CIRCULATION: ENCODING-PARTITIONING-DECODING

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The present treatise arose out of a curiosity to delve deeper into the nature of obstacles or obstructions that prevent a single, uniform, and linear approach to dealing with systems, entities or, more generally, beings, and their communication. The current motivation comes from a certain degree of dissatisfaction with the overflow of scientific production on what is called "science of complexity" and the qualification of what counts as "information of complex systems" based exclusively on specific "target-oriented approaches".

The thread of this amphiboly starts from questioning what is considered to be "complex" in contradistinction to what is considered "simple". Usually these terms are implicitly pre-loaded with an ontological meaning, which essentially identifies complexity with certain aggregations of elementary sharply distinguishable constituents developing emergent properties that are behaviourally observed under specific interactions or conditions. This approach is assumed to be valid irrespectively of scale, depending on what is axiomatically baptized as an elementary constituent, so that certain statistical patterns can be applied upon them targeting the simulation of their behaviour. A natural set of questions in this setting is the following: What makes an entity a constituent, how an entity can be characterized as elementary, and most important, how can an entity be sharply distinguishable?

The afterthought of these considerations is that the epithet "complex" arises from the opposition to what is called "simple", where what is "simple" is identified with the "axiomatic elementary" in the context of the vast majority of these "target-oriented approaches". Notwithstanding these scientific tendencies of the present, this is not actually the way that humankind came to terms with complexity. The targets were not predefined, but rather they always emerged out of a necessity to cope with obstacles in communication of every particular sort. In this respect, the meaning of complexity is altered dramatically if theorized from an "obstacle-oriented" standpoint instead of a "target-oriented" one. More precisely, the ability to locate obstacles or obstructions forcing a deviation from some standard condition of uniformity becomes the primary task. In contrast, "target-oriented" approaches are based on the shaky foundations of some pre-given axiomatic elementarity, usually identified with the foundations of set theory, on the top of which, pre-designed statistical methods claim to provide universal truths as the feeding source of policy-making on a larger scale. In this type of approach, where the form of the target is well-defined itself *ab initio*, precluding any chance for genuine novelty, even the standards of congruence effecting the condition of inertial variation in some domain are not determined with respect to the singularities of this domain, but on the basis of general classification methods applied in a more or less ad hoc manner.

In an "obstacle-oriented" approach the presence of an obstacle necessitates, firstly, the localization of this obstacle, and secondly, the unfolding of the obstacle into an appropriate partition spectrum that allows its indirect embracing by means of resonance with the marked frequencies of this spectrum, or reciprocally, but equivalently, synchronization with its corresponding temporal periods. By inversion, the first of the above, points to a viable understanding of what intelligence is about, whereas the second points to a viable understanding of the notion of an irreducible duration, together with its role in what memory refers to. Of course, the ability to obtain such a partition spectrum is always conditioned to the ability to identify a suitable *gnomon*, which actually provides the means to resonate or synchronize with the obstacles in question. This leads inevitably to a valuable association of *gnomonics* with the old art of *harmonics*. We will examine this association in detail in what follows, together with the intervening notion of *canonics*.

At present, a basic fact worth pondering at is that the localization and unfolding of an obstacle, requires a process of metaphora around this obstacle, i.e. a potential circular flow around it that allows its embracing. If the obstacle is considered as merely an obstacle in space, then the potential flow clearly takes place within the space where the obstacle is located. The drawback of this is that the notion of space is pre-assumed in relation to the localization of the obstacle, whereas the opposite should be the case; rather the notion of space should emerge out of the nature of the obstacle, or else, should be cohomologous to the nature of the obstacle with respect to the employed gnomon of measurement. Put simply, the labyrinth is not pre-existing in any sense, but it is the space opened up by all these meandering paths which serves to embrace its sacred center. This is at any rate the route we have to take in case the obstacle is an obstacle in time.

Actually this is always valid, even in the simplest case whereby we think of an obstacle as a hole in a single linear dimension, where the latter is considered as the line of inertial variation, i.e. the straight path of motion with constant velocity. The attendant need for an imaginary dimensionality, orthogonal to the initial real dimension, to bring into central focus the "unit circle" enunciated in potential or power terms via the exponential function, and culminating in the arithmetics and geometry of the complex numbers, should be properly thought of in temporal terms and not in spatial ones. In other words, the imaginary unit, or imaginary ring, supplying the complex structure, is a precise *symbolon* of metaphora through another level of *hypostasis* that allows the embracing of the obstacle localized at the zero point of the complex plane.

The notion of a symbolon bears a periodic temporal connotation, as opposed to its static reflective spatial manifestation as a symbol, which is further deteriorated to a mere formal sign. This connotation amounts to the instantiation of a rhythm that bears the potential to persist covariantly and resolve a moment of time, as manifested in the linear real dimension, into a vertical, orthogonally placed, spectrum of imaginary-valued frequencies. This resolution induces a *metabole* in both the qualitative and quantitative conceptions of time around the obstacle that spatially are manifested as relative phases on the unit circle, functioning in this sense as the homeostatic symbolic imaginary locus, i.e. the base sign-recording indexing of the metabole. Simply put, this is the subtle difference between an icon as a temporal and rhythmic symbolon and its spatial image as a semiotic indexing snapshot. From this viewpoint, an indexing real-valued sign should be never considered independently of its spectral depth in terms of the associated imaginary-valued frequencies. In all interesting cases, this frequency spectrum is actually quantized, i.e. it is displayed in terms of discrete quanta engulfing the integer multiples of some fundamental period, for instance, the period of circulation around the obstacle.

From a purely topological standpoint, the localization and unfolding of an obstacle may be equivalently thought of as a process which exhaustively encompasses and universally covers the obstacle. In the case of a single obstacle, manifested as a hole on the plane, the universal covering space is displayed as a helix, whose different layers correspond to the different frequencies comprising the universally covering or unfolding spectrum. The underlying topological idea is that an obstacle is the source of multiple-connectivity, enacted by all different types of paths embracing it, which are classified by their characteristic winding number. The universal covering space in this case may be thought of as a discretely fibred space, i.e. over any real-valued point on the base there is a discrete fiber above it consisting of the frequencies of the spectrum. A fiber is the carrier of the spectral depth of a base point in the sense that a loop based at this point and embracing the obstacle is uniquely lifted to the universal covering space, such that its starting and ending point belong to the same fiber above the base point and their relative spectral difference on this fiber qualifies the winding type of the base loop. All fibers of the universal covering space are not disconnected from each other, but cohere; there is a connectivity structure that binds them all together, giving rise to a helix. The important thing is that the helix is simply connected, i.e. the universal covering space is actually a geometric space, meaning that it bears a geometric form, and as such it resolves the multi-connectivity issue of the base induced by the obstacle. It is precisely in this spectral manner that the simply-connected obstacle-covering geometric form topologically unfolds the obstacle.

The topological unfolding process of universally covering an obstacle, giving rise to a simply-connected geometric spectral form, can be properly supplemented with the complex structure induced fiber-wise by the imaginary unit bearing the function of the symbolon of metaphora through another level of hypostasis. The setting here pertains to a multi-valued complex function deemed to be analytic; there exists a power series expansion of this function locally, and the purpose is to extend this local domain of definition by a method called analytic continuation, effectively furnishing a connection expressed locally by means of imaginary-valued gauge potentials. Singularities appear, for instance, in terms of poles where the function becomes infinite. Then, the unfolding procedure can be implemented in this context with the exception of these singular points of ramification, where branching behavior appears. Intuitively, the fiber over this point becomes degenerate collapsing to a single point, through which different branches collide together and reciprocally open up into well-defined multiplicities.

The significance of the complex structure in this setting, or equivalently, the conformal structure, properly lies in the domain of harmonics. The imaginary ring, i.e. the gnomon qualifying the conformal structure, gives rise to a type of resonator, where the unfolded branches, like bounded helical strings of a musical instrument vibrate under the action of the imaginary gauge potentials. In this way, a discrete series of harmonics is instantiated that have the capacity to resolve a moment of real-valued time, together with their consonances and dissonances. This rhythmic enunciation of the imaginary ring together with the harmonic series admits a choreographic interpretation in terms of oriented angle-preserving transfigurations, called conformal morphisms, looking like global metamorphoses from a geometric perspective that respect only the relative angles so as to maintain the rhythm. Another viable way to think of this choreography is in the terms of a helical standing wave and its associated harmonics, where this wave is not thought of as being in space in any sense, but rather, space arises out of its vibrations, periodicities, and resonances conformally in historical time. The latter is not comprised by the real-valued moments of events, but rather, it pertains to the coherent aggregation of all fibers bearing the harmonics over these moments, or equivalently, the quanta of resonance and synchronization. In a nutshell, complex harmonics is an expression of the economy of historical time in its entirety as it is unfolding through metaphora.

In this manner, and referring to the domain of complex harmonics, the imaginary ring is like the translocal imaginary umbilical cord, which as a symbolon of metaphora through another level of hypostasis, forces the translocal *homeosis* of the local with the global in terms of angle-preserving periodically repeating, and thus, homeostatic iconic tessellation of the whole by the part; In the two-dimensional case of an unfolded *epiphaneia*, this gives rise to only three distinct universal covering, simply-connected, geometric forms characterized by constant positive, zero, or negative curvature respectively. Simply put, the geometric form arises out of complex harmonics as a homeostatic crystalline epiphaneia of the choreography subordinate to the imaginary rhythm. It is a crucial fact that the domain of complex harmonics does not bear any distinctive ontology, rather its nature is akin to a magma. Notwithstanding this fact, emanating from the reciprocal and complementary relation between the chords and the potentials with respect to the imaginary resonator-symbolon in the acoustic articulation of the pure harmonics, a magma has *entelechy*, which is precisely manifested geometrically in one of three universal distinctive curvature forms.

