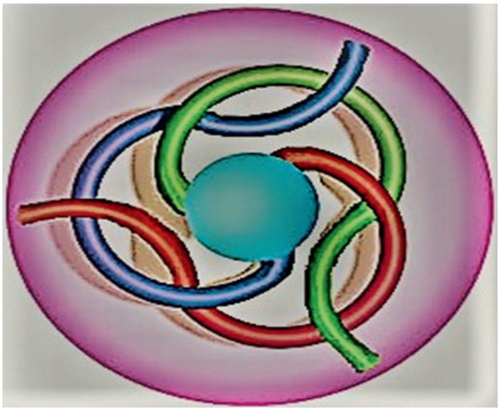
We realize that in the initial setting arithmetic is exemplified through its algebraic or operational aspect, whereas in the latter setting it is exemplified through its spectral discrete aspect. Analogously, in the initial setting geometry is exemplified through its spectral, quantum qualification, in the form of points of stasis, whereas in the latter setting it is exemplified through its kinematic qualification whose forms are spiral progressions. It is not difficult to realize that there exists a further symmetry, which can be considered a metaphora from arithmetic to geometry via encoding and decoding bridges belonging to the domain of harmonics. The encoding bridge is the extraction of the roots of unity, whereas the decoding bridge is the utilization of the harmonics as powers for synchronization of the points of stasis with the unity. We conclude that there exists a threefold canonical metaphoric communication among the domains of harmonics, arithmetic, and geometry, such that any one of them gives rise to a pair of encoding/decoding bridges with respect to a metaphora establishing the canonics between the other two. In this case, we say that these three domains constitute a *static tripod*, in accordance to the ancient Greek term.

### 3.3 TRIPOD OF TIME: WINDING-MEASURING-BOUNDING

Since a tripod of this nature and function is balanced on the notions of armonia, arithmos, and arachne, according to the above, it is worth revisiting briefly the cosmogonical context of the pre-socratic philosophy, where these threefold canonical communication relations were first conceived and explicated. The notion of the static tripod is of *Delphic* origin, where it stands stably over the *omphalos* symbolizing the primordial obstacle. In this sense, the static tripod is the generator of a threefold stable communication relation embracing the obstacle.

The very first abstract tripod incorporating the characteristics described above, is the tripod [*thauma*(state of wonder)/*ananke* (necessity)/ *aletheia*(truth)]. *Thaumazein* is the act of looking upward to the sky and wondering in the attempt to unveil what is true. The encoding and decoding bridge between these two levels, of thauma and aletheia is enacted by necessity. Note the bidirectionality of necessity in this metaphora, which is not restricted to some type of efficient causality, but it incorporates the entelecheia inversely as well. Moreover, aletheia is not identified with what we call today logical truth. Aletheia refers to unveiling what is diachronically true, where the latter is identified with what should not be forgotten after being unveiled; in other words aletheia is tantamount to *mnemosyne*, the faculty of memory and remembrance, personified by the mother of the nine muses.

Since thaumazein is the act of looking upward and aletheia is meaningful with respect to *lethe* which comes first, we can easily consider them as bidirectional bridges between the two other legs of the tripod, thereby completing the circuit of the threefold metaphoric communication. We may think of the tripod as follows:



Concomitantly, we think of each leg of the above tripod as a *melos*, meaning a breaking in the continuity associated with the *omphalos* in the middle. Each melos in its rendering as a bidirectional bridge with respect to the other two, may be thought of in terms of an umbilical cord emanating from the omphalos, serving as a means of establishing its diachronic identity. A melos is distinguishable as such, i.e.

independently from its participation in the tripod, by bounding and cutting the cord, breaking the continuity in this way, to be re-established by its re-habilitation within the tripod that re-activates its cord. The re-activated cord bears the mark of its breaking as a root of its unity that allows a melos to synchronize with its cord via powers, to be thought of as harmonic frequencies giving rise to a spectrum that translates a cord into a chord.

Each distinguishable melos of the initial tripod gives rise to another tripod embracing the obstacle that its associated discontinuity imposes. The state of wonder gives rise to the tripod [philosophy/ mathematics/ architectonics], which is resolved further to the tripod [*logos*/ *orthos doxa*/ *arche*], to the tripod [*ratio*/ *arithmos*/ *architecture*], and to the tripod [*armonia*/ *arithmos*/ *arachne*], we encountered previously.

The aletheia as mnemosyne gives rise to a triad of tripods capturing the metaphoric communication among the nine muses, the first composed of [epic poetry/ lyric poetry/ hymn], the second of [choral dance/ comedy/ tragedy], and the third of [music/ history/ astronomy]. We come finally to necessity, the melos from which the tripod of time emerges out of its spindle and the thread it spins as an *axis mundi*, i.e. as a cosmic helical axis of unfolding perpendicular to the epiphaneia.

The tripod of time consists of [*Clotho/ Lachesis/ Atropos*], together called the *Moirai*. Clotho is the one who winds the thread, Lachesis is the one who unfolds and measures the thread, and Atropos the one who bounds and cuts the thread. Especial care is needed in comprehending the tripod of time specifically, as a means of threefold metaphoric communication among Clotho, Lachesis, and Atropos, and not as a literal spatial representation of different actions. The key is especially provided by the domain of harmonics, without which a literal geometric representation becomes untenable and distorting.

In particular, the discrete marking or quantization of the thread by Lachesis in terms of the integer periods of winding around by Clotho is not accidental, but it is subordinate to the bounding of the thread by Atropos, so as to reveal the harmonics. In this manner, the winding of the thread as a temporal diastasis of unfolding is complemented by the harmonics of a bounded helical standing wave appearing in the caduceus (*kerykeion*) of Hermes, the father of the Moirai and personified God of communication, which in turn, qualifies the measurement of the thread by Lachesis as a spectral measurement. This brings us back to the tripod [harmonics/ arithmetic/ geometry], which has been already analyzed in detail. It is an amusing realization that the tripod of time harbours such an articulating relevance for these mathematical considerations pertaining to metaphora and natural communication.

### 3.4 SINGLE TEMPORAL DIASTASIS: CHANGE OF TIME AS CHANGE OF PHASE

It is worth examining the major difference between the notion of a temporal diastasis, i.e. the notion pertaining to a helical temporal axis of unfolding perpendicular to the epiphaneia, and the standard notion of a real temporal dimension totally ordering events. An interesting analogia is emerging from the domain of music pertaining to the difference between *harmony* and *melody*. Whereas melody refers to the sequential horizontal ordering *of* notes, harmony refers to the vertical, consonant or synchronized listening *to* the notes at each *present*.

We recall that the concept of the epiphaneia can either be thought of as a disk in the complex plane equipped with the polar grid and bounded by the unit circle, identified with the imaginary ring, or under a conformal transfiguration via a branch of the complex logarithm, as a rectangular strip on the complex plane, where the imaginary axis bears the information of spatialized spectral intervals. The important thing to keep in mind is that the imaginary ring or the imaginary axis, model the present diachronically. The reason is that a spatialized spectral interval is not merely a geometric interval, but bears topological and harmonic information that manifests in discrete or quantum terms with respect to the temporal unfolding diastasis.

To recapitulate, at each present, the harmonics lead to the cyclotomy of the circle, bearing the status of powers for harmonic resonance in each present. This is why the meaning of the *imaginary axis*, as an axis spatializing time spectrally in each present, is very different from a real dimension ordering events. In the latter case, events appear as geometric points on this axis, which are sequentially totally ordered from the past to the future on a geometric linear continuum. In the former case, the geometric points on the imaginary axis are points of stasis bearing temporal depth, characterized by their power to synchronize or resonate with the present. Notice that from the perspective of the present, and given that the same helical temporal axis applies to each present isomorphically, the difference between past and future is a difference in orientation, which is to say that past and future appear symmetrically with respect to the present differing in orientation.

From a topological viewpoint, we have the phenomenon of multiple-connectivity of the past with the future with respect to the present. In particular, the past can be connected to the future in a multiplicity of possible ways in the present, if we take into account the symmetric appearance of roots of unity in the present differing in orientation, as well as the significance of the primitive roots of unity for this purpose. We argue that the notion of a temporal diastasis presents interesting results even in the case, where we restrict our attention to the spectral interval referring to a single harmonic. Within such an interval change of time may be simply thought of as a continuous change of phase. If we assume that the same interval applies for each present, then we have to distinguish only two cases. The first case applies when the continuous rate of change is the same for each present, whereas the second applies when this continuous rate of change differs from present to present.

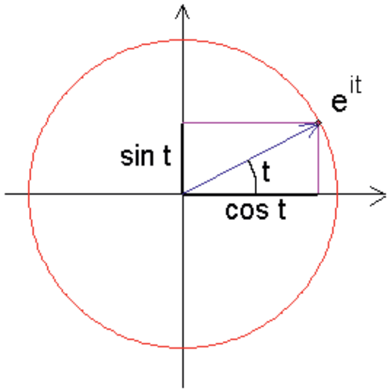
### 3.5 CONSTANT RATE OF PHASE CHANGE AND CONTRACTION OF LENGTH

First, let us consider a kinematical model of Special Relativity in the complex plane, where the real horizontal axis is a spatial axis, whereas the imaginary axis is a spatialized temporal axis. According to this theory, the maximal speed of electromagnetic signal transmission is defined by the speed of light $c$, which is constant in all directions. Moreover, the spatiotemporal metric relations are constant at every point-event leading to the group of Lorentz transformations as the kinematical symmetry group of the theory. The metric measuring distances is expressed as $dS^2 = dx^2 - c^2dt^2$, which takes the form $dS^2 = dx^2 + (icdt)^2$ on the complex plane, where the imaginary unit $i$ is used in the conversion of the temporal factor into the spatialized form, which the metric relation refers to.

Note that the temporal metric factor appears in imaginary spatialized form by the adoption of the speed of light $c$ through the intervention of the imaginary unit $i$. The metric relation $dS^2 = dx^2 + (icdt)^2$ on the complex plane is subordinate to the upper bound in geometric information signaling defined by the speed of light $c$, and thus, it pertains to phenomena approximating that speed. Furthermore, this type of continuous temporal unfolding at very high speeds takes place at a constant rate in each present given by the speed of light $c$.

We would like to study the connectivity between the past and the future in the present in this case. The basic idea is not only that the speed of light is constant, but that the chrono-geometric relations induced by this upper bound are constantly the same at each present, since the metric is not variable. This means that the imaginary ring, thought of as the unit circle with the complex structure, by normalizing the speed of light to unity, persists isomorphically at each present regarding both its shape, and the continuous rate of change upon it. It follows that, from the viewpoint of each present the difference between future and past is a difference in orientation only, subsumed by the imaginary unit and its complex conjugate.

Thus, in this case, change of time from the view of the present amounts to continuous change of phase, and this is isomorphically the same in each present for both the past and the future, differing only in orientation with respect to the present. As a consequence, we obtain the Lorentz contraction of lengths in the direction of motion. If we consider motion along the horizontal real spatial direction at a high speed below the speed of light, then spatial extension in the real horizontal linear dimension by $1$ unit of length will appear contracted with respect to the present at "0", i.e. with respect to the imaginary ring centered at "0", since it amounts to a change of phase on the unit circle equal to the passage of spatialized time, measured by the angle with respect to the real horizontal dimension. Hence, the length contraction (depending on the speed of motion) with respect to the present at "0" is just the projection on the horizontal linear spatial dimension of the corresponding phase change on the unit circle. Interestingly, at the speed of light, the length is contacted to zero with respect to the present at "0".