The geometric, hence visual, form of the universal covering epiphaneia in the setting of complex harmonics, requires for its articulation a *canon* of *metalepsis*, effecting a transgression from the acoustic domain of pure harmonics to the visual domain of colours. This requires the detuning of the pure harmonics within equally-partitioned chromatic intervals. In this sense, the canonics from the acoustic domain to the visual domain amounts to a process of imaginary logarithmization with respect to equally-distanced angular intervals, so that any pitch in the chromatic domain is situated at equal distance from its nearest neighbours. The canonics of this *heteromorphism*, which essentially arises out of the adjunction of the chromatic to the harmonic domain, transfuses an affine character to the rhythm in its chromatic manifestation that allows its generation and progression infinitesimally and differentially in a continuous manner. The suppression of the pure harmonics determined by the type of the obstacle, i.e. the quotient of the induced chromatic spectrum by the module of pure discrete harmonics, leaves a trace for imprinting a memory element on the imaginary ring of metaphora revealing thereby its global role. More precisely, this trace is a global non-integrable relative phase factor, i.e. a global irreducible residual phase marking the *anholonomy* of the metaphora due to the embraced obstacle. As such, it incorporates both topological and geometric information. On the other side, the suppressed pure harmonics, as the implicit harmonic invariants, guide the extraction of the complex roots of unity on the imaginary ring. In a well-defined way, the complex roots of unity negate the punctual character of any real-valued moment, in the sense that they open up channels of potential resonance via the harmonics with other non-locally related moments, recorded as geodesic paths of connectivity in the corresponding universally unfolded curved geometric form.

In all cases concerning the embracing of an obstacle, the metaphora is effective if the cyclic flow is communicative, i.e. it establishes bidirectional bridges connecting any two hypostatic levels. It is in this sense that an "obstacle-oriented" approach is not a directly "problem-solving" approach, but an indirectly "problem-embracing" one. The different levels are not in any relation of *hypotaxis* of one to another, but only in a relation of *parataxis*, and what really matters is the facility in the passage from one level to another in a bidirectional way via the bridges. This facility requires that the bidirectional bridges should not be ad hoc, i.e. they should not be designed as referring to particular subjective choices, but they should apply under covariant variations on their respective inversely related hypostatic domains and codomains. This requirement is captured by the adjective *"natural"*, which pertains to a double articulation of an algebraic nature: First, the hypostatic domains and codomains should be though of as *categorical*, and second, the bridges between them should be *functorial*, i.e. not depending on particular choices of objects in the interrelated categorical domains and codomains of the bridges.

Generally speaking, algebra pertains to a structural enunciation of the "obstacle-embracing", communicative process of metaphora between any two hypostatic levels. This enunciation is formulated operationally in terms of symbolic algebraic structures like groups, rings, modules, and categories. The notion of an algebraic symbol does not bear, neither the temporal connotation of a symbolon, nor the spatial connotation of a sign. For this reason, an algebraic structure maintains an independence from both the harmonic, and the geometric connotation of its symbols, although it may properly mediate between them and abstract from both of them. The key idea is that an algebraic structure, in the context of the "obstacle-oriented approach", plays the role of a structure expressing both the invariant, and the covariant characteristics of the metaphora with respect to some specified notion of unity or equivalence displayed as an algebraic identity. For instance, in the case of a group structure it is the notion of a neutral element with respect to the implied operation that characterizes the algebraic identity. This becomes prominent in the case of structure-preserving morphisms, like homomorphisms and isomorphisms that transfer the group structure in some particular way. In this sense, the concept of a unit implicated by a gnomon, like a gnomon of discrete counting or rational measuring, is imprinted symbolically in the neutral element of the associated group structure. This is indispensable, since inversion of an operation is not feasible without the specification of the neutral element in any group-type structure. The ability to form inverses in achieving the closure requirement of a group-type structure is conditioned on the extension of elements into algebraic power domains, which elevates the exponentiation operation to a place of prominance, subsuming the operations of addition and multiplication. Its inversion, i.e. the logarithmization operation, cannot be performed unconditionally.

More precisely, the expression of the irrational numbers via the real logarithm requires a topological continuity condition, whereas the expression of the imaginary numbers via the complex logarithm requires a local topological simple-connectivity condition. In other words algebraic operational definability and topological continuity or connectivity become inextricably intertwined and inseparable in the transcendental realm. This is exactly the reason that a group-type of structure is, in principle, capable of expressing invariants of a topological nature.

The transcendental realm, as the cumulative realm of algebraic powers and their inversion or reciprocation, is particularly suited to express symbolically potential circular flows embracing obstacles of any type. This leads to the idea that there should exist a minimal, in the sense of the most economical, algebraic symbolic description of a metaphora between any two hypostatic levels that can be qualified as structural. In other words, any metaphora with respect to an obstacle should, in principle, be expressible in terms of a symbolic *"motivic key"* capable of unlocking operationally the communication capacity between the domains it is applied to. Thus, its function is doubly significant: First, since the key is symbolic, it always allows the structural encapsulation of the metaphora in terms of invariants, which can be qualified effectively by means of standard algebraic structures; Second, since the key is motivic, its economy is not subordinate to artificial choices pertaining to the bridging of the hypostatic domains, and thus, it should ne natural, fulfilling the requirement of covariance.

Regarding the issue of "naturality" in the specification of this symbolic motivic key, in the general case that the communicative domains-levels are heteronymous if hypostasized structurally, it is essential that the structural qualification follows from a deeper algebraic categorization that admits the possibility of heteronymous bridging via heteromorphisms. This is not imposed on the basis of some universal axiomatic system, but it arises out of necessity when operating in the heterogenous symbolic. The reason is that heteronymous domains cannot be bridged together directly, but require a certain canonics that makes them partially structurally adaptable to each other. We have already introduced this notion in relation to bridging together the harmonic with the geometric domain, or the acoustic with the visual domain. Here, algebraic canonics of metalepsis assumes a particular type of heteromorphisms between the concomitant categorical characterization of these domains, which can be internalized homonymously, i.e. expressed in terms of homomorphisms within each category, under the existence of initial or terminal objects via which the potential cyclic flow factors through. In categorical language, they are called *adjunctions*, expressed in terms of natural isomorphisms arising from the bidirectional bridging of the relevant categorical domains via adjoint functors. It is important to emphasize that this is how "natural communication" inverts the usual understanding of category theory, since it is the canonics between heteronymous domains that entails the category-theoretic characterization of those domains. In other words, the latter appears out of the *metaleptic economy* characterizing the "natural communication" between heteronymous domains in the terms of adjoint functors, and not as an appeal to any type of structuralist foundations.

Since the central issue is the notion of an obstacle and the metaphora required to embrace it via different hypostatic domains capable of entering into "natural communication", the notion of a foundation is totally misconceived. Instead, what is crucial always is the notion of an architectonic scaffolding that potentially is able to bridge together these domains so that the metaphora leading to the communication between these domains can be elucidated. The framework of adjoint functors in category theory pertaining to the symbolic and operational aspects of this metaphora can be appreciated if evaluated properly in its function as a sophisticated abstract architectonic scaffolding. The harmonic, topological, and geometric aspects of the metaphora, as explicated concisely above, also bear an indispensable role in the articulation of "natural communication" and the efficacy of abstract gnomonics in this fashion, which would be impossible by restricting only to the categorical and the symbolic realm. Put equivalently, the algebraic symbolic *"morphe"* should be consistently elaborated and ingrained by the harmonic symbolon of the metaphora, together with its topological schematism, and eventual geometric spectral form.

At a further stage, and as a result of this elaboration, the bridges between different hypostatic domains in the most economical articulation of "natural communication" in the terms of a motivic key can be thought of as *cobounding* the conditions of communication between these domains. Since cobounding is always of a local or even infinitesimal nature, these conditions transcribe the norms of harmonic congruence that can be eventually unfolded multi-periodically in some simply-connected geometric form. The important thing is that the norms of harmonic congruence allow the evaluation of any other possible metaphora between the domains involved with respect to this norm, i.e. *cohomologically*. This means that the qualification of information under a process of metaphora involving the communication between two domains is essentially cohomological, or else, entropy as a measurable magnitude of this information is cohomologically quantifiable.

The model of "natural communication" envisions an architectonics of communication as the most prominent conceptual stance in tackling the problem of complexity. The founding realization is that such an architectonics of structural relations, which are based on communication between appropriate correlated domains, poses itself as a necessity in all these cases, where obstacles and obstructions of any particular type prohibit or prevent the direct accessibility to hypothetical sharply distinguishable elements of complex domains. The shaping of objects in these domains takes place according to foamy patterns, characterized by topological plasticity, emergent properties and generically probabilistic attributes. The application of pre-specified, ready-tailored constitutional and elemental design ontologies to these complex domains, according to analytic methods designed for those ideal cases where sharp elemental distinguishability is feasible, not only distorts the architectonics of their intricate connectively weaving patterns, but limits and restricts, even inadvertently, their potential computational capacities.

The basic idea to address and utilize the architectonic modelling of these domains is the following: Instead of following the standard analytic method of dissecting ontologically complex objects, i.e. objects situated in non-directly accessible domains, in terms of the collections of their hypothetical set-based elements and their concomitant absolute relations, we adopt a synthetic method: We let them unveil themselves by adjoining to them other adequately-understood, or directly accessible domains, which can provide pointers and open up communication channels with the complex ones. The difficulty in this synthetic act rests on the fact that an adjoined domain should be capable of opening up such a communication channel, otherwise its adjunction is vacuous. The crucial condition that should be fulfilled for this approach is the viability of setting up a bidirectional bridging scaffolding of encoding/decoding relations between these domains.

These relations may be thought of as giving rise to a sieve through which the eventual unveiling of the connectivity patterns characterizing the objects of the complex domain becomes effective. In this sense, the study of the architectonics of communication is tantamount to the realization of the possibilities of unveiling a complex or obstacle-laden domain through an appropriate weaving sieve. A sieve with this function bears the capacity to reveal the intricate bonds beyond direct access, which give rise to the coherence of the complex domain. Therefore, complex objects are unveiled via percolation through a sieve of communication relations, an invariant process which is depicted precisely by the ancient Greek term *"aletheia"*. In this setting, the synthetic act of adjoining a domain for communication should not be ad-hoc, and should not depend on artificial choices, meaning that it should be designed to respect any structure encountered, at least, locally or partially. This refers to the “naturality requirement” of the model, a term which belongs to the register of category theory. Technically, as it will turn out as the argument develops, the adjunction process is concretely modelled in terms of a pair of adjoint functors between the involved categorical domains.

In practice, the process of adjoining a controllable or directly comprehensible domain to a complex, or obstacle-laden, categorical domain, is tantamount to considering these domains as different categorical levels in a broad stratified universe of discourse, which are amenable to bi-directional correlation by means of oppositely or reciprocally oriented bridges. The architectonics of communication targets precisely the conception, explicit construction, and manifestation of these bridges, once suitable probing domains have been structurally delineated, due to their partial or local congruence properties, to be adjoined to the directly inaccessible complex domain. The bridge directed from the latter to an adjoined controllable domain plays the role of an encoding bridge, whereas its inverse or reciprocal plays the role of a decoding bridge. These level-interconnecting bridges effect the communication between the involved domains, in the sense that they bear the capacity to open up and establish natural and universal bidirectional communication channels through which a schema of metaphora can be accomplished, which is based on the notion of an “obstacle-encircling” flow.

From a topological standpoint, the initially inaccessible categorical domain is being spectrally resolved in the fashion of a spiral, i.e. by a process of cyclic unfolding with respect to the various probing domains that have been adjoined to it, corresponding to different layers of spectral resolution. In this manner, the invariants emerging by the process of unfolding depict the substantive characteristics of the reciprocal communication flow between the domains involved. Consequently, the complexity of the non-directly accessible, or obstacle-laden, domain is not specified constitutionally on the basis of a pre-assumed or axiomatic elementary ontology, but relationally and functorially, in terms of the spectral invariants emerging in the bidirectional communication flow established with suitable probing domains. From an algebraic viewpoint, these invariants can be described in terms of structural group-type ciphers for the symbolic encryption/decryption of the induced flow.

The obstacle-embracing schema of metaphora giving rise to this transitory multi-levelled cyclic flow, called the “logical conjugation method”, is always implemented on the basis of adjoining controllable or directly accessible domains to an obstacle-laden domain in their role as markers, or pointers, or more generally gnomons, providing eventually the means of unveiling it indirectly through communication. This constitutes a legitimate synthetic logical manoeuvre in the specification of a complex domain that has the capacity to unveil it by means of metaphora through the weaved sieve of encoding/decoding relations. The temporal percolation due to this sieve constitutes the –aletheia– of the complex domain, that is, what is unveiled and should not be forgotten about the coherence of this domain. The logical conjugation method effectuating the metaphora plays the functional role of a motivic key that bears the potential to harmonically unlock the complexity of the inaccessible domain, depending on the nature and type of the obstacles located. The creative art consists in the innovation of the reciprocal encoding/decoding bridges acting as the means of communication between an obstacle-laden domain and a conjugate accessible domain.

The function of the gnomon, enunciated through the encoding/decoding bridges, is instrumental in the eventual schematism of a partition spectrum pertaining to the obstacle-laden domain. Each partition block or cell of this spectrum is characterized completely by the equivalences induced by this pair of encoding/decoding bridges utilized for communicating an aspect of the non-directly accessible domain with respect to the probing conjugate domain. An important stipulation for the role of a probing domain, bearing a gnomon of partial congruence with respect to the obstacle-laden one, is that it is not to be considered, in any sense, as the foundational background of the latter, namely in terms of absolute elemental constitution, as would be the case in mathematical set theory. Rather, the functional role of a probing domain is to open up a communication channel with the complex domain, such that the partial congruence between them can be properly thought of as a resonance within certain intervals of frequencies comprising the induced partition spectrum. In other words, a probing domain, endowed with a gnomon enunciating the encoding/decoding capacity of the associated communication bridges with the complex domain, gives rise to an architectonic scaffolding for the qualification and quantification of the information gained by the embracing of an obstacle characteristic of the complex domain.

The simplest possible articulation of the proposed schema invites us to consider a problem in the context of a complex domain, i.e. a domain whose objects and relations are non-directly accessible due to various types of obstacles. First, we have to move out of the context of the initially posed problem, formulated at the level of this domain, since it is not directly accessible, due to obstacles preventing any possible type of sharp analysis. For this purpose, we act synthetically by adjoining to the complex domain another controllable probing domain at least locally. In order to accomplish this, we have to set-up an encoding bridge from the level of the inaccessible domain to the level of the accessible probing domain. Once, we have succeeded in setting up this bridge, we are able to transfer the initial problem, even locally, at the level of the controllable domain, where the means to resolve it effectively exist. This is tantamount to the construction of a resolving partition spectrum that covers the complex domain, and groups its communicable attributes with the probing domain into distinctive blocks, i.e. communication channels. The schema is completed by setting up an inverse decoding bridge from the level of the controllable domain to the level of the inaccessible one. In this way, the resolving spectral capacity of the controllable domain can be raised at the initial context of the problem, and thus, the problem is indirectly resolved through decoding in the context of its initial formulation. The reciprocal encoding and decoding bridges constitute the means of a novel architectonics of communication. It should be emphasized that the procedure implemented above can be iterated by the involvement of more than one controllable domains, which are adjoined respectively to the inaccessible domain. The skeleton of this algorithmic procedure of resolution remains invariant under the adjunction of deeper spectral levels. In all cases, there emerges a partition spectrum of the inaccessible domain, whose cells are indexed by the respective pairs of encoding/decoding bridges. The procedure of resolution conducted by synthetically adapting motivic keys to an inaccessible domain, according to the above schema, is technically called logical conjugation, whence the communicating domains, represented by levels inter-connected by the encoding/decoding bridges, are called conjugate domains. The adjective "logical" adopted for the conjugation method, is intrinsically related to the fact that the unveiling of the complex domain in this way, constitutes its -aletheia-, a notion that extends and enriches conceptually the standard bare logical notion of "truth".

1.3 THE "OBSTACLE-ORIENTED" APPROACH: INVARIANCE, ACTION, AND *GNOMONS*

First of all, it is worth focussing on two interrelated aspects of what we term the "obstacle-oriented" standpoint of enquiry. The first issue refers to the nature of metaphora around an obstacle. Simply stated, in what way is a cyclic flow initiated, giving rise to communication between two heterogeneous levels of hypostasis by connecting them through bidirectional bridges? The second issue refers to the specification of an obstacle as a source of invariance with respect to the specific context of its localization. These two issues are closely related by the temporal notion of action. In turn, it is the notion of action that qualifies the notion of connectivity independently of any spatial instantiation.

We start following the thread that identifies an obstacle as a source of invariance. The notion of invariance is not absolute, but it is modular, meaning that invariance is meaningful only within the specific context of localization of an obstacle. Invariance can be operationally characterized only through action directed initially away from the level or context of the obstacle. The effect of action is to initiate a stream flow that is capable of retracting the inaccessibility or obstruction imposed by the obstacle to some generic situation at another level through which a passage becomes viable, and then re-direct the flow black toward the initial level, so that the obstacle can be embraced. Successfully embracing an obstacle always leaves a residue, to be thought of in terms of "countable quanta of *metabole* by periodic action". These quanta are spectral quantities, denoting rhythmic arrangements within regular temporal cycles, to be thought respectively as frequencies. Most important, these quanta encode the invariance of the obstacle they refer to with respect to all possible embracing circular flows initiated by temporal actions.