### 3.6 VARIABLE RATE OF PHASE CHANGE AND METRIC ANHOLONOMY

In the case of General Relativity, the spacetime metric becomes variable from point-event to point-event, depending on the distribution of matter in its vicinity. In this way, General Relativity is reducible to Special Relativity only in the infinitesimal vicinity of every point-event. Thus, the metric, and therefore, the chrono-geometric relations are not constant as in Special Relativity, but become variable. In turn, the variability of the metric requires that a standard of comparison is required at each point-event. This gives rise to the infinitesimal process of *parallel transport*, induced by a connection, involving small round trips around each point according to a prescribed rule of parallelism, characterized by the metric-compatibility of the connection. These round trips detect the change of orientation of a transported vector, expressed by means of a relative geometric phase factor, called the *metric anholonomy* of the connection, explicating locally the curvature associated with uneven matter distributions.

In this case, in the infinitesimal vicinity of any point-event the metric assumes the form, $dS^2 = dx^2 + (icdt)^2$, but this form is not retained constantly as we move from point-event to point-event. Thus, the rate of phase change is not constant between the past and the future with respect to the present. Concomitantly, although change of time amounts to continuous change of phase from the view of the present, the rate of phase change is differentiated between the past and the future. In this manner, past and future differ not only in *orientation* with respect to the present, but there is also a relative *geometric phase difference* between them, i.e. their metric anholonomy.

We conclude that the notion of a temporal diastasis presents interesting results even in the case, where we restrict our attention to a single spectral interval, qualified metrically. Within such an interval change of time corresponds to a continuous change of phase, and assuming that the same interval applies for each present, then, either the continuous rate of change is the same for each present, or the continuous rate of change differs from present to present giving rise to a relative geometric phase factor.

Actually, the analogous type of effects considered in the case of Special Relativity and General Relativity, that have been concisely discussed above from our perspective, may be thought of as emerging naturally in seemingly unrelated contexts, just by abstracting the relevant constraints. In the kinematical case of Special Relativity the constraint emanates from the constancy of the speed of light as an upper bound characterizing the propagation of electromagnetic signals. The crucial thing is that this upper bound speed is used as a universal metrical factor for the spatialization of time in the present along the imaginary axis on the complex plane in two dimensions. In this way, another type of constant speed pertaining to an upper bound for a different kind of propagation characterized metrically as above, would also correspond to a change of time as a change of phase, giving rise to an analogous effect of length contraction in the present referring to the direction of propagation.

Let us consider the case of propagation of an army in the battlefield. Initially, the notion of an armored vehicle was simply conceived as a means of protecting the infantry following it. As such the upper bound in the speed of propagation was set by the infantry. The strategic transmutation of the role of an armored vehicle into a unit of armored vehicles moving independently from the infantry amounts to a change in the syntaxis of time in the battlefield. This is because the speed of propagation of the army is altered by the upper bound set by the unit of armored vehicles. Thus, at speeds of propagation near this upper bound, change of time amounts to change of phase in the battlefield, where time in the present is spatialized on the imaginary axis due to this upper bound. Then, assuming that the continuous rate of change remains constant for each present, we obtain the phenomenon of length contraction in the direction of motion of the army in the battlefield from the view of the present of the resting infantry. In a similar fashion, effects of local curvature out of a relative geometric phase factor may appear in the present at the battlefield. This is the case if the continuous rate of change of phase does not remain constant for each present. For example, consider the case of a cavalry unit in comparison to an armored unit. Although change of time amounts to continuous change of phase from the point of view of the present in both cases, the rate of phase change is differentiated between the past and the future, e.g. at the present where a cavalry unit is substituted by an armored unit. In terms of this example, past and future do indeed differ not only in orientation with respect to the present, but there is also a relative geometric phase difference between them, which locally curves the battlefield to the advantage of the armored unit. In this case, we may think of the change of time as a *"massive"* one in analogy to the geometrized gravitational effect.

### 3.7 PAIR OF LINKED TEMPORAL DIASTASES: DOUBLY-PERIODIC SPECTRAL WEAVING

From a topological viewpoint, the most interesting phenomenon accompanying the notion of a temporal diastasis is the multiple-connectivity of the past with the future with respect to the present. We examined previously only the metrical case involving a single spectral interval, where change of time corresponds to a continuous change of phase in the ring of the present. Additional challenges naturally arise in the case of listening and comprehending a piece of music, or in the case of reading a book and trying to understand its content.

Clearly, the process of acquiring meaning and understanding cannot be reduced to the sequential order of time. More precisely, at each and every present, we rather instantiate temporal bonds between the memorized past and the anticipated future, where the present plays the role of a modulus for this type of bond. Due to topological multiple-connectivity these bonds allow information amalgamations irrespectively of any notion of distance or proximity in the text.

In particular, things in the very far past may form a temporal bond in the present with anticipated things in the very near future. If both the past and the future comply to the same temporal diastasis, in our terms if the ring in the epiphaneia of the present persists, through unfolding on the same helical axis perpendicular to the epiphaneia, then a temporal bond in the present can be formed from the primitive roots of unity on the persisting ring between the past and the future. In other words, there are roots from the past, which are relatively prime with anticipated roots in the future. Both past and future roots are elicited bidirectionally to the same diachronically persisting ring of the present, as seeds bearing corresponding powers, capable of forming a bond in the present. This bond is articulated in the same ring by the demarcation of the primitive roots. As such, change of time amounts to the relative phase difference that is subordinate to new root primitivity. This is how, the present is synchronized with the whole constellation of consonant spectral intervals, whence the primitive roots account for temporal bonds.

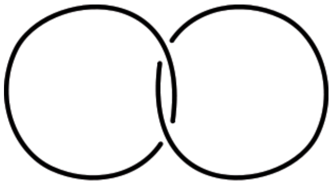
We will now think of the general case, where the same temporal diastasis does not persist diachronically, in the sense that change of time in the present cannot be accounted for in terms of a relative phase difference. This possibility already presents itself where we have two different temporal diastases that act jointly, in the sense of imposing two different periodic rules. We have to be careful to distinguish two possibilities in this setting.

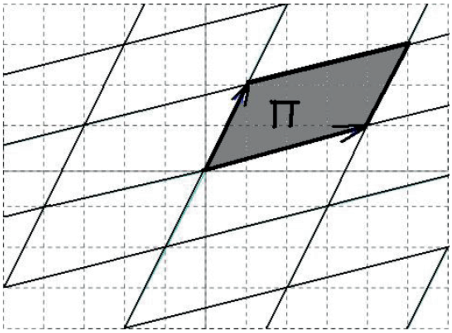
The first refers to the case that these temporal diastases are characterized by different, but directly interlinked rings in the present. The second refers to the case that these temporal diastases are unlinked in any direct way to each other, but at some present they get linked together through a third ring, such that if any of these three rings is eliminated then everything becomes totally unlinked. The latter case will present the major interest for us, since here, thinking from the view of the present, change of time is not merely a change of relative phase, but a change of circle. Additionally, this case depicts the depth of a temporal bond, and a posteriori captures the essence of a tripod based on a diachronically stable natural communication constituted by means of tripartite metaphora.

Before we study in detail the synergetic metaphora that gives rise to a change of circle, it is worth making some remarks concerning the first paradigm, where two directly linked temporal diastases are acting jointly at the same place. Recall that in the case of a single temporal diastasis, meaning a helically unfolding temporal axis, the imaginary ring, or its conformal logarithmic transfiguration to the imaginary axis of the complex plane serves to model the present on the epiphaneia. In those cases where we have another temporal diastasis directly linked with the former, then the only way that we can model it in the context of the two-dimensional epiphaneia is in terms of the orthogonal axis to the former one. These diastases are linearly independent over the real numbers and the whole complex plane tesselated by rectangles, whose orthogonal sides correspond to the periodic rules of the two diastases, i.e. their spectral measures, models the present.

In this manner, $\mathbb{Z}^2$ gives rise to a lattice on the complex plane, and we obtain instead of spectral measures on the imaginary axis, spectral measures on the integer lattice on the complex plane. The functions defined on the complex plane bearing this spectral lattice are doubly periodic, called *elliptic functions*. Analogously to the trigonometric functions culminating to the simply-periodic complex exponential function parameterizing the circle, the doubly-periodic elliptic functions parameterize two directly linked circles. In this fashion, note that complex analytically the unit circle is identified with the imaginary ring in the first case. In the second, due to the direct chain linkage, only one of the rings is thought of as imaginary. This is also clear if we think of the parametrization of the spectral lattice taking place in the complex plane bearing one imaginary and one real dimension. If we denote these two periods by $\omega_1$, and $\omega_2$, then their ratio is imaginary.

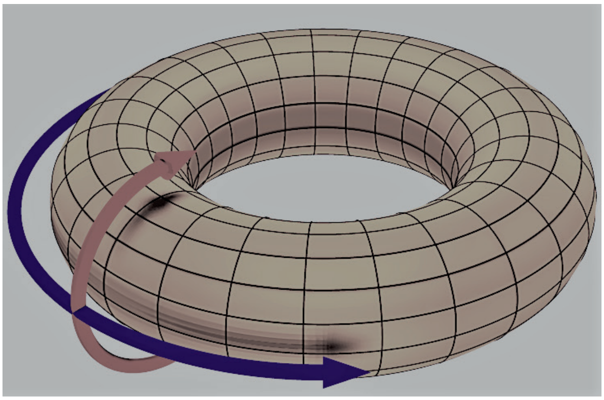
The fundamental parallelogram on the complex plane is the one with vertices $0$, $\omega_1$, $\omega_2$, $\omega_1+\omega_2$.





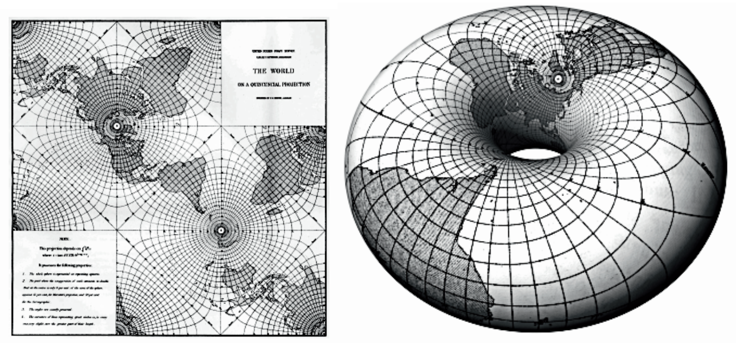

### 3.8 TORICS: THE QUINCUNCIAL PROJECTION

From a topological perspective, the above case of the two directly linked chain-like temporal diastases, descends from the torus. Topologically, we may think of them as two linked cycles on the torus constituting the homology basis of its first homology group.



Analogously to the case of a single temporal diastasis descending from the sphere, two directly linked temporal diastases descend from the torus. Note that the torus is topologically different from the sphere, since it bears the central hole in the middle, i.e. it has *topological genus* $1$ in comparison to the sphere whose genus is $0$. So once more, we may think of the topological genus temporally, as emanating from the notion of a temporal diastasis. If we think of this link in terms of the sphere, then we would need a sphere bearing four poles. This is explained complex-analytically by saying that the torus constitutes a *double branched covering* of the sphere.

The above has been conceived by Charles Sanders Peirce as a way of mapping the sphere on the epiphaneia, called the *quincuncial projection*. If we think of the above as the fundamental parallelogram on the complex plane, we see that it bears two poles whose residues cancel each other. Considering the elliptic functions determined by their values on a fundamental parallelogram as above, we realize that they can not be holomorphic, since in that case they would be constant as bounded functions; called for this purpose, *meromorphic* functions instead.



It is interesting to compare the stereographic with the quincuncial projection. The latter enfolds two copies of the sphere bearing four branch points. These can be identified with the points where the displayed axis pierces the torus. The two different projections on the epiphaneia of the complex plane are correlated as follows: Using the inverse quincuncial projection we obtain a conformal mapping of the fundamental region on the torus, which then can be projected to the Riemann sphere by means of the branched double cover of the sphere by the torus. Then, the stereographic projection of the sphere accomplishes the sought after metaphora.

### 3.9 PAIR OF NON-DIRECTLY LINKED TEMPORAL DIASTASES AND TEMPORAL BONDS

Next we consider the general case of two temporal diastases, which have no direct link to each other, but at some present they get linked together indirectly giving rise to a temporal bond. On the epiphaneia, this type of linking of two rings is possible only through a third ring, such that if any of these three rings is eliminated then all linkage is completely lost. The first important thing to realize is that a temporal bond cannot be constituted within two dimensions; an additional dimension is needed.

Alternatively, we have to consider two unlinked imaginary rings, each of which lies on its own copy of the complex plane. The feasibility of a temporal bond in three imaginary dimensions implies that two imaginary rings can be amalgamated together with respect to a third imaginary ring, otherwise they remain unlinked. Thus, a temporal bond cannot be modelled on the complex analytic plane, requiring rather the four dimensional *quaternionic* analytic setting, where three of the four axes are imaginary. In much the same way that a single imaginary axis is meaningful together with a real axis orthogonal to it, three imaginary axes are meaningful together with a real axis being orthogonal to all three of them. Note that the three imaginary axes are orthogonal to each other, but any one of them emerges from the product of the other two. As such this product is not commutative, showing that the algebraic and analytic modelling of a temporal bond is non-commutative. This means that the articulation of a temporal bond is based on non-reversible temporal actions. It is due to this irreversibility that the bond becomes stable. Since we do not intend to enter the domain of non-commutative quaternionic analysis, we call the site involving three imaginary axes a *crystalline* site.

We are going to restrict ourselves to the simplest topological rendering of a temporal bond not involving any analytic considerations. Keeping the temporal perspective, and in particular, thinking in terms of the present, change of time is not merely a change of relative phase, but a change of circle, where each circle is viewed here as a topological circle. Since the circle at present is assumed to amalgamate together the two other unlinked a priori circles, which entails the linking should take place in modular relation to the circle of the present, it is necessary to examine the conditions that make this feasible. We note that topologically no distinction pertains between the circles with respect to the temporal before and after relation that enters only by identifying a particular circle with the present, according to our previous analysis. In other words, topologically speaking, a temporal bond is threefold symmetric with respect to three circles linked in this way. The distinction comes from identifying one circle as the circle of the present, so that the other two can be temporally amalgamated together in modular relation to the marked one. The topological distinction that matters is whether two circles are directly linked or not, but even when not is the case, if they can be linked through a third one appropriately.

### 3.10 CHANGE OF TIME AS A SYNERGETIC CHANGE OF CIRCLE IN THE LIVING PRESENT

In the course of this problematics, the change of time as a change of circle from the past to the future in modular relation to the present, should be thought of as a synergetic change of circle belonging to the conceptual domain of *synectics* in the *Aristotelian* meaning of this term. At a first stage, we may conceptualize a synergetic change of circle as a higher connectivity interface, binding cohesively together the past with the future, independently of their metrical linear distance, in modular relation to the present, considered as their unity. At the event level, such a symmetric treatment of the past and the future with respect to the present is not possible. But the connectivity interface we examine currently does not refer to the level of events, where past and future stand in an asymmetrical ordered relation with respect to the present.

This interface capable of giving rise to a temporal bond mediates the metaphora of both the past and the future with respect to the present at this present. In this sense, the metaphora of the past to the future through the present via change of circle by means of a temporal bond should qualify these terms as three different circles capable of being linked together. Since the linking is meaningful in the present where a change of circle takes place, a metaphora of both the past and the future must be in play at this present. Of course, this metaphora is not in the nature of a metaphora of events that would be in any case impossible. The mental strain concerns especially the notion of the future appearing absurd from a standard perspective if not conceived through its metaphorical essence.

More precisely, one of the main functions of the human brain is to act as a metaphora from the future to the present, in the sense of gnomonically anticipating and envisioning the future. This is possible exactly because of the symmetry of a metaphora in terms of encoding and decoding bridges. Thus the vision of the future in relation to a change of circle from the past that takes place in the present is qualified topologically as the metaphora of this envisioned circle in the present. The same actually holds symmetrically for the past through the faculty of memory, which functions topologically as the metaphora of the circle of the past in the present. Consequently, memory and vision are symmetrically articulated with respect to the circle of the present, from the viewpoint of gnomonics and natural communication, although it is not obvious at all how it is possible to form a bond in the present that will allow the concomitant change of circle from the past to the future.

Since we have established, that this bond pertains to a higher connectivity interface in the present it cannot be expressed at the level of events. In other words, from the naive view of a simply-connected linearly ordered one-dimensional continuum of events a temporal bond amounts to a wormhole that can connect very distant events non-locally, something impossible given only the conceptual apparatus of, say, a *Markov chain* connecting these events. Essentially, such wormholes qualify as metrically non-local bridges of connectivity that can instantaneously bind the very far past with the very near future. Although this is paradoxical if considered at the ordered event level, the human brain utilizing mental metaphora performs it unproblematically.

The issue is that the metaphora takes place not at the level of events, but at the level of topological *germs*, or simply elicited *seeds*, of events that bear a *meteoric* nature. For this reason, the present does not play the role of a pathetic point on a line ordered by means of succession, but its role becomes energetic in the sense of binding the remembered past with the gnomonically or canonically envisioned future that precisely characterizes its invisible depth. The claim is that this notion of the present is a living one, whereas the former is a dead one. It is the same with the articulation of some given history at any present as a living or as a dead one.

In other words, the qualification of the living present is metaphorically equivalent to its ability to give rise to temporal bonds. It is clear from this discussion that this does not apply to any past and to any envisioned future in the present. Put differently, there are certain conditions that allow a change of circle in the present from a circle of the past to a circle of an anticipated future. From the standpoint that harmonics takes prior place to topological considerations, these circles may be though of as unlinked chords bearing their harmonics as elicited seeds in the present, and the issue is how they can be tuned together in the present so as to give rise to a temporal bond through which change of circle takes place.

The notion of a temporal bond is qualified as follows: First, it should not be conditioned by relations of metrical proximity of the elicited seeds from the past and the future in the present, where a change of circle can take place; Second, a temporal bond between elicited seeds from the past and the future, always bears a modular relation with the present acting as their unity. Equivalently, a temporal bond should not be thought of as a direct pair-wise gluing, but as a modular gluing, that takes place in relation to the present, and together with the present; Third, a temporal bond induces a synergetic circle change, if and only if the pertinent elicited seeds from the past and the future, in their capacity for amalgamation together in modular relation to the present, are both relatively prime with respect to the present. Equivalently, they should be neither analyzable nor localizable to any other common factors with respect to the present; Fourth, a temporal bond as a synthetic unity that takes place in the present, characterized symmetrically and bidirectionally by relative primeness, or primitivity, with respect to the present, specifies the syntaxis of a compounded temporal unfolding encapsulated by synergetic circle change in the present; Fifth, a temporal bond should be a Tripodal link of least action, since it corresponds to an inseparable tripartite correlation, which cannot be analyzed to any pairwise correlations.

It is crucial to make some further remarks in relation to the property of primitivity, or relative primeness, required bidirectionally from the elicited seeds in their capacity to enter into a modular gluing relation with the present, characteristic of a temporal bond. This notion is analogous to the one in integer modular algebra conceived by Gauss, where the absolute notion of an integer prime number becomes relativized with respect to a modulus. The significance of this generalization, in the case of integer modular systems, is that any integer can assume the role of a prime, but only in relation to another integer acting as a modulus. The idea here is that the quality of relative primeness in relation to the present is crucial for the realization of a temporal bond. In more intuitive terms, the assertion is that, relatively to the present, a seed from the past becomes spectrally spontaneously recognizable, and thus, capable of being elicited in the present, without any possible factorization through anything else. The same, symmetrically, holds for an envisioned seed in the anticipated future in its capacity to enter into a temporal bond with a seed from the past in the present, which acts as the modulus of unity for this Tripodal link.

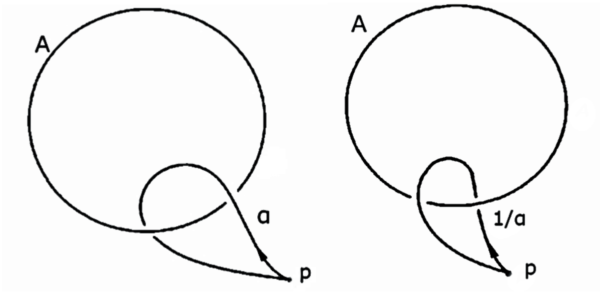
It is instructive to recall that the realization of such a temporal connectivity interface becomes effective only on the condition of topological non-degeneracy of the temporal unfolding between the past and the future in the present, acting as their modulus of unity.

Equivalently, the different winding stairs of the helices corresponding to the a priori unlinked temporal diastases should be spectrally distinguishable in the present. Topologically, this spectral distinguishability amounts to quantized action. In other words, the winding stairs indexed by the group of the integers, count quanta of action. Again, spectral distinguishability should be always relativized with respect to the pertinent present, where a temporal bond is instantiated as a least action solution to the indirect linking of the past with the future.

The most important consequence of relative primeness conceived this way, is that the pertinent seeds of the past and the future entering into a temporal bond, become, both symmetrically, relationally inverse with respect to the present, and relationally conjugate with respect to each other, in the present. In turn, the present is qualified through an Archimedean fulcrum relative to these seeds or, more precisely, relative to their respective a priori unlinked topological circles. Thus, seeds from the past and the future become eliciting seeds in their power to enter into a temporal bond in the present, if and only if they can be leveraged metaphorically to the present, relationally to each other, with respect to the fulcrum of the present.

### 3.12 RESONANCE OF TEMPORAL CHORDS: THE TRIPODAL LINK OF A BOND

If we consider a seed either in the past or in the future, it becomes spectrally spontaneously recognizable from the fulcrum in the present, i.e. not factorizable through any other simpler common factor, by means of a loop, and more precisely, a simple tame closed curve, which is based at the fulcrum. This means that it starts and ends at the fulcrum, and passes through the topological circle corresponding to its temporal diastasis. Since we refer to a seed, it is better to consider the whole equivalence class of such loops that can be continuously transfigured to each other. It is enough to recognize a single representative of this class by means of a based loop at the fulcrum. If we denote the relevant topological circle by $A$, then a based loop at the fulcrum passing through $A$, may admit two distinct orientations: If the loop passes through $A$ with direction away from the fulcrum $(+)$, it is denoted by $\alpha$, whereas if it passes with direction toward the fulcrum $(-)$, it is denoted by $\alpha^{-1}$. Therefore, oriented loops based at the fulcrum of the present and crossing unlinked topological circles encapsulate the reflexive principle of seed recognition from the past and the future, depending on their orientation.




In other words, reflexive recognition of a seed in the past or in the future, in its power to enter into a temporal bond in the present, is not enough for the establishment of the bond. What is required additionally is the metaphoric leveraging of these seeds to the fulcrum of the present by utilizing the property of relative primeness, so that they become elicited seeds in the present capable of modular amalgamation. This condition leads to the notion of a temporal chord in the present, through which the expression of a temporal bond becomes explicit.