The crucial issue is that a residue of a cyclic stream flow is something associated with a differential, i.e. it is the result of an integration procedure along a temporal cycle surrounding the obstacle. This has the following consequences:

i Temporal cycles can be distinguished only by the countable number of winding actions around the obstacle, and thus, the frequency spectrum is indexed or quantized by an integer number;

ii A differential is a quantity that does not assume any value at a point, meaning that it is viable only in germinal form with respect to a cloud or foam surrounding any point around the obstacle;

iii As a consequence of the above, an indistinguishability or ambiguity is induced in relation to the stream flow, expressed by the notion of the multiple-connectivity associated with an action;

iv In turn, this multiple-connectivity is annihilated by unfolding the domain of action continuously in successive branches, giving rise to a helicoidal staircase until uniformity or simple-connectivity prevails;

v The universal helicoidal staircase as the unfolded domain of all temporal actions to embrace the obstacle constitutes the quantized spectrum of these actions;

vi The nature of metaphora is explicated by the connectivity bridges between the level of the obstacle and any level of the helicoidal staircase together with the associated countable quanta of metabole;

vii In terms of metaphora, the invariance of the obstacle is expressed in a unequivocal way upon perpetual completion of all temporal action in terms of the anholonomy of metaphora;

viii The anholonomy of metaphora serves as the memory element of the process of encompassing the obstacle.

In context of the obstacle-oriented approach, there are two further intertwined issues that deserve special attention. The first has to do with the fact that the notion of quanta requires something that distinguishes among them, meaning a mark or a gauge or a boundary. The second has to do with the conditions of qualification of a quantum as a spectral information unit. These two issues open up the vast subject we address by the term *"gnomonics"*, i.e. creatively devising gnomons suited to the *physis* of obstacles that allow flow streams of communication, or else chains of connectivity, to bind together different levels of hypostasis.

The gnomonic enrichment of the obstacle-oriented approach is inseparable from the conception of obstacles as modular sources of invariance. This has been the case since the beginning of natural philosophy and natural science initiated by the magnitude measurement of the height of a pyramid by Thales. It was his innovation to use a vertically placed measuring stick as a gnomon, which to the theory of *homeothesis* connecting the level of actual objects with the level of their shadows by means of proportionality, or invariance of angle, under the temporal action of light fixed at the same time of the day. The notion of an appropriate gnomon in the context of any type of obstacle is always instrumental for obtaining a spectrum, consisting of distinct equivalence classes, or partition cells, or fibers, or finally, distinguishable orbits of a multiply-connected temporal action generating a stream flow around an obstacle.

The abstraction of the initial connotation of a gnomon from its association with the sun-dial of a stick emerged early during the flourishing period of the Alexandrian school of mathematics. A gnomon with respect to an obstacle becomes any suitable form with the following property: If the gnomon is adjoined to the obstacle-laden form it gives rise to a new form self-similar to the original one. This conception of the function of a gnomon in relation to an obstacle is not only ingenious, but it paves the way to a gnomonic derivation of the whole framework of category theory, which represents undoubtedly the most abstract part of modern mathematics. At this early stage, we mention for the sake of the curious and eager reader that the notion of adjoint functors forming an adjunction in category theory is nothing else than a technical elaboration of the way a specific category of frames serves as a gnomon with respect to another obstacle-laden and non-directly accessible one. For the time being, it is enough to highlight that the invention of a suitable gnomon with respect to an obstacle always gives rise to bidirectional bridges binding together the level where the obstacle is located with another level of hypostasis whose meaning derives only from relation to this gnomon. This constitutes the algebraic manifestation of the function of a gnomon. In cases where the bridges are exact inverses to each other, a group structure is generated, where the gnomon is enciphered as the neutral element of this group. In case that the bridges are only conceptually inverse, or else, adjoint to each other, what is generated is rather a categorical *monad* structure.

It is necessary now to examine in more detail the relation between these different conceptions of obstacles as modular sources of invariance and gnomonics. We consider the case that the algebraic manifestation of the function of a gnomon gives rise to a group of temporal actions. The notion of a group is associated with the conception of symmetry relations established by the action-elements of this group. Symmetry means common measure with respect to a standard of measurement, or a standard of demarcation, which is to say a gnomon. In other words, the idea of symmetry is subordinate to the function of a gnomon. Coming back to the context of our inquiry, symmetry of temporal actions can be established only with respect to a standard of partitioning for these actions into distinct classes of equivalence with respect to this standard. Given that these temporal actions initiate cyclic stream flows around the obstacle, such a standard of partitioning can be enunciated only in terms of boundaries for these flows. As such, the notion of a boundary serves as the cipher, or equivalently, the neutral element of the group structure partitioning symmetric temporal actions into distinct cells, and thus, forming a spectrum.

In more technical terms, we obtain for our purposes, a *metabasis* from *homothesis* to *homology* and *cohomology* by inventing a new gnomon bearing the property of topological deformation invariance with respect to the obstacle. This new gnomon is more potent in power than the homothetic one, since

a boundaries form an Abelian group structure, meaning that the cipher becomes of a structural type, and
b the gnomon detects and operates on germs of actions due to the deformation invariance property.

Due to these properties, the modular source of invariance forced by the obstacle is qualified in terms of symmetric, and thus, equivalence classes of germs of temporal actions with respect to the gnomon giving rise to a spectrum. Therefore, the pairing between homology and cohomology is spectrally induced, providing countable means of distinguishing among different

> quanta. Each quantum, namely each symmetric class of germs of temporal actions, specifies a concrete rhythmic arrangement within a regular temporal cycle surrounding the obstacle; it is of a certain harmonic nature.

The above brings us to the second fundamental issue of gnomonics in the context of the obstacle-oriented approach, the one pertaining to the conditions of qualification of a quantum as a spectral information unit. More precisely, we have seen how the function of this deformation invariant gnomon induces a spectrum, each cell of which is indexed harmonically in terms of "countable quanta of metabole by periodic action". The first crucial thing to stress is that the whole spectrum appears only through the simply-connected universal helicoidal covering of the obstacle. Since this universal covering is the global and uniform unfolded domain of all temporal actions embracing the obstacle, we conclude that a quantized spectrum becomes manifest only in a simply-connected domain.

In other words, the manifestation of a spectrum necessitates the gnomonic annihilation of multiple-connectivity associated with a temporal action. In turn, multiple-connectivity is of a foamy and ambiguous nature, that is, it prevents the distinction among streams of flows around the obstacle. Given that these streams amount to chains of connectivity, it is only through boundaries that this fog of objective probabilistic ambiguity may be gradually lifted in time. Note that this form of ambiguity is of an objective character, since it is not based on any type of subjective ignorance. This lifting is enacted by the perpetual completion of all germs of temporal action until simple-connectivity prevails gnomonically and the spectrum of distinguishable quanta becomes manifest.

The second crucial thing to stress is that a distinguishable quantum of the spectrum does not constitute a unit of spectral information yet. It becomes such if and only if it is actually distinguished. Put differently, the spectrum is only the pre-condition for conveying information in terms of quantum units, but it does not qualify any of its elements as pre-existing units of information without any act of actual distinction among them. This can be explained by the fact that the spectrum constitutes the articulation of the symmetry of the gnomon, and as such, the spectrum is precisely the bearer of the simple connectivity emancipated by the gnomon. On the other side, the objectification of a quantum of a certain frequency from the spectrum as the bearer of in-formation, requires an actual distinction that breaks its connectivity bridges with all other symmetrically connected ones with respect to the gnomon. We conclude that a spectral information unit amounts to breaking the symmetry in the connectivity pattern of the gnomon, the latter being the price for information. Note that, by the principle of gnomonic constitution of the spectrum, a spectral information unit pertains to germs of temporal actions, and thus, it is a modular unit of actual distinction between different cells. Henceforth, maximal symmetry amounts to minimal information, whereas minimal symmetry amounts to maximal information. Put differently, connective symmetry and spectral information constitute a *Galoisian* complementary pair with respect to the function of the gnomon in the context of the obstacle-oriented approach.

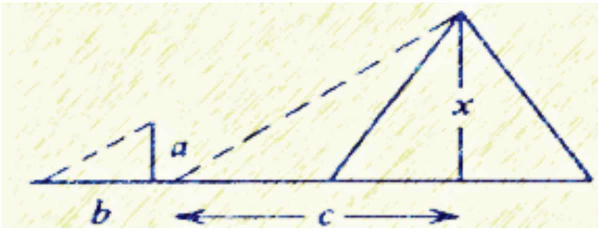
1.4 *HOMEOTICS* OF COMMUNICATION: OBSTACLE-EMBRACING *METAPHORA*

The notions of *analogia* and *metaphora* are considered in their broadest possible meaning, where a quantity, or an object, or a structure, or even a category, is amenable to a certain process of comparison with another of the above kinds. In this sense, analogia and metaphora are detached from the restricted cognitive and linguistic connotations that the widely used notions of analogy and metaphor carry with them. Notwithstanding this fact, the former notions are included as special cases of analogia and metaphora, which are endowed with a well-defined mathematical content, being at the basis of what we call "-natural communication-". The underlying basic idea is that the purpose of this process of comparison is to embrace the obstacles, which are associated in some specific manner with some domain occupied by any of the above kinds through transference to another conjugate domain. The transference is thought of bidirectionally, which means it involves both an encoding and a decoding bridge from some domain to its conjugate domain. The obstacle-embracing function of analogia and metaphora is articulated through congruence or similarity of relationship between instances and not as similarity between the instances themselves pertaining to any of the above kinds. This is a subtle difference that needs to be emphasized, since it discloses the basic characteristic of any analogia or metaphora underlying the power of this function and binding it with the notion of communication between conjugate domains.

Thinking in terms of instances in two different domains between which a congruence or similarity of relationship is established, one is generally not directly comprehensible or accessible, while the other is assumed to be better or more easily tractable. It is important to clarify that according to the above, an analogia pertains to a congruence relation not between two instances, but between the *relations* of these two instances. Thus, an analogia as a congruence relation between relations, involves (at least) two terms, each of which is itself a relation. This congruence relation between relations is called a *homeotic* relation.

As a simple first example, if we use the scaffolding of naive set theory, such that a simple relation is thought of as a binary relation between two sets, then an analogia requires four terms in order to be expressible. The four terms are distributed in two distinct levels or domains, where two of the four terms are placed on the same level so as to express a simple relation. Furthermore, three of the four terms are assumed to be known or directly measurable, or accessible, or more generally, determinable by some method, and the purpose is to determine the fourth.

The primary example of the notion of analogia emanates from Thales' theory of homeothesis or proportionality. It is important to emphasize that the purpose of Thales' theory of proportions had been the measurement of non-directly accessible magnitudes. More concretely, the objective of Thales was to find the directly inaccessible height x of a pyramid, given the length c of its accessible shadow, as well as the height a, and the shadow length b, of an accessible object placed vertically as a reference stick, which plays the role of a measurement rod in homeothesis. The analogia devised by Thales for the resolution of this problem is based on the idea that light coming from the sun induces a congruence relation between the level of heights and the level of shadows for each specific time recording the magnitudes of all the four variables involved. The analogia of homeothesis is expressed symbolically as follows:



(a to b) is as (x to c)

In algebraic terms, the above analogia is expressed by the simple equation $\frac{a}{b}=\frac{x}{c}$, from which the non-directly accessible magnitude x can be determined indirectly as $x=\frac{ac}{b}$. Note that the four terms of this proportion between magnitudes are arranged into two distinct levels according to some qualifying characteristic, i.e. a and x occupy one level as vertical heights, whereas b and c occupy the other level as horizontal shadows.

In point of fact, Thales provided a geometric solution to the problem addressed by homeothesis, since the set-up involving the algebraic equation of proportionality of magnitudes together with its simple algebraic solution presented above, was not available at that time.

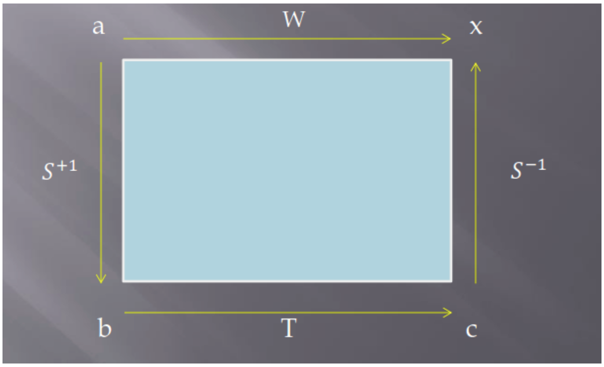
The important fact is that the solution of the proportionality equation involves the group-theoretic operations of multiplication and, inversely, division of positive integer magnitudes. Thus, from the viewpoint of natural communication, the geometric theory of proportions, i.e. the theory of homeothesis, contains all the seeds of abstraction leading to the conception of the modern algebraic structure of a multiplicative group.

In particular, given the multiplicative monoid structure of the positive integers, the solution of the Thalesian problem of proportionality or analogia of magnitudes requires the option to invert the multiplication operation, i.e. it requires the operation of division. In turn, since this is not possible in the context of positive integers, the operation of division entails their algebraic extension to the wider context of the rationals, culminating in the multiplicative group structure of the rationals.

Conclusively, the determination of an unknown magnitude in the Thalesian setting, by analogia, interpreted now algebraically, requires the introduction of the multiplicative group structure of the rational numbers in order to provide a solution to the associated proportionality of magnitudes equation expressing that analogia. In a suggestive manner, we can rewrite the solution of this equation as follows:

$$x = M_a c M_b^{-1}$$

meaning that to obtain the non-directly accessible magnitude x, "multiply by a" (denoted by M_a) the magnitude c, and then, divide by b (denoted by M_b^{-1}). Thus, the determination of inaccessible magnitudes by means of analogia, algebraically necessitates the introduction of the group-theoretic closure structure on magnitudes, equipped with the operation of multiplication and possessing an inverse, which is division.



By extrapolating, we may assume that the resolution of a more general problem, based on analogia (not restricted to the situation of proportionality of magnitudes) implicitly requires for its algebraic manipulation the following:

Firstly, the distribution of the four terms of an analogia into two distinct levels, two of the four on each level, where three of the four terms are assumed to be directly determinable, and the purpose is to determine the fourth.

Secondly, the introduction of an appropriate closed algebraic structure with respect to a process that bridges together the two distinct levels, playing a similar role to the operation of multiplication (between magnitudes at different levels). This multiplicative adjunctive process can be thought of as a directed bridge which connects the upper level with the lower one, where each level is occupied by things belonging to the same class, or domain, or universe of discourse.

Thirdly, the possible determination of the inverse to the multiplicative adjunctive process, called the division process. In many of the cases an exact inverse process (being suggestive of the global schematism of reversibility via another level) may not be attainable, and thus, partially or locally inverse processes should be employed, satisfying appropriate conditions.

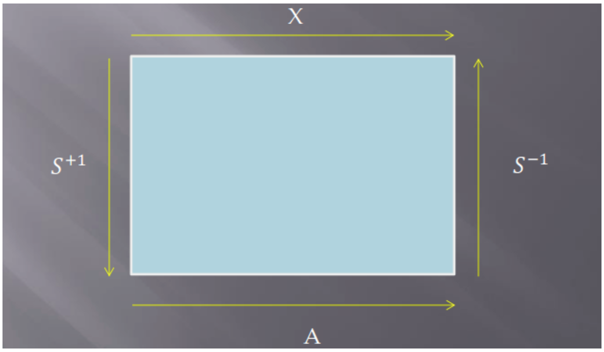
According to the above, in the case that an exact inversion process is available or globally constructible, facilitating an effective exact round-trip between two delineated levels, we call the analogia a metaphora. This conception has an *Aristotelian* origin, formulated in the statement in *Poetics*, according to which: "Metaphora is the substitution of the name of something else, and this may take place from genus to species, or from species to genus, or from species to species, or according to proportion."

Projecting this statement back to the general environment of analogical relations, we conclude that a general analogy between instances may be concerned with class membership or class characterization.

In a nutshell, an analogia, formulated as a relation among four terms distributed at two distinguished domains or levels follows a unifying conceptual thread: Starting from a term at some level the determination of an inaccessible term with respect to the first, at the same level, via a cyclical global round-trip process through another understandable level, involving three stages:

First, setting up an encoding multiplicative adjunctive bridge of correspondence of the initial term at the first level with another term conceived as an instance at the other level. Second, processing the required task at this other level. Third, devising a decoding bridge of correspondence, inverse to the multiplicative one, which facilitates the return at the initial level, and simultaneously resolves indirectly the problem of inaccessibility, or equivalently embraces the obstacle encountered at the initial level.

Subject to the above, characterizing the general thought pattern of an analogia or metaphora, as an attempt to extract the conceptual essence of the Thalesian theory of proportions of magnitudes, and then, abstract it algebraically, we express an analogia or metaphora in terms of the following symbolic relation:



$$X = SAS^{-1}$$

where, the unknown X, at the obstacle-laden level, may be specified by an ordered three-stage process, through some quite easily determinable A at another viable level mediated via the opposite pointing bridges S and S^{-1} connecting the two levels. In case where the bridges S and S^{-1} are exact inverses, and A is considered to be noise-free or *homeorhetic*, we say that the analogy is effective, characterized as a metaphor. In the general case, where the bridges S and S^{-1} are not exact inverses to each other, but only conceptually inverse, they are called adjoint.

The underlying idea in all these cases is that a communication is established between these two levels capable of embracing the obstacles encountered at the initial level. The characteristic feature of an analogia is that a problem, located at a domain or level, requires for its effective treatment to move away from the context of this domain, i.e. to transfer the problem into another non-obstacle-laden domain. This is possible by designing appropriate encoding and decoding bridges between these two domains and following the three-stage process indicated previously, in the specified order. The effect of this, is that, as a result of the established congruence between the domains involved, the return to the initial domain, carries within itself the indirect solution to the problem that has been actually derived at the non-obstacle laden domain. Of course, the designation of the encoding/decoding bridges, being either exactly inverse or adjoint, is of fundamental significance for the operational or computational manifestation of a round-trip between these levels, interpreted as a process of communication. Note that communication is possible if and only if a specific global, or local, or even partial, congruence becomes attainable between the involved domains. The requirement is that the bridges are somehow stable, i.e. they are not dependent on ad hoc choices or oversimplifying assumptions. This is expressed by the adjective natural, which obtains a well-defined meaning in mathematical category theory, as we will discuss later.

From an algebraic viewpoint, the symbolic relation $X = SAS^{-1}$ admits a dual interpretation, namely one in terms of substances and another in terms of operations. In the context of communication, the operational interpretation is preferable, since it places the emphasis on the process devised for overcoming the initial inaccessibility. In this sense, the *symbol* X (indirectly determinable by analogia), followed by the sign of equality, may be interpreted as signifying the total ordered series of the three actions needed for its effective determination via another level, connected to its own by two bridges in opposite directions. It is also instructive to notice that the meaning of the operational interpretation can be captured even from its dual substantive viewpoint, under the convention that the symbolic relation of analogia can be extended in the notational form:

$$X(l_1, l_2) = S(l_1)A(l_1, l_2)S^{-1}(l_2)$$

where, the symbols l_1 and l_2 denote some kind of base locality or base indexing parameter.