Consider a recognized seed from the past, identified either with the based oriented loop at the fulcrum, $\alpha^{+1} := \alpha$, or with $\alpha^{-1}$, by means of crossing the topological circle $A$, depending on the orientation. Analogously, consider a recognized seed in the future, identified either with the based oriented loop at the fulcrum, $\beta^{-1}$, or with $\beta$, by means of crossing the topological circle $B$, depending again on the orientation. For instance, if $\alpha$ and $\beta^{-1}$ are recognized, they both become eligible to be elicited seeds in the present by metaphoric leveraging with respect to the fulcrum. For this purpose, they should be relationally conjugate to each other by the requirement of relative primeness with respect to the present. This means that $\beta$ and $\beta^{-1}$ should play the role of bidirectional bridges for the leveraging of $\alpha$, and also symmetrically that $\alpha$ and $\alpha^{-1}$ should play the role of bidirectional bridges for the leveraging of $\beta^{-1}$. It is elicited seeds that give rise to temporal chords in the present. Therefore, a temporal chord in the present is enunciated by interpolating a recognized seed from the past, for example, $\alpha$, between the bridges $\beta$ and $\beta^{-1}$, i.e. by metaphora through $\beta$ and $\beta^{-1}$, which leverages the seed $\alpha$ with respect to the fulcrum, e.g. $\beta\alpha\beta^{-1}$.

The significance of temporal chords in the present is that they can resonate together harmonically in the present. More precisely, a temporal chord emanating from a recognized seed in the past can be fused together with a temporal chord from a recognized seed in the future by harmonic resonance in the present. The latter is instantiated by means of new topological circle in the present capable of amalgamating together the circles $A$ and $B$ in a non-pairwise fashion.

For this purpose, we consider a recognized seed from the past, identified with the based oriented loop at the fulcrum, $\alpha$, by means of crossing the topological circle $A$ in the prescribed orientation, and symmetrically, a recognized seed from the future, i.e. an envisioned seed in the present, identified with the based oriented loop at the fulcrum, $\beta$, by means of crossing the topological circle $B$ also in its respective prescribed orientation. Note that $A$ and $B$ are a priori unlinked corresponding to two different temporal diastases. These oriented fulcrum-based loops $\alpha$ and $\beta$ can be composed, either in the order $\alpha\beta$, or in the order $\beta\alpha$, and these compositions are non-commutative. In this manner, composed actions of seed recognition from unlinked temporal diastases are order irreversible with respect to the fulcrum.

Let's consider the composition in the order $\alpha\beta$. The basic idea is to extend this composition in consecutive stages and express it in terms of temporal chords with respect to the fulcrum. If we adjoin by composition $\alpha^{-1}$ to $\alpha\beta$, we obtain the temporal chord $\alpha\beta\alpha^{-1}$, which amounts to the metaphora of $\beta$ with respect to the fulcrum, utilizing the bridges $\alpha$ and $\alpha^{-1}$ for this recognized seed in the future. Next, we adjoin $\beta^{-1}$ to the temporal chord $\alpha\beta\alpha^{-1}$, to obtain $\alpha\beta\alpha^{-1}\beta^{-1}$, which can be read either as the composition of the temporal chord $\alpha\beta\alpha^{-1}$ with $\beta^{-1}$ or as the composition of $\alpha$ with the temporal chord $\beta\alpha^{-1}\beta^{-1}$, due to the associativity property of non-commutative composition. In the same way, by continuing the process of adjoining as above, the objective is to generate a cycle based at the fulcrum. A cycle is generated when the process of adjoining ends with the composition $\alpha\beta$ that has been utilized at the initial stage. A cycle based at the fulcrum is generated by the resonance of a temporal chord from a recognized seed in the past with a temporal chord from a recognized seed in the future, as follows:

$$\alpha\beta \to \alpha\beta\alpha^{-1} \to \alpha\beta\alpha^{-1}\beta^{-1} \to \alpha\beta\alpha^{-1}\beta^{-1}\alpha \to \alpha\beta\alpha^{-1}\beta^{-1}\alpha\beta.$$

In the above process, the cycle generated is given by

$$C = \alpha\beta\alpha^{-1}\beta^{-1},$$

since starting from the ordered non-commutative composition $\alpha\beta$ we ended at:

$$(\alpha\beta\alpha^{-1}\beta^{-1})(\alpha\beta) = C(\alpha\beta).$$

The cycle

$$C = (\alpha\beta\alpha^{-1}\beta^{-1}) := [\alpha, \beta],$$

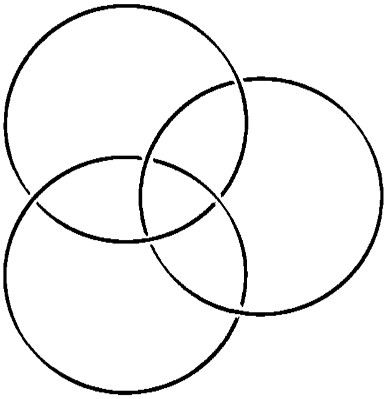
i.e. the commutator of $\alpha$ and $\beta$, is generated by the resonance of the temporal chord $\alpha\beta\alpha^{-1}$ with the temporal chord $\beta\alpha^{-1}\beta^{-1}$. Therefore, the new topological circle $C$ leads to the indirect gluing of the topological circles $A$ and $B$ in modular relation to the present, identified thereby, as the circle of the present. It is only through the Tripodal link instantiated in the present by $C$ that a synergetic change of cycle from $A$ to $B$ takes place.

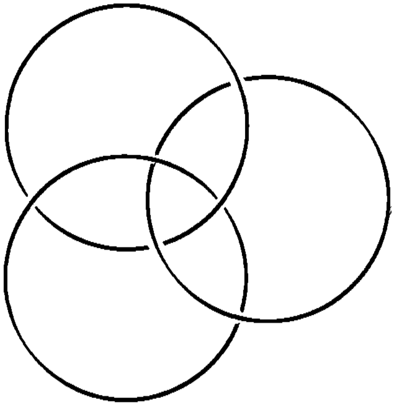
The topological circle $C = (\alpha\beta\alpha^{-1}\beta^{-1}) := [\alpha, \beta]$, involves four crossings of the circles $A$ and $B$, more concretely, two of the circle $A$ and two of the circle $B$, with opposite orientations and in an alternating order. Notice that the structure of $C$ does not depend on what we consider as an initial composition, like $(\alpha\beta)$ in the case presented. If we consider any other initial composition from all possible ones, we will again arrive at a cycle of the same structure, i.e. to a resonance of a "temporal chord" from the "past" with a "temporal chord" in the "future" with respect to the fulcrum. In other terms, the compositional structure of the topological circle $C$, amalgamating $A$ and $B$ in modular relation with respect to the present, is the invariant of resonance between a temporal chord from the past with a temporal chord from the future, that qualifies change of time as a synergetic change of cycle from $A$ to $B$ in the present, and giving rise to a temporal bond.

The crucial observation is that a temporal bond induces a particular type of topological linking of the cycles $A$, $B$ and $C$, which we call a *Tripodal topological link*. Equivalently, a synergetic change of cycle from $A$ to $B$ in the present is tantamount to a Tripodal link of the cycles $A$, $B$ and $C$, qualified by the property that if any one of the cycles is removed from this link the remaining two come completely apart.

Topologically, a Tripodal link may be simply thought of as an interlocking family of three loops, such that if any one of them is cut, then the remaining two become completely unlinked. Each loop is considered a tame closed curve. The property of tameness means that the closed curves considered can be deformed continuously and without self-intersections into polygonal curves, which are those formed by a finite collection of straight-line segments.

Moreover, a loop, as a topological object, discloses the following properties: First, a loop is not separated into two pieces by cutting it at a point, which is rather achieved by cutting it at two points; Second, a loop is an intrinsically one-dimensional object though of as a figure in three dimensions; Third, a loop is bounded, i.e. it is contained in some sphere of sufficiently large radius. Moreover, a loop is called *knotted* if it cannot be continuously deformed into a topological circle in three dimensions without self-intersection. Therefore, each one of the three interlocking loops of the Tripodal link should be considered as an *unknotted* tame closed curve. We refer to them simply as loops keeping in mind that each one of them is unknotted. To sum up, in terms of loops, a Tripodal link is depicted as the configuration displayed on the left below, which is to be contrasted with a different type of configuration consisting of three interconnected loops displayed on the right.

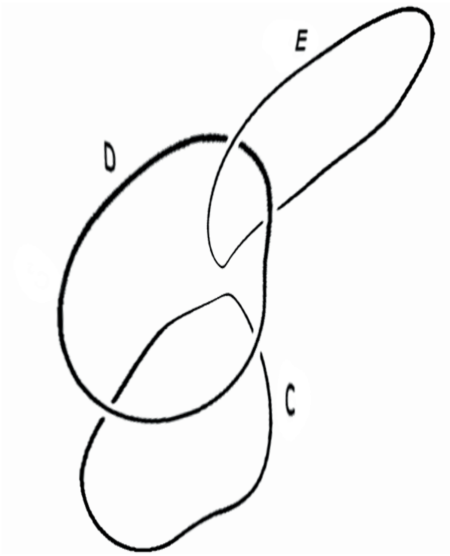




The Tripodal link configuration of loops on the left is such that if any of the loops is cut at a point and removed, then the remaining two loops become completely unlinked. In contrast, the configuration on the right is such that each loop actually links each of the other two.

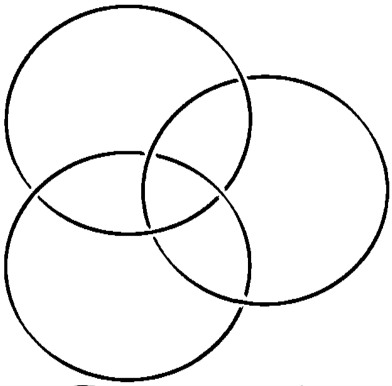
The topological notion of a link pertains to the connectivity among a collection of loops. In general, an $N$-link is a collection of $N$ loops in three dimensions, where $N$ is a natural number. Regarding the connectivity of a collection of $N$ loops, the crucial property is that of splittability of the corresponding $N$-link. We say that an $N$-link is *splittable* if it can be deformed continuously in three dimensions, such that part of the link lies within $B$ and the rest of the link lies within $C$, where $B$, $C$ denote mutually exclusive solid spheres (balls) in three dimensions.

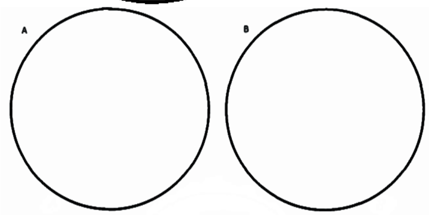
Intuitively, the property of splittability of an $N$-link means that the link can come at least partly apart without cutting. Complete splittability means that the link can come completely apart without cutting. On the other side, non-splittability means that not even one of the involved loops, or any pair of them, or any combination of them, can be separated from the rest without cutting. As an illustration, we consider the following $3$-link:




The above 3-link consists of three loops, denoted by $C$, $D$ and $E$. Clearly, this is a splittable 3-link, which is not completely splittable. As can easily be seen in the above figure, the loops $D$ and $E$ cannot be split apart without cutting. Notwithstanding this fact, it is a splittable 3-link because the loop $C$ can be separated from the rest without cutting. Thus, the above 3-link can come at least partly apart, and therefore is splittable.

The property of splittability of a topological link, is adequate to completely characterize the Tripodal link. First, the Tripodal link is a 3-link, since it is consists of 3 loops. Second, the connectivity of this 3-link in terms of the splittability property, implies that the Tripodal link is a non-splittable 3-link, such that every 2-sublink of this 3-link is completely splittable. More precisely, it is a non-splittable 3-link because not even one of the three loops, or any pair of them, can be separated from the rest without cutting. A 2-sublink is simply any sub-collection of two loops obtained by removing the loop that does not belong to this sub-collection. Since, the Tripodal link is characterized by the property that if we erase any one of the three indirectly interlocking loops, then the remaining two loops become unlinked, it follows that every 2-sublink of the initial 3-link is completely splittable, according to the figure below:

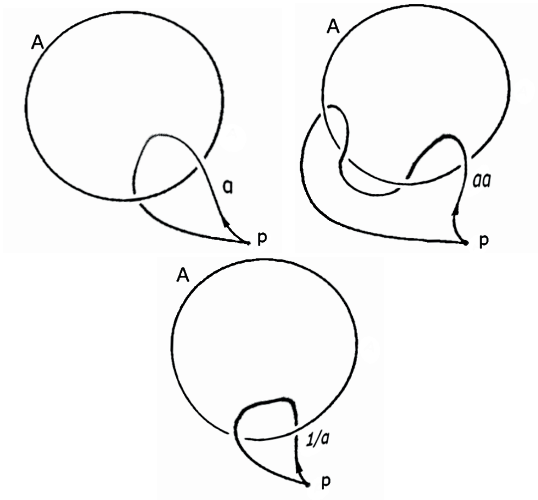





### 3.14 THE FREE NON-ABELIAN GROUP STRUCTURE OF ORIENTED BASED LOOPS

Our objective is to discover an appropriate algebraic structure capable of encoding the connectivity type of the Tripodal link, that is to say we seek a metaphora from the domain of topology to the domain of algebra, which will enable us to disclose the essence of a Tripodal link.

First, we consider an unknotted tame closed curve in three dimensions. Since any such curve can be continuously deformed to a topological circle it is enough to consider such a circle in three dimensions, denoted by $A$. Second, we think of a based oriented loop which may pass through this circle a finite number of times, each one with a prescribed orientation. A based loop means simply that it starts and ends at a fixed point $p$. The orientation of the loop is defined as follows: If it passes through the circle one time with direction away from $p$ it is denoted by $\alpha^{+1}$, whereas if it passes one time with direction towards $p$ it is denoted by $\alpha^{-1}$. Thus, in the algebraic symbols of the generic type "$\chi$" we encode: First, the passage or not of a based loop through a circle $A$, which qualifies or not the naming of the loop by the corresponding symbol $\alpha$. Second, the number of times that this loop passes through the circle $A$, which is encoded as a power of the symbol $\alpha$. Third, the orientation of the loop with respect to $p$, which is encoded by a “$+$” sign if a passage through the circle takes place away from $p$ and by a “$-$” sign if a passage takes place towards $p$.

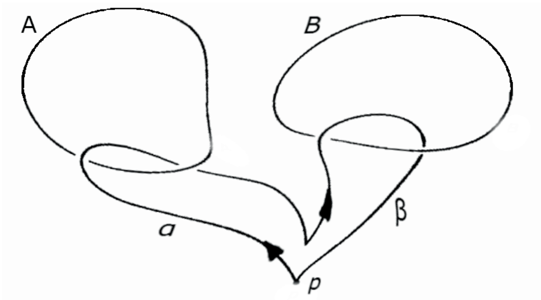



The first figure from the left depicts a loop based at $p$, beginning at $p$, then passing through the circle $A$ once directed away from $p$, then curving around the circle $A$, and finally returning to $p$. According to the above, this loop in relation to the circle $A$ should be denoted by $\alpha^{+1}$, which we write simply as $\alpha$. Note that any other loop with the same behavior can be continuously deformed to the loop $\alpha$. Thus, the algebraic symbol “$\alpha$” actually denotes a partition block, i.e. the equivalence class $[\alpha]$ of all loops of the kind $\alpha$, passing through the circle $A$ once with the prescribed orientation. Any loop in the block $[\alpha]$ can be continuously deformed to an equivalent one in the same class. Taking into account this remark, we still keep using the symbol $\alpha$ as above, where $\alpha$ is thought of as a representative of the equivalence class $[\alpha]$.

In the middle figure, we have a loop based at $p$, such that; it starts at $p$, then passes through the circle $A$ twice directed away from $p$, then it curves around the circle $A$, and finally returns to $p$. This loop, in relation to the circle $A$, should be denoted by $\alpha^{+2} = \alpha \circ \alpha$, which we write simply as $\alpha^2$.

In the last figure from the left, we find a third loop based at $p$, such that; it starts at $p$, then curves around the circle $A$, then passes through the circle $A$ once directed towards $p$, and finally it returns to $p$. This loop, in relation to the circle $A$, should be denoted by $\alpha^{-1} = 1/\alpha$.

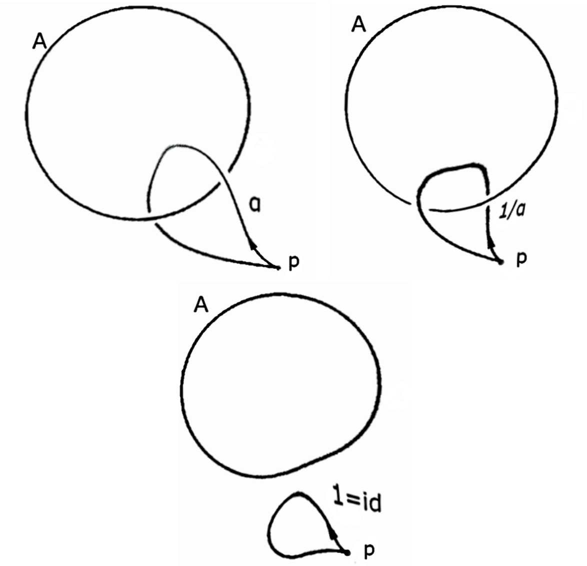
Next, we need to consider the composition of based oriented loops of the generic type "$\chi$" in relation to circles of the generic type "$X$". The composition of two loops is viable if both of the loops are based on the same point $p$. Then, the composed based oriented loop should also be a loop of the same generic type in relation to the two circles of the composing ones.




In more detail, let us consider two based oriented loops, which are both based at the same point $p$. Taking into account the orientations, we denote the first loop by $\alpha$ (in relation to the circle $A$) and the second loop by $\beta$ (in relation to the circle $B$). Then, we can define their composition denoted by $\alpha \circ \beta$ respecting the order of tracing the loops, viz. we first trace $\alpha$, and then we trace $\beta$. Thus, the rule of composition produces a based oriented loop $\alpha \circ \beta$ in relation to the circles $A$ and $B$, which is interpreted as follows: It starts at $p$, then passes through the circle $A$ once directed away from $p$, then it passes through the circle $B$ once directed away from $p$, and finally returns to $p$. We note that it is allowed topologically to remove the end of $\alpha$ and the beginning of $\beta$ from the base point $p$, and then join them together at a nearby point. We interpret the composition rule $\alpha \circ \beta$ as the multiplicative product of the oriented loops $\alpha$ and $\beta$ based at the same point $p$, which we denote simply as $\alpha\beta$. This establishes the closure of the elements of the generic type "$\chi$" under multiplication as above.

We note also that the above multiplicative product is not commutative, i.e. $\alpha\beta \neq \beta\alpha$. This is due to the fact that the rule of composition of based oriented loops at a point is order dependent, such that $\alpha \circ \beta \neq \beta \circ \alpha$. The underlying topological reason is that the based oriented loop $\alpha \circ \beta$ cannot be continuously deformed to the based oriented loop $\beta \circ \alpha$, meaning that the order-dependence of the composition rule makes the corresponding multiplicative product non-commutative. Notwithstanding this fact, multiplication is an associative operation, i.e. $(\alpha \circ \beta) \circ \gamma = \alpha \circ (\beta \circ \gamma)$, so we skip the parentheses in multiple compositions of based oriented loops.

Next we look for the existence of a neutral element, and inverses with respect to this product operation. Clearly, for each based oriented loop $\alpha$, there exists the inverse loop $\alpha^{-1}$, such that both compositions $\alpha \circ \alpha^{-1}$ and $\alpha^{-1} \circ \alpha$ give as their multiplication product the loop based at the same point that does not pass through any circle at all. Thus, we call the latter loop the neutral element, or equivalently the multiplicative identity $1$, such that $\alpha\alpha^{-1} = \alpha^{-1}\alpha = 1$. We verify immediately that $1\alpha = \alpha 1 = \alpha$, where the equality sign is interpreted as an equivalence of based oriented loops under continuous deformation, according to the preceding.



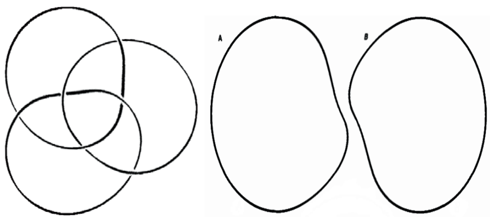

We conclude that the set of symbols of the generic type "$\chi$" representing based oriented loops in relation to topological circles $X$, endowed with the non-commutative multiplicative product expressing the ordered composition of loops based at the same point, bears the algebraic structure of a non-commutative group, denoted by $\Theta$. Since this group is generated by two non-commuting elements, and there are no further relations imposed on its algebraic structure, $\Theta$ is identified with the non-abelian group in two generators.

### 3.15 FROM TOPOLOGY TO ALGEBRA: ENCODING-DECODING THE TRIPODAL LINK

Using the multiplication operation we may form any permissible string of symbols in the group $\Theta$, which can be shortened into an irreducible form by using only the standard group-theoretic relations $\alpha\alpha^{-1} = \alpha^{-1}\alpha = 1$, $1\alpha = \alpha 1 = \alpha$, $\alpha\alpha = \alpha^2$ and so on. Two arbitrary strings of symbols, i.e. words in the group $\Theta$, are equal if they can be brought into the same irreducible form in $\Theta$, meaning that the corresponding product loops, are equivalent under continuous deformation.

The property of irreducibility of a string of symbols in the group $\Theta$, which amounts to the irreducibility of a product loop in $\Theta$, is the leading idea for the algebraic encoding of the Tripodal link in terms of the group structure of $\Theta$. Note that any multiplicative concatenation of symbols in the group $\Theta$, when translated in product loop terms is always thought of in relation to corresponding circles, forming the collection of all circles that a product loop is associated with.

We proceed by investigating what kind of topological information the property of irreducibility of a string of symbols in the group $\Theta$ encodes in algebraic terms. We will show that algebraic irreducibility encodes the topological property of non-splittability of a link. Bearing in mind that a link corresponds generally to a collection of loops. The topological connectivity of a link is expressed by the property of splittability. In particular, the Tripodal link is a non-splittable 3-link, such that every 2-sublink formed by erasing one of the three loops of this 3-link is completely splittable.




The idea is to encode the Tripodal link group-theoretically in terms of an appropriate product loop in the group $\Theta$, which is associated with two circles $A$ and $B$. Note that erasing this hypothetical product loop would leave the two circles unlinked, since that removal results in a completely splittable 2-sublink. In algebraic terms, this situation depicted by the above figure on the right is described by the neutral element, i.e. the identity $1$ of the group $\Theta$. Hence, complete splittability of this 2-sublink is encoded by the identity $1$ of $\Theta$. For reasons of symmetry, the same behavior appears if we erase any of the circles $A$ or $B$, since the neutral element of $\Theta$ is unique. Nevertheless, in order to prove it algebraically we need the explicit formula describing the product loop in the terms of elements of $\Theta$.