1.5 ALGEBRA OF *METAPHORA*: CONJUGATION BETWEEN HYPOSTATIC DOMAINS

In general mathematical terms, the presentation of an effective analogia, or metaphora, in the symbolic form

$$X = SAS^{-1}$$

defines X to be conjugate to A under S, where S^{-1} is considered to be the conceptual inverse of S. This is a useful observation because it associates the algebraic principle of conjugation with the functional role of a metaphora. Since a specific algebraic structure is not pre-supposed ab initio, we call the principle leading to the algebraic expression of a metaphora as the principle of logical conjugation operating between two hypostatic domains.

The algebraic expression $X = SAS^{-1}$ consists of two basic organic structural parts: The first part is delineated by the two conceptually inverse vertically displayed arrows S and S^{-1}, forming the outer part, or the boundary of the analogia or metaphora, interpreted abstractly as a bidirectional bridge of information encoding/decoding between two different levels entering into a communication with each other. The second part is constituted by the horizontally displayed arrow A, forming the inner part of the analogia, and interpreted as a directed process of *rhesis* or *stasis*, i.e. transfer or storage, within the level specified by the first vertical directed bridge. Rhesis or stasis always give rise to a partition spectrum at the obstacle-free level, which contains the blueprint for embracing the obstacle of the initial level under the action of the decoding bridge. Note that the functionality of an analogia or metaphora is always crucially dependent on the interpolation of some appropriate inner part A between the succession of the actions of the inversely pointing bridges. More precisely, if the inner part A is absent, then the outer part simply collapses since it cancels out. Based on this fact, we can formulate the basic properties of logical conjugation as pertaining to an effective analogia as follows:

1 Horizontal Extension of Metaphora in Length: This is equivalent to the juxtaposition of two metaphoras, i.e. two metaphoras sharing the same bridges can be combined horizontally simply by juxtaposing one with another as follows: if $X_1 = SA_1S^{-1}$ and $X_2 = SA_2S^{-1}$, then $X_1X_2 = SA_1A_2S^{-1}$;

2 Vertical Extension of Metaphora in Depth: This is tantamount to the stacking of two metaphoras arising from the substitution of the inner part of a metaphora by another metaphora, such that, the initial metaphora can be accomplished via a splitting into a deeper level of hypostasis, and so on, as follows: if $X = SAS^{-1}$ and $A = TBT^{-1}$, so that, $X = STBT^{-1}S^{-1}$, then $X = (ST)B(ST)^{-1}$;

3 Inversion of Metaphora: This means that if a process X is conjugate to a process A at another level under the action of a bridge S, then A is conjugate to X under S^{-1}, as follows: if $X = SAS^{-1}$, then $A = S^{-1}XS$.

An interesting type of logical conjugation arises in cases where a bridge S equals its own inverse, that is $S = S^{-1}$. An immediate consequence is that if the inter-level transformation S is repeated twice in succession, then it gives the identity, viz. $S^2 = 1$. In this case the bridge S is called an involution bridge. The most well known example of an involution bridge is provided by any device operating strictly between two states, represented by the simplest Boolean algebra containing two truth values (True and False, or 0 and 1).

Then, if the bridge S represents the transformation from the one state to the other (acting like a Boolean negation operator between the levels of truth and falsity), its repeated application for a second time brings us back to the original state. In logical terms, the negation of negation is equivalent to the identity, and therefore, an involution bridge functioning between two states distributed in two distinct levels is a picturesque way of expressing the law of excluded middle in Boolean logic.

Due to the properties listed above, and making temporary use of the scaffolding of naive set theory, an effective analogia expressed by means of logical conjugation can be presented in the form of an equivalence relation, namely as:

$$X \sim_S A$$

stating that X is conjugate to A under S. This is an equivalence relation because it is reflexive, transitive and symmetric: First, due to the property of metaphora extension in length if $X_1 \sim_S A_1$ and $X_2 \sim_S A_2$, then $X_1X_2 \sim_S A_1A_2$. Second, due to the property of metaphora extension in depth, the transitivity condition is established since, if $X \sim_S A$ and $A \sim_T B$, then, $X \sim_{ST} B$. Finally, due to the property of metaphora inversion, the symmetry condition is established since, if $X \sim_S A$, then $A \sim_{S^{-1}} X$.

1.6 PARTITION SPECTRUM AND CIPHERS: VIRTUAL AND ACTUAL DISTINCTIONS

The equivalence relation $W \sim_S T$ stating that the process W is conjugate to T under S, gives rise to a finite partition of all possible processes into blocks or cells constituting the observable spectrum of this partition. Each block of the partition is algebraically the equivalence class of W under some S, denoted by $E(W)$. In other words, $E(W)$ is the class of all processes V that are equivalent to W under S. In this sense, V is in the equivalence class of W if and only if $E(W) = E(V)$. Note that the cells of a partition are non-overlapping, and more precisely, they are mutually exclusive and jointly exhaustive. This is the crucial property that characterizes the notion of the induced observable spectrum, indexed or classified below in terms of different colours:

It is important to point out that a metaphora involving two hypostatic levels pre-supposes implicitly that these two levels can be in principle differentiated. Of course, this is only possible in terms of the inversely pointing bridges S and S^{-1}. Concomitantly, two processes W and T, which are equivalent under S, i.e. they belong to the same cell of the partition, can be differentiated within this cell by the intervention of S and S^{-1}, although they cannot be actually distinguished. The underlying reason is that a cell of the partition should be properly thought of as an intrinsically indistinguishable element by itself, which however, can be potentially differentiated internally (e.g. by enforcing the encoding and decoding bridges S and S^{-1}), and most important, can actually be resolved, refined, and distinguished externally (e.g. by external acts of distinction).

For this purpose, in the context of a finite partition, it is instructive to introduce the difference between virtual and actual distinctions. An actual distinction is characterized by pairs (W, W'), where W and W' belong to distinct cells of the partition. In contrast, a virtual distinction is a differentiation internally within some cell of the partition, which is invoked by the explicit enforcement of S and S^{-1}. The interesting question, which has far reaching consequences in relation to the notion of information, is when a virtual distinction becomes an actual distinction. This is only possible by means of refining the partition. Equivalently, partition refinement requires a decoupling of the encoding and decoding bridges, since for a pair (W, W') qualified as an actual distinction W and W' belong to distinct cells, which entails that there are no bridges between them in force. If we adopt the provisional definition that information emerges through actual distinctions of a partition in the present context, then the procedure of partition refinement amounts to decoupling bridges of metaphora, and in this way, obtaining new information.

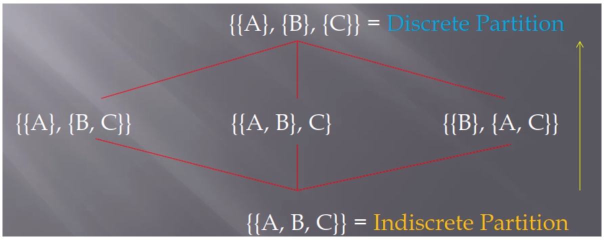
As a consequence, the notion of metaphora expressed through conjugation, incorporates three distinct types of structural ciphers. The first is the structural cipher of an algebraic group that expresses the notion of symmetry. In particular, an encoding bridge S together with the inverse decoding bridge S^{-1} differentiating a process W from a process T within a cell of the partition they enforce, make W and T symmetric to each other. The second is the structural cipher of a partial order that expresses the notion of distinguishability. In particular, external acts of refinement, i.e. acts of refining the grain of resolution in a spectrum to obtain actual distinctions -and thus- discern new information, induce a partial ordering relation among partitions. The third is the structural cipher of a category as a common abstraction between the notions of a group and a partial order. In particular, in our context the notion of a category is the algebraic cipher required to express the duality between sets and partitions, which emerges simply from qualifying the differentiation within a cell of a partition, given the bridges S and S^{-1}, as a set of virtual distinctions.

It is worth focussing, at this stage, on these structural ciphers in more detail. The structural cipher of an algebraic group is tantamount to the identification and classifications of the cells of a partition defined by metaphora, i.e. through conjugation. A group G is closed with respect to an operation, i.e. addition or multiplication, and it always contains an identity element, called the neutral element of the group. In this manner, the structural action of a group as a cipher is encoded in its neutral element. It is because of the neutral element that for any S in the group G, there exists a unique inverse S^{-1} such that their operational composition results in this element. A group is characterized via its action, i.e. it is the notion of a group action that structurally expresses the notion of equivalence in the cells of the partition. In this manner, the notion of a partition cell, under a group action that realizes it, is equivalent to the notion of an orbit or fiber of this group action.

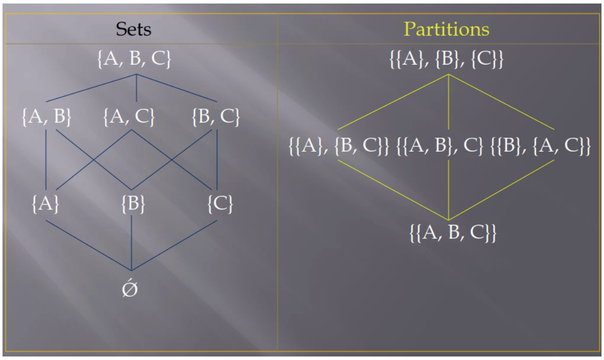
Next, we may consider the totally indiscrete partition that can be potentially differentiated under a group action, which essentially involves the enforcement of the encoding and decoding bridges in this partition. Since this partition is indiscrete by hypothesis, all virtual distinctions within it are symmetric to each other. In other words, we derive that the state of maximal symmetry equals a state of maximal indistinguishability. Equivalently, since there are not any actual distinctions in the indiscrete partition that constitutes a single block of the same colour, e.g. black colour, there is zero information that is extractable from it without any refinement.

The procedure of refinement of the indiscrete partition is possible by symmetry breaking; in other words, actual distinctions in a refinement of the indiscrete partition require the breaking of the maximal symmetry of that partition. Symmetry breaking makes information discernible in terms of the actual distinctions of the refined partition. On the other side, maximal refinement destroys all symmetry, i.e. it breaks all the bridges, and in this way is equivalent to maximal information, since everything becomes actually distinct in the discrete partition. Refinement is definable as a relation of partial order among partitions, where the indiscrete partition is the minimal element, and the discrete partition is the maximal element of this partial order.

To sum up, given a group action, we think of differentiation within a cell of the induced partition in terms of the set of virtual distinctions belonging to an orbit or fiber of this action. Moreover, we consider the partial order of refinement in terms of virtual and actual distinctions. Then, in the simple case comprising three elements amenable to actual distinctions, the order of refinement looks as follows:



The partial order of partition refinement is dual to the partial order of subsets of a set, and this fact renders the respective categories dual to each other. Thus, starting from metaphora and its properties as expressed through conjugation, we derive the categorical duality between partitions and sets. Note that the dual partner of an element of a subset is given by an actual distinction of a partition. Clearly the indiscrete partition does not have any actual distinctions, whereas all the possible actual distinctions are the actual distinctions of the discrete partition.



Moreover, since differentiation within a partition cell is qualified by the set of virtual distinctions that can be made within this cell, and due to the fact that a partition cell is identified with an orbit of a corresponding group action, based on the notion of symmetry induced by the inverse bridges of a metaphora, the duality between sets and partitions arises from the inverse correlation between symmetry and information. In particular, this inverse correlation boils down to the existence of two partial orderings that are in cadence with each other but the first is increasing whereas the other is decreasing. Note that both of these partial orders are bounded from above and below, such that the state of maximal symmetry in the first ordering corresponds to the state of null actual distinctions, and thus minimal information, in the second ordering, and so much is equally true of the converse. These two inversely correlated partial orders may be thought of as orthogonal to each other, if depicted jointly, since symmetry and information are complementary in the context of metaphora.

Now, suppose that $M \subseteq K \times K$ is the equivalence relation induced by logical conjugation on a set of processes or relations K. We may consider a category (K, M) in which K is the set of objects (standing for processes), M is the set of arrows, and the source and target maps $M \to K$ are given by the first and second projection. Then given X and A in K, there is precisely one arrow (X, A) if X and A are in the same equivalence class, viz. they are metaphorically related by conjugation, while there is none if they are not. Then transitivity assures us that we can compose arrows, while reflexivity tell us that over each process X in K there is a unique arrow (X, X), which is the identity. Finally symmetry tells us that any arrow (X, A) has an inverse (A, X). Thus, (K, M) is a *groupoid* (category in which all arrows are isomorphisms) such that, from a given object of this category (process) to another there is at most one arrow (if they are metaphorically related). Conversely, given a groupoid, such that from a given object to another there is at most one arrow, if we denote by K the set of objects and by M the set of arrows, the source and target maps induce an injective morphism $M \infty K \times K$, which gives an equivalence relation on K with the concomitant interpretation.

1.7 INTELLIGIBILITY OF THE COSMOS: STRUCTURAL EXTENSION OF ALGEBRAIC SCALARITY

Let us now examine the functionality of logical conjugation from a structural algebraic standpoint. We have already claimed previously that the resolution of the Thalesian problem of determination of an inaccessible magnitude by the method of proportions, implicitly contains the seeds of discovery of the multiplicative group structure of the (positive) rationals. More precisely, multiplication is an essential operation that can be performed on the integers endowing them with the closed structure of a multiplicative *monoid*. Division, the inverse operation to multiplication, is nevertheless not a total operation on integers, and thus, the determination of inaccessible magnitudes on the basis of proportion cannot be effectively performed within the reference domain of integers.

To achieve a total operation of division, to resolve the Thalesian problem, we are obliged to extend the initial domain into a new domain of numbers, where the required inverse operation can be always implemented. This means that the resolution of the problem requires an appropriate extension of the initial closed structure (integers) with respect to the operation of multiplication into a new structure (rationals), which is closed with respect to both multiplication and its inverse operation of division. This is a recurring theme in universal algebra and thus it deserves a closer analysis in order to explain the particulars of its implementation by means of the logical conjugation strategy.

For this purpose, let us state explicitly the ordered series of three processes that have to be performed, according to the general pattern characterizing metaphoras, for the construction of the field of rationals from the ring of integers. We recall that the rationals constitutes the set of all fractions a/b, a and b integers, $b \neq 0$ with the usual relation $a/b \equiv c/d$ if $ad = bc$, which makes invertible every non-zero element of the integers.

The basic ingredient for the construction of the field of fractions is the fact that the set of non-zero elements of the integers is multiplicatively closed. The structural metaphora characterizing completely this construction is technically called the process of localization of the commutative unital ring of the integers $\mathbb{Z}$ with respect to the multiplicative closed subset of the non-zero integers. The whole purpose of this structural metaphora by conjugation is to make every element of the multiplicative closed subset of non-zero integers invertible, such that the new structure of numbers obtained in this manner, fulfills the following objectives: First, it bears a structural similarity to the initial domain of numbers, viz. it is also a commutative unital ring with respect to addition and multiplication. Second, the operation of division (inverse to multiplication) can be performed by the existence of inverses of non-zero integers, which have been incorporated in the new extended closure domain of numbers. Third, as a consequence of the above, the initial domain of numbers together with their arithmetic can be embedded in the new one.

We consider the commutative unital ring of integers $\mathbb{Z}$ and let $S \subseteq \mathbb{Z}$ be the multiplicative closed subset of non-zero integers. The first step is to set up a directed bridge from the level of commutative unital rings to the level of sets, encoding the process of extending the underlying set-theoretic domain of integers $\mathbb{Z}$ into a new domain formed by the cartesian product of sets $\mathbb{Z} \times S$. Note that the ordered pairs of integers (a,s) with $s \neq 0$, are not supposed to have any a priori structure, since their existence is required at the level of sets by means of the encoding directed bridge connecting the structural levels involved. In this extended new set-theoretic domain the initial task can be facilitated by imposing the homological equivalence criterion, according to which the ordered pair of integers (va, vs) should be equivalent to (a,s) for any non-zero integer v. Technically this condition is described in the following way:

In the set $\mathbb{Z} \times S$ we define the following binary relation: (a,s) $\Diamond$ (b,t) if and only if there exists $v \in S$ such that: $v(at - bs) = 0$. The relation $\Diamond$ is an equivalence relation, partitioning the set $\mathbb{Z} \times S$ into equivalence classes. We will denote the quotient set by $\mathbb{Z}_S$, and the equivalence class of (a,s) by the fraction symbol a/s. Thus, the quotient set $\mathbb{Z}_S$ contains elements which can be interpreted as fractions, bearing the semantics of numbers allowing division by non-zero integers.

The structural metaphora is completed via logical conjugation by setting up an inversely directed decoding bridge from the level of sets to the level of commutative unital rings, effectuating the indirect round-trip as follows: We set $a/s + b/t := (ta + sb)/st$, $(a/s)(b/t) = (ab/st)$ for every a/s, $b/t \in \mathbb{Z}_S$. The operations are well defined and endow $\mathbb{Z}_S$ with the structure of a ring. The zero and unit elements are, respectively, $0/s$ and s/s, for every $s \in S$. Finally, we define the canonical morphism of rings $h:\mathbb{Z}\to\mathbb{Z}_S$, given by $h(a)=a/1$, for every $a \in \mathbb{Z}$. Note that for any $s \in S$ we have that $1/s$ is the inverse of $h(s)$ in $\mathbb{Z}_S$. Hence, $\mathbb{Z}_S$ is the smallest ring containing $\mathbb{Z}$, in which every element of the multiplicative closed subset of non-zero integers S is invertible.

Thus, the extension of scalars of the commutative unital ring of integers $\mathbb{Z}$ by means of algebraic localization, with respect to the multiplicative closed subset of non-zero integers, is understood as a structural algebraic metaphora implemented by logical conjugation. The structural effect of this metaphora by conjugation is the addition of multiplicative inverses to the elements of the multiplicative closed subset $S\subseteq\mathbb{Z}$, such that the extended ring $\mathbb{Z}_S$, consists of fractions a/s, where $a\in\mathbb{Z}$, $s\in S$. Moreover, the conceptualization of algebraic localization as a structural metaphor for the resolution of the general problem of making division a total operation by congruent extension of structure via the logical process of conjugation, permits its application in generalized structural environments as we shall see in the sequel.