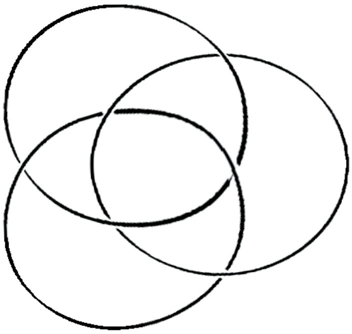
At the next step, since the product loop should be expressed in relation to the circles $A$ and $B$, it is necessary to involve at least a string of symbols consisting of $\alpha$, $\beta$ and their group inverses $\alpha^{-1}$, $\beta^{-1}$ in some specific order, which does not involve any consecutive appearance of $\alpha\alpha^{-1}$, $\alpha^{-1}\alpha$, $\beta\beta^{-1}$, $\beta^{-1}\beta$, since all of the latter are reduced to $1$. The reason for the appearance of both $\alpha$, $\beta$, and their group inverses $\alpha^{-1}$, $\beta^{-1}$, lies in our expectation that erasing any of the circles $A$ or $B$ would collapse the product loop to the neutral $1$. This is the desired case referring to the Tripodal link because every 2-sublink is completely splittable. If the circle $A$ is erased, for example, then in the sought after product loop formula both instances of $\alpha$ and $\alpha^{-1}$ should be deleted, since both $\alpha$ and $\alpha^{-1}$ have a meaning with respect to $A$. The same holds symmetrically for $\beta$ and $\beta^{-1}$ in relation to the circle $B$. Finally, since the fact that every 2-sublink of the Tripodal link is completely splittable is encoded algebraically by reducibility to the neutral element of $\Theta$, the requirement is that the non-splittability of the total 3-link should be encoded by the irreducibility of the product loop formula.

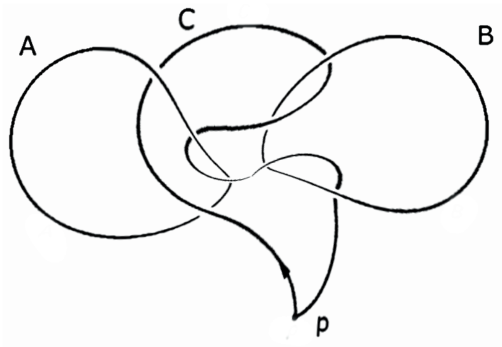
We conclude that only one combination of symbols exists that fulfills our requirements, namely:

$$\gamma = \alpha\beta^{-1}\alpha^{-1}\beta.$$

Thus, the irreducible formula $\alpha\beta^{-1}\alpha^{-1}\beta$ represents the loop $\gamma$ as a product loop composed by the ordered composition of the four based oriented loops $\alpha \circ \beta^{-1} \circ \alpha^{-1} \circ \beta$. We call the product loop $\gamma$ the *"Tripodal loop"* and the formula or multiplicative string $\alpha\beta^{-1}\alpha^{-1}\beta$ the

"Tripodal loop formula". The algebraic irreducibility of $\alpha\beta^{-1}\alpha^{-1}\beta$ in the group $\Theta$ encodes the non-splittability of the Tripodal link. Deletion of both $\alpha$ and $\alpha^{-1}$ (corresponding to removal of the circle $A$) reduces the formula to the identity $1$ (and the same happens symmetrically for both $\beta$ and $\beta^{-1}$ in relation to the circle $B$). Thus, every 2-sublink of the Tripodal 3-link is completely splittable.






In the above figure, we imagine that we continuously pull apart the two upper topological circles of the Tripodal link displayed on the left. Then, we obtain the configuration on the right, which is interpreted in group-theoretic terms as a product loop, that is, the irreducible "Tripodal loop" associated with these two circles. Hence, we have a geometric representation of the "Tripodal loop formula". The algebraic irreducibility of this formula $\alpha\beta^{-1}\alpha^{-1}\beta$ in the group $\Theta$ encodes the non-splittability of the 3-link in the Tripodal topological configuration. If we cut the "Tripodal loop", or remove any of the circles $A$ or $B$, we obtain a completely splittable 2-sublink. The "Tripodal loop formula" reads as follows: First, it passes away from $p$ through $A$ (represented by $\alpha$); Second it passes towards $p$ through $B$ (represented by $\beta^{-1}$); Third it passes again towards $p$ through $A$ (represented by $\alpha^{-1}$); Fourth, it passes away from $p$ through $B$ (represented by $\beta$).

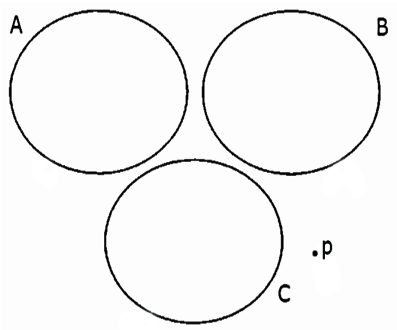
Thus, the topological information of the Tripodal link has been completely encoded in terms of the algebraic structure of the non-commutative multiplicative group $\Theta$. In this way, we have obtained a bi-directional bridge between the topological connectivity model of the "Tripodal rings" expressed in terms of links and the algebraic algorithmic information model expressed in terms of the structure of the group $\Theta$. This is of fundamental significance because it allows the translation of a hard topological problem into algebraic terms, which becomes the encoding of the problem in group-theoretic terms, where it can be solved quite easily, and then inversely, the decoding of this solution into topological terms, which provides the solution of the topological problem posed initially. An illustration of this powerful method, which generalizes the case of the "Tripodal link" to higher non-splittable links whose all sublinks are completely splittable, in analogy to the Tripodal case, will be presented as we progress.

### 3.16 HIGHER TEMPORAL BONDS FROM THE TRIPODAL LINK

It is instructive to clarify that the algebraic structure of the group $\Theta$ is not only restricted to the typical Tripodal configuration, explained in the previous section, but it can encode the topological information of higher links since we are free to construct product loops composed of any number of factors according to the composition rule we have defined. This presents the challenge of using the group $\Theta$ in order to solve the harder topological problem of identifying a non-splittable 4-link, all of whose 3-sublinks are completely splittable. Clearly, this problem constitutes the immediate higher generalization of the Tripodal link, which involves a non-splittable 3-link for which all 2-sublinks are completely splittable. The main interest in such a generalization lies in the intuition that the Tripodal link acts as a kind of a building block for the substantiation of higher order links of this type.

The method we will follow in order to attack this topological problem is the use of the bi-directional bridge between topology and algebra we have established in this context. Namely, we will translate the problem in terms of the algebraic structure of the group $\Theta$, we will try to solve it in group-theoretic terms, and then decode the solution back into topological terms. Intuitively, the notion of a link involves the gluing conditions among its constituents. It is precisely these gluing conditions that are expressed algebraically in terms of the group $\Theta$, as the fundamental case of the Tripodal link has revealed by means of the "Tripodal loop formula" $\gamma = \alpha\beta^{-1}\alpha^{-1}\beta$ in relation to the circles $A$ and $B$.

The starting point is the analogous one to the standard Tripodal link case. Namely, since all 3-sublinks of the sought after non-splittable 4-link are completely splittable we will consider three circles $A$, $B$, $C$ and look for a product loop composed of the products of $\alpha$, $\beta$, $\gamma$ and their group inverses $\alpha^{-1}$, $\beta^{-1}$, $\gamma^{-1}$, in some specific order, which does not involve any consecutive appearance of $\alpha\alpha^{-1}$, $\alpha^{-1}\alpha$, $\beta\beta^{-1}$, $\beta^{-1}\beta$, $\gamma\gamma^{-1}$, $\gamma^{-1}\gamma$, because all of them are reduced to the identity $1$. The crucial point again is that the product loop formula should reduce to $1$ in the group $\Theta$ in case of removal of any of the circles $A$, $B$, or $C$, which is encoded algebraically by the deletion of all instances of both $\alpha$, $\alpha^{-1}$, or $\beta$, $\beta^{-1}$, or $\gamma$, $\gamma^{-1}$, which follows whenever $A$, or $B$ or $C$ respectively are erased. This is again the algebraic encoding of the fact that every 3-sublink of the total non-splittable 4-link should be completely splittable. Clearly, the non-splittability of the 4-link is again encoded by means of irreducibility of the product formula describing this 4-link.




Algebraically, this problem can be solved quite easily. The most elegant solution, which also trivializes the algebraic encoding of even higher links of this type, is to use the Tripodal link, viz. the algebraic "Tripodal loop formula" $\alpha\beta^{-1}\alpha^{-1}\beta$ in the group $\Theta$ as a building block and iterate it self-referentially. We will explain how this works for the case at issue. First, by inspecting the "Tripodal loop formula" $\alpha\beta^{-1}\alpha^{-1}\beta$ we realize that it can be written as the commutator in the group $\Theta$, that is defined as follows

$$[\alpha,\beta^{-1}]=\alpha\beta^{-1}\alpha^{-1}\beta.$$

This means that the commutator $[\alpha,\beta^{-1}]$ of the elements $\alpha$ and $\beta^{-1}$ in the group $\Theta$ producing the "Tripodal loop formula" encodes algebraically both the gluing condition of the non-splittable 3-link as well as of the completely splittability of all 2-sublinks, according to the preceding analysis. We may also re-define the element $\beta^{-1}$ as b, viz. $\beta^{-1}:=b$ in the group $\Theta$ in order to obtain the commutator:

$$[\alpha,b]=\alpha b\alpha^{-1}b^{-1}$$

in the group $\Theta$ equivalently. Thus, the idea of using the Tripodal link as a building block for analogous links of a higher type means employing the group commutator iteratively as an encoding device for these higher links of the same type. Therefore, in the case of a total non-splittable 4-link all 3-sublinks of which are completely splittable that involves the gluing of the three circles $A$, $B$ and $C$ of the above figure by a "higher Tripodal loop" we proceed as follows:

First, we glue the circles $A$ and $B$ by the standard "Tripodal loop" and then we glue this product analogically with $C$. Algebraically speaking, the first step is simply the commutator $\xi=[\alpha,b]=\alpha b\alpha^{-1}b^{-1}$. The first iteration of this procedure, which involves the gluing of the product $\xi$ with $\gamma$ (in relation to the circle $C$), reads simply as the commutator of $\xi$ with $\gamma$. We conclude that a "higher Tripodal loop" that solves the problem is given in the structural terms of the group $\Theta$ simply as follows:

$$\delta=[\xi,\gamma]=[[\alpha,b],\gamma].$$

If we expand this formula, by using the definition of the group commutator as well as the group theoretic relation

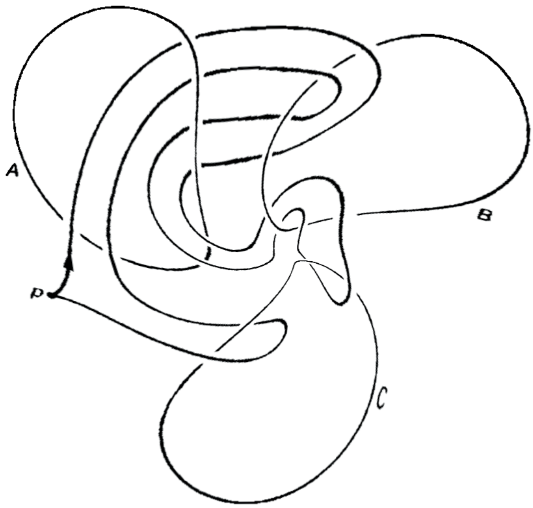
$$(\chi\psi)^{-1}=\psi^{-1}\chi^{-1},$$

where $\chi$, $\psi$ may stand for arbitrary strings of elements of the group $\Theta$, we obtain the following unfolded expression for the "higher Tripodal loop formula":

$$\delta=[\xi,\gamma]=[[\alpha,b],\gamma]=\{\alpha b\alpha^{-1}b^{-1}\}\gamma\{b\alpha b^{-1}\alpha^{-1}\}\gamma^{-1}.$$

From the above expanded "higher Tripodal loop formula" it also becomes clear how the Tripodal link becomes a building block via terms of the form $\lambda\mu\lambda^{-1}\mu^{-1}=[\lambda,\mu]$ for expressing higher order links of the Tripodal type. We can also see that deletion of all incidences of any of the symbols (which involves the simultaneous deletion of the inverse symbol as well, as we have seen) reduces the formula to the identity $1$ in the group $\Theta$.

As a final step, we decode the obtained algebraic solution back into topological terms by using the inverse bridge, and the obtained topological solution of the problem of finding a non-splittable 4-link whose all 3-sublinks are completely splittable by means of "Tripodal building blocks" is illustrated as follows:




We conclude with the observation that although the topological solution of the problem is quite hard to obtain in a straightforward manner, as evidenced by the above figure, the same problem can be solved quite easily by using the algebraic structure of the group $\Theta$, and in particular, the notion of the group commutator and its iterations. It is a remarkable fact that the Tripodal link is encoded in terms of the commutator of $\Theta$. In this way, the Tripodal link can be efficiently used as a building block for the encoding of higher-order links of the type described above, by iterating the formation of commutators for product loops.

## 3.17 DEPTH OF HIGHER TEMPORAL BONDS: NESTED STACKING OF TRIPODAL LINKS

In the previous Section we proposed the idea of using the Tripodal link as a building block for analogous links of a higher type by making higher order iterations of the group $\Theta$ commutator. We have explained already how this method works in the case of a total non-splittable 4-link all 3-sublinks of which are completely splittable. The crucial insight is that the group commutator acts as an encoding device for these higher links of the same type in two ways: First, the commutator provides the gluing scheme of link-formation by means of "Tripodal loops". Second, due to fact that deletion of all incidences of any of the involved symbols reduces the commutator to the identity $1$ in the group $\Theta$, the commutator also encodes the information of complete splittability of any remaining sublink after removing any of the constituents of the total non-splittable link.

In order to proceed more efficiently, we need to systematize our terminology as follows: The notion of the commutator of the simple oriented based loops $a$, $b$, that is $[a,b]$, is used as synonymous to the algebraic "Tripodal loop formula" in the group $\Theta$ and it is decoded in topological terms as the concept of a Tripodal link, equivalently identified as a Borromean link. We denote the latter by $\Sigma(3,2)$ meaning that it is a total non-splittable 3-link all 2-sublinks of which are completely splittable. In this way, the symbol $\Sigma(4,3)$ denotes a total non-splittable 4-link all 3-sublinks of which are completely splittable.

By induction, the symbol $\Sigma(N,N-1)$, where $N \geq 3$, denotes a total non-splittable $N$-link all $(N-1)$-sublinks of which are completely splittable. We have shown that a $\Sigma(4,3)$ link can be constructed in terms of the "Tripodal link building block" simply by one iteration of the commutator formation. This means that starting with three symbols $a$, $b$, $c$, we first glue a with b together by means of the commutator $[a,b]$, and then we glue their glued product $[a,b]$ with $c$ to obtain the stacked commutator $[[a,b],c]$. This final glued product gives the required fourth symbol in the group $\Theta_2$, which decodes topologically as a $\Sigma(4,3)$ link. In an analogous manner, by iterating the commutator formation twice, starting with four symbols $a$, $b$, $c$, $d$, we obtain a $\Sigma(5,4)$ link. The same process can be clearly repeated inductively, so that we finally can construct any $\Sigma(N,N-1)$ link by means of Tripodal building blocks, or more precisely, "Tripodal connectivity units", where $N \geq 3$. We may summarize this process in the following table:

| | | | | |
|---|---|---|---|---|
| Borromean Link | Σ(3,2) | [a, b] | Gluing of a with b | 3-link |
| 1st Iteration | Σ(4,3) | [[a, b], c] | Gluing of [a, b] with c | 4-link |
| 2nd Iteration | Σ(5,4) | [[[a, b], c], d] | Gluing of [[a, b], c] with d | 5-link |
| By Induction N≥3 | Σ(N,N-1) | Repeat process with (N-1)-symbols. | Gluing via commutator stacking | N-link |

We note that the process of iterating the commutator formation in the group $\Theta$, so as to obtain any link of the form $\Sigma(N, N-1)$, can be realized as an algorithmic procedure of commutator stacking in consecutive nested levels. Semantically, this procedure may be thought of as an operation of self-referential unfolding. The reason is that if we start iterating the commutator formation from level-0 (Tripodal link $\Sigma(3,2)$) which involves simple loops, then already at level-1 (link $\Sigma(4,3)$), the symbol $[a,b]$ in the composite stacked commutator $[[a,b],c]$ plays a dual role: First, it is the symbol of a loop, namely the product "Tripodal loop" of $a$ and $b$, and second, it is the symbol of a gluing operator acting on $a$ and $b$. Thus, the unfolding from level-0 to level-1 takes place self-referentially by identifying a loop as an argument of the stacked commutator at level-1 with the result of a gluing operator at the previous level-0. Clearly, the same phenomenon repeats at all higher levels.

It is instructive to explain in more detail the algebraic operation of commutator stacking. Recall that a commutator of two symbols $a$ and $b$ produces a new symbol $[a,b]$ in the group $\Theta$, where $[a,b]$ denotes the gluing of $a$ and $b$ together to produce a new symbol, such that the triad of symbols $a$, $b$ and $[a,b]$ constitute a Tripodal link of the type $\Sigma(3,2)$. Thus, a $\Sigma(3,2)$ link involves a commutator in 2 symbols standing for the gluing operator of these two symbols according to the Tripodal constraint. Similarly, a $\Sigma(4,3)$ link involves a stacked commutator in 3 symbols. The commutator is stacked because first we have to glue $a$ with $b$, and then we have to glue their product $[a,b]$ with $c$ in order to produce a new symbol $[[a,b],c]$, such that the tetrad of symbols $a$, $b$, $c$ and $[[a,b],c]$ constitute a $\Sigma(4,3)$ link.

We stress again that deletion of any of the symbols involved in the stacked commutator collapses it to the unity of the group $\Theta$, meaning that erasing any one of them causes the rest to come apart. Thus, by induction a $\Sigma(N, N-1)$ link involves a stacked commutator in $(N-1)$ symbols, where $N \geq 3$. For convenience, we call it a stacked commutator of order $(N-1)$. Note that the order of the stacked commutator in any link of the form $\Sigma(N, N-1)$ coincides with the number of symbols that separate if we remove any symbol from the total non-splittable $N$-link. For example, a $\Sigma(7,6)$ link is expressed via a stacked commutator of order 6, meaning that it should be a commutator in 6 symbols of the form $[[[[[a,b],c],d],e],f]$. For reasons of simplicity, we define a stacked commutator of order $(N-1)$ as a "Tripodal stack" of order $(N-1)$.

### 3.18 TEMPORAL MULTIPLICATION: CHAINS OF TRIPODAL LINKS

First, we introduce another definition to our series for terminological convenience. This refers to the characterization of a link of the general form $\Sigma(N,K)$. A link of the form $\Sigma(N,K)$ is defined as a link of $N$ loops in 3-d space, such that each $K$-sublink is completely splittable, but each $(K+1)$-sublink, $(K+2)$-sublink, $\ldots$, $(N-1)$-sublink up to the $N$-link itself, is non-splittable. For example, a $\Sigma(7,3)$ link is a link of 7 loops, such that each 3-sublink is completely splittable, but each 4-sublink, 5-sublink, 6-sublink and the 7-link itself, is non-splittable. The natural question emerging in this context is if it is possible to express a general link $\Sigma(N,K)$ in terms of "Tripodal building blocks", or equivalently "Tripodal functional units" encoded algebraically by the gluing operator of symbols, that is, by the commutator in the group $\Theta$. We already know the answer in case that $K=(N-1)$. Namely, we have shown that the algebraic operation of commutator stacking of order $(N-1)$ is enough to express any $\Sigma(N,N-1)$ link. In other words, an arbitrary $\Sigma(N,N-1)$ link is simply a "Tripodal stack" of order $(N-1)$. So we need to consider what happens in the general case, where $K \neq (N-1)$.

We will show in the sequel that there exists another natural operation on "Tripodal building blocks", which is described by taking an appropriate product of commutators in the group $\Theta$. Intuitively speaking, this natural operation should express a procedure of Tripodal extension in length, or simply the formation of a "Tripodal chain" of some appropriate length. In order to motivate the notion of a "Tripodal chain" it is necessary to start with the simplest example of this type, namely the $\Sigma(4,2)$ link. This is a link of 4 loops, such that each 2-sublink is completely splittable, but each 3-sublink and the 4-link itself, is non-splittable. From this definition, we immediately deduce that if we remove any loop from a $\Sigma(4,2)$ link we obtain a 3-sublink which is non-splittable. Moreover, since each 2-sublink is completely splittable, we deduce that if we remove any loop from a $\Sigma(4,2)$ link we actually obtain a $\Sigma(3,2)$ link, viz. a Tripodal link. Furthermore, if we remove any two loops from a $\Sigma(4,2)$ link the remaining two fall completely apart because again each 2-sublink of a $\Sigma(4,2)$ link is completely splittable. Therefore, by encoding this information in the group $\Theta$, we attack the problem as follows: Consider three symbols $a$, $b$, and $c$. We seek a formula expressing the fourth symbol, such that deletion of all incidences of any of the symbols $a$ or $b$ or $c$ causes the formula to reduce to the "Tripodal loop formula" (that is the commutator of the remaining two symbols), whereas deletion of all incidences of any two of the three symbols, viz. $(a,b)$, or $(a,c)$, or $(b,c)$ causes the formula to reduce to the unity 1.

It is instructive to emphasize that the algebraic encoding of the problem referring to a $\Sigma(4,2)$ link paves the way to its solution. The problem is whether it is possible to express a $\Sigma(4,2)$ link in terms of "Tripodal building blocks", that is, in terms of suitable operations on commutators in the group $\Theta$. By the defining properties of a $\Sigma(4,2)$ link, if a formula in three symbols $a$, $b$, $c$ actually existed fulfilling the two requirements laid out in the previous paragraph, and also expressed exclusively in terms of commutators built from these three symbols, then it would be true that the $\Sigma(4,2)$ link can be constructed in terms of "Tripodal building blocks". Now, considering the symbols $a$, $b$, and $c$, we may construct the "Tripodal stack" of order 3, viz. the stacked commutator formula $[[a,b],c]$. Clearly, although this expresses a $\Sigma(4,3)$ link as we have seen in the previous Section, it is not an appropriate formula to express a $\Sigma(4,2)$ link because deletion of any of the three symbols causes the formula to reduce to 1. What we need is another operation, which hopefully can involve only commutators and have the desired properties. A simple observation is that given three symbols $a$, $b$, and $c$, we may construct out of them three distinct commutators, namely $[a,b]$, $[a,c]$ and $[b,c]$. Since each of these commutators gives a new symbol in the group $\Theta$, we may take their product which is also a new symbol in the group $\Theta$.

Notice that each of the commutators $[a,b]$, $[a,c]$, $[b,c]$, gives separately a Tripodal link. Thus, their product $[a,b][a,c][b,c]$ is actually a composition of three separate "Tripodal links" in the group $\Theta$:

$$\rho = [a,b] \circ [a,c] \circ [b,c],$$

which gives rise to a "Tripodal chain" of length 3. The formation of this "Tripodal chain" $\rho$ provides the sought after operation on "Tripodal building blocks" to express a $\Sigma(4,2)$ link, and therefore solve the posed problem. We can immediately see this as follows: First, we notice that deletion of any one of the symbols $a$, $b$, $c$, in the "Tripodal chain" $\rho$ of length 3, $[a,b]\circ[a,c]\circ[b,c]$, reduces this chain to a Tripodal link. For instance, if we delete the symbol $a$, what remains is the Tripodal link $[b,c]$, and analogously for the other two cases. Second, we notice that deletion of any two of the symbols $a$, $b$, $c$, reduces this chain to unity. Hence, we conclude that the "Tripodal chain" of length 3, defined by the product of commutators $[a,b][a,c][b,c]$ , provides the formula for the fourth symbol $\rho$ in the group $\Theta$, such that the defining properties of a $\Sigma(4,2)$ link are satisfied, and moreover, this link is expressed in terms of "Tripodal building blocks". An interesting observation that we will put to use as we progress is that the length of the "Tripodal chain" solving the problem is given by the number of combinations of 2 symbols out of 3, where a combination is simply the formation of the commutator of 2 symbols in this case.

### 3.19 PRIMAL ROLE OF THE TRIPODAL LINK IN ALL HIGHER TEMPORAL BONDS

Regarding the possibility of expressing arbitrary links in 3-d space of the general form $\Sigma(N,K)$ in terms of "Tripodal building blocks", or equivalently "Tripodal connectivity units" we have proved up to present the following: First, the algebraic operation of commutator stacking of order $(N-1)$ is enough to express any $\Sigma(N,N-1)$ link. In other words, an arbitrary $\Sigma(N,N-1)$ link is simply a "Tripodal stack" of order $(N-1)$. For instance, a $\Sigma(4,3)$ link is simply a "Tripodal stack" of order 3. Second, we have shown that the expression of a $\Sigma(4,2)$ link requires the consideration of another operation on "Tripodal building blocks", which is interpreted as the operation of extension of length 3, called the formation of a "Tripodal chain" of length 3. Based on these findings, the next question posing itself naturally in this context is if these two operations on "Tripodal building blocks", namely the formation of "Tripodal stacks" of some suitable order and the formation of "Tripodal chains" of some suitable length are adequate in order to express any arbitrary link in 3-d space of the general form $\Sigma(N,K)$.

This would be certainly of significance in our understanding of the whole universe of links, because it would prove that any $\Sigma(N,K)$ link can be constructed by means of "Tripodal connectivity units" via the combinatorial formation of "Tripodal stacks" and "Tripodal chains". Moreover, due to the algebraic modelling scheme instantiated structurally by the non-commutative group $\Theta$, the process of analysis and synthesis of arbitrary links in terms of prime elements, which is to say in terms of “Tripodal connectivity units” would be implementable algorithmically, and thus at hand as a valuable tool for making evaluations and predictions.

Before we consider the general case of a $\Sigma(N,K)$ link, it will embellish our intuition to examine the case of a $\Sigma(5,3)$ link. The reason is that a $\Sigma(5,3)$ link has enough complexity so as to pave the way for the treatment of the general case of a $\Sigma(N,K)$ link. From the definition of a $\Sigma(5,3)$ link, the crucial observation is that if we remove any of the loops, what remains is a $\Sigma(4,3)$ link, which we already know is expressed by means of a “Tripodal stack” of order 3, by the stacked commutator formula $[[a,b],c]$ in 3 symbols. Thus, in order to express the formula of a $\Sigma(5,3)$ link, if we consider 4 symbols $a$, $b$, $c$, $d$, we require a formula such that deletion of any of them causes it to reduce to one of a $\Sigma(4,3)$ link, to a “Tripodal stack” of order 3.

The important concept solving this problem is based on the observation that we can form “Tripodal chains” of arbitrary length using “Tripodal stacks”. In the particular case of a $\Sigma(5,3)$ link considered, since we require that deletion of any of the four involved symbols $a$, $b$, $c$, $d$, reduces the formula to a “Tripodal stack” of order 3, we just need to form a “Tripodal chain” of “Tripodal stacks” of order 3, where the length of the chain should be 4. This is explained easily by the fact that the length of the “Tripodal chain” is given by the number of combinations of 3 symbols (which is the number of symbols involved in a “Tripodal stack” of order 3) out of 4 symbols $a$, $b$, $c$, $d$. We immediately conclude that the sought after formula expressing a $\Sigma(5,3)$ link is given by the “Tripodal chain” of length 4, composed by “Tripodal stacks” of order 3, and described explicitly by the following formula:

$$\chi = [[a,b],c] \circ [[a,b],d] \circ [[a,c],d] \circ [[b,c],d].$$

In more detail, we see that the above formula is given by the composition of 4 “Tripodal stacks” of order 3 (since they involve 3 symbols each), and thus produces a “Tripodal chain” of length 4, such that deletion of any of the four involved symbols $a$, $b$, $c$, $d$, reduces this chain to a “Tripodal stack” of order 3 as required. Thus, we have completely resolved the problem of a $\Sigma(5,3)$ link in terms of prime “Tripodal connectivity units”.

Now, having understood in detail the case of a $\Sigma(5,3)$ link, we are ready to state the following theorem:

An arbitrarily complex link of the general form $\Sigma(N,K)$, where $1 \le K \le N$, can be enunciated solely in terms of Tripodal links, by means of combining nested stacking and multiplicative chaining of Tripodal links of appropriate depth order and length respectively.

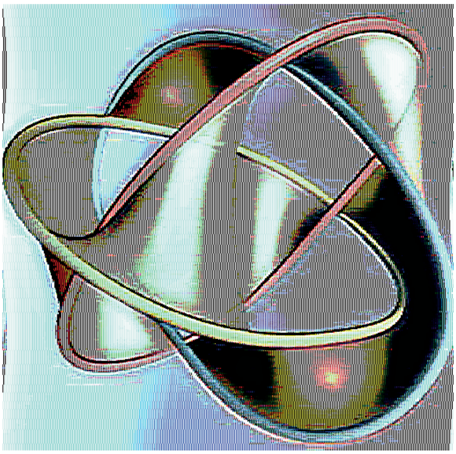
We consider an arbitrarily complex link of the general form $\Sigma(N,K)$, where $1 \le K \le N$, and prove that it can be constructed solely in terms of "Tripodal building blocks" within the group $\Theta$. For any $K$, we already know that the link $\Sigma(K+1,K)$ is expressed by means of a "Tripodal stack" of order $K$. Next, we consider $(K+1)$ symbols in $\Theta$, and we wish to construct a $\Sigma(K+2,K)$ link. The crucial observation is that if we remove any topological circle from a $\Sigma(K+2,K)$ link, what remains is a $\Sigma(K+1,K)$ link. Thus, we treat this case in complete analogy to the case of a $\Sigma(5,3)$ link, discussed previously. More precisely, we form a "Tripodal chain" out of "Tripodal stacks" of order $K$, where the length of this chain is given by the number of combinations of $K$ symbols out of $(K+1)$ symbols. The formula expressing this "Tripodal chain" provides the sought after $(K+2)$ symbol. Now, we consider $(K+2)$ symbols, and we wish to construct a $\Sigma(K+3,K)$ link. We just have to form a "Tripodal chain" out of "Tripodal stacks" of order $K$, where the length of this chain is given by the number of combinations of $K$ symbols out of $(K+2)$ symbols. The formula expressing this new "Tripodal chain" provides the sought after $(K+3)$ symbol in $\Theta$. We continue the same process of formation of new "Tripodal chains" of appropriate combinatorial length composed by "Tripodal stacks" of order $K$, stage by stage, until we reach $N$. This completes the proof of the theorem that an arbitrarily complex link of the general form $\Sigma(N,K)$ can be constructed solely in terms of "Tripodal building blocks", or equivalently, "Tripodal connectivity units".

We may consider as an application of this theorem the case of a $\Sigma(7,4)$ link. The link $\Sigma(5,4)$ is expressed by means of a "Tripodal stack" of order 4. Next, we consider 5 symbols, and we wish to construct a $\Sigma(6,4)$ link. Let us call these symbols $a$, $b$, $c$, $d$, $e$. Next, we form a "Tripodal chain" of "Tripodal stacks" of order 4, where the length of this chain is given by the number of combinations of 4 symbols out of 5 symbols, which is 5. Let us denote by f the new symbol provided by this "Tripodal chain" of length 5. Thus, we have constructed a $\Sigma(6,4)$ link. Now, we consider these six symbols $a$, $b$, $c$, $d$, $e$, $f$, and we wish to construct a $\Sigma(7,4)$ link. We just have to form a "Tripodal chain” of “Tripodal stacks” of order 4, where the length of this chain is given by the number of combinations of 4 symbols out of 6 symbols, which is 15. The product formula expressing this new “Tripodal chain” of length 15, provides the sought after 7th symbol. Therefore, we have constructed a $\Sigma(7,4)$ link by means of prime “Tripodal connectivity units” using only the combinatorial formation of “Tripodal stacks” and “Tripodal chains”.

### 3.20 IMAGINARY SURFACE OF TEMPORAL COHESION AND ENTANGLEMENT

The modeling of a prime temporal bond as a topological “Tripodal link” enables us to comprehend the process of “synergetic cycle change”, effected by the “modular gluing” of a seed from the “past” with an anticipated seed in the “future” with respect to the “present”, upon establishment of this temporal bond. This threefold metaphora provides the necessary topological means to elucidate how a holographic boundary of temporal cohesion can be adjoined to 3-d spatial space in the present that this bond is realized. The adjunction of this synectic boundary of temporal cohesion in 3-d space permit the holographic connectivity and entanglement of a seed from the “past” with an envisioned seed in the “future” topologically, independently of their proximal distance. This is achieved by demarcating an imaginary oriented compact and connected surface of temporal cohesion in the present.

We consider the compact, connected and oriented surface with boundary to constitute the “Tripodal link”. This surface is visualized as follows:



Thus, the imaginary surface of temporal cohesion generated by a prime temporal bond is equivalent to a torus bearing three punctures (corresponding to the *aphaeresis* of three disks). The significance of this imaginary surface of temporal cohesion instantiated by a temporal bond is that it gives rise to a global curvature topological effect characterized as a minimal surface. In other words, it is the least-action connectivity solution, and thus, the most economical solution to the modular amalgamation instantiated by a prime temporal bond.

A simple method to visualize this surface spatially is to consider the "minimal surface" formed by a soap film, when three wire rings linked together as the "Tripodal link" are immersed into a solution of soapy water and then taken out. This surface is a "least-action" solution to the shape that a soap film acquires in this case, since it minimizes the area. Interestingly enough, every point in this "surface of cohesion" is locally similar to a saddle, i.e. its local curved geometry is of the hyperbolic type, whereas its global topology is of the toroidal type.