1.8 STRUCTURAL *METAPHORA*: ADJOINING - PARTITIONING - QUOTIENTING

It is instructive to explicate in more detail the conjugation strategy related with the efficient functioning of the above structural metaphora. First, we observe that the encoding process of the underlying set-theoretic domain of $\mathbb{Z}$, utilized as an architectonic scaffolding, into the new domain formed by the cartesian product of sets $\mathbb{Z}\times S$ takes place by means of extending the scalars of $\mathbb{Z}$ with respect to the scalars of the multiplicative closed subset S of $\mathbb{Z}$. This means that the extension of scalars of the set-theoretic domain of $\mathbb{Z}$ is effectuated by adjoining to $\mathbb{Z}$ the scalars of a well-defined internal algebraic part S of $\mathbb{Z}$ distinguished by its anticipated operational role.

Second, the level of sets can be thought of as a temporary underlying scaffolding via which logical conjugation can be effectively applied. More precisely, at the level of sets the operational role of the distinguished part S of $\mathbb{Z}$ can be implemented by the imposition of an appropriate homeotic equivalence relation on the previously extended set-theoretic domain $\mathbb{Z}\times S$. The conceptual underpinning of this process is the identification of those elements of the extended domain $\mathbb{Z}\times S$, which exhibit a certain homeotics of behaviour, which we symbolize by the relation R. Any suitable criterion of homeotic indiscernibility must lead to a partition of $\mathbb{Z}\times S$ into disjoint classes of elements bearing the imposed homeotic relation R, and hence R must be an equivalence relation. Since, the imposition of such a relation R effectuates a classification of the elements of $\mathbb{Z}\times S$ into disjoint classes of equivalent elements, partitioning it in the particular way determined by R, the latter can be thought of as a homeotic perspective. It follows that an equivalence class modulo the homeotic perspective R, consists of all the elements of $\mathbb{Z}\times S$, indiscernible with respect to R, and thus homeotically identical.

More specifically, the homeotic perspective R imposed on $\mathbb{Z}\times S$, requires that the ordered pair of integers (va,vs) should be homeotically identical to (a,s) for any non-zero integer v, under the intended interpretation of the homeotic class of (a,s) by the fraction symbol a/s. Note that the homeotic classes (a,s) are metaphorically interpreted as elements a/s, being assigned a new name, viz. fractions, of a new set, namely of the quotient set $\mathbb{Z}_S$. It is important to notice that consequent to the transition from $\mathbb{Z}\times S$ to $\mathbb{Z}_S$ is the replacement of equivalence modulo R, viz. R-perspective homeotics, by equality (identity) of elements in the quotient $\mathbb{Z}_S$.

Third, the structural metaphora realizing the result of the applied logical conjugation is completed by means of the inversely directing bridge from the level of sets back to the initial level of commutative unital rings. The semantic aspect of this bridge amounts to a re-casting of the elements of the quotient set $\mathbb{Z}_S$, as elements of a new ring, viz. as elements of the same closed structural genus as the initial $\mathbb{Z}$. This is accomplished by modifying appropriately the addition and multiplication operations referring to these new elements (fractions). This modification takes place according to the principle that the new operations should incorporate and reproduce the effect of the old ones, when restricted to the old elements, being dressed in the new form imposed by the adopted homeotic perspective.

The important thing to notice is that the completion of the structural metaphora according to the logical conjugation strategy described above, accomplishes the task of making the operation of division total, and thus, resolves the geometric problem of homeothesis in a structural way. In this way, from the standpoint of the ring of integers, the structural metaphora permitting the unconstrained action of the division operation on magnitudes, belonging now to an extended closed partially congruent structure of the same algebraic genus (ring of rationals), accomplishes the interpretation of division as an emergent well-defined total operation. This is due to the fact that the operation of division acts properly on this new kind of species (fractions), which remains closed with respect to its action. The logical conjugation resolves the original Thalesian problem structurally because fractions are formed at the set-theoretic level, and then lifted at the ring-theoretic level by means of encoding/decoding bridges. In particular, fractions are formed by the inverse processes of extending the set-theoretic domain $\mathbb{Z}$ to the larger one $\mathbb{Z} \times S$ with respect to the part S, and then restricting this extended domain by collapsing it, viz. by partitioning it homeotically into disjoint classes, with respect to the imposed internal homeotic perspective subsumed.

In more general terms, the above algebraic localization structural metaphora is a particular application of the logical conjugation strategy designed for the resolution of a specific problem involving (at least) two delineated structural hypostatic levels, and based on the existence of a pair of inversely pointing bridges connecting these two levels, as follows: First, by means of an extension bridge, encoding the information of a structural domain into a new extended one assuming existence at a different level. Second, performing the required task at that level by realizing an appropriate equivalence relation, and subsequently forming the associated quotient structure. Finally, by means of a reciprocal bridge, decoding the acquired information in a structural form congruent to the form of the structural domain we started with, according to the specification of the initial level.

1.9 INDIRECT SELF-REFERENTIAL *METAPHORA* AND *HOMEOTIC* CRITERION

At a further stage of development of these ideas, we realize that the successful implementation of the conjugation strategy, concerning structural metaphoras, necessitates primarily the investigation of the meaning of an effective analogia within the same algebraic structural genus. This task is important, because it clarifies the nature of an indirect analogical self-referential relation taking place within a certain closed structural genus. From the general context of the preceding analysis, it has become clear that at least, referring to the set-theoretic level of magnification, a set can be related to a distinguished part of it by the imposition of an equivalence relation on their jointly formed cartesian product with respect to a homeotic perspective, which reciprocally required the delineation of that distinguished part, in the first place. The total process can be cast into the pattern assumed by a self-referential structural metaphora as follows:

Initially, we assume that a set of elements, considered as an individual object within the genus of sets (characterized by the membership relation), can relate to itself by separation of a well-defined part of it, viz. a subset bearing the functional role subsumed by a particular homeotic perspective. In turn, this homeotic perspective can be applied to the extended object obtained from the initial object by adjoining the distinguished part. Finally, using the quotient construction, we collapse the extended object into a new partitioned object belonging to the same genus. Of course, this is only possible if all of the following conditions can be fulfilled: First, if the initial object can split its substance between two internal levels or hypostases within the same genus, such that the latter, formed by extension with respect to a part, is also an object of the same genus encoding the former. Second, if the application of the homeotic perspective on the extended object partitions it into equivalence classes, forcing in this way a homeotic criterion of identity, or equivalently, an indiscernibility relation with respect to this homeotic perspective, at the same level. Thirdly, if the equivalence classes of the quotient can be re-interpreted as elements of a new object of the same genus, being formed at the initial level by identifying equivalent elements with respect to the homeotic perspective.

It is important to realize that an indirect self-referential relation, implicated by logical conjugation within the same genus, accomplishes precisely the satisfaction of the above conditions. This is possible by means of two inverse internal bridges connecting these two separate levels of hypostasis into a non-contradictory circular pattern as follows: the first bridge carries out the extension process of an object to another level of hypostasis, being formed by adjoining to it a distinguished part, delineated by the functional role subsumed under a homeotic perspective. At the new level, an appropriate equivalence relation on the extended object implements the functional role of the homeotic perspective, that is, implements a homeotic criterion of identity. As a result, we end up with a partitioning of the extended object into a set of equivalence classes constituted by indiscernible elements with respect the imposed criterion. Finally, an inverse bridge performs the transition back to the initial level, by collapsing the extended object with respect to the homeotic perspective, and thus, transforming the homeotic relation into an equality (identity) of elements in the quotient set, formed back at the initial level. Notice the crucial point that the quotient structure formed by returning to the initial level has to be again a set-theoretic object, that is, it must be congruent to the structural specification of the initial object we started with.

After this series of remarks, there arises the natural problem of applying the logical conjugation strategy realizing an indirect self-referential metaphora into the context of objects belonging to some algebraic structural genus, like groups, rings and algebras. This becomes possible, if we formalize the notion of a homeotic perspective as an equivalence kernel of a comparison morphism (homomorphism) between structures of the same algebraic genus. Note that the functional role subsumed by a homeotic perspective, elevates the relation of equivalence among elements belonging into the same equivalence class at the level reached by descending the first bridge, to a relation of equality (homeotic identity) at the initial level regained by ascending back through the inverse bridge. In turn, this constitutes the precise implementation of what we call a *homeotic criterion of identity*.

Set-theoretically speaking, this amounts to the implication that if two elements α and β of the extended set, at the new internal level of hypostasis, are equivalent with respect to a homeotic perspective R, viz. $\alpha R \beta$, then their images inside the quotient set, interpreted as new elements, at the initial level, are identical, viz. $[\alpha]_R = [\beta]_R$. Based on this argument, we can deduce the modeling of the notion of a homeotic perspective between structures of the same algebraic genus, by passing into some appropriately restricted type of equivalence relation by means of logical conjugation, depending on the algebraic genus considered.

1.10 EQUIVALENCE AND *HOMEOTIC* KERNELS OF ALGEBRAIC *GENUS*

In a general context, the minimum requirements for an algebraic system include the existence of a set S with an equality relation for which there is defined a binary law of composition, namely, a single-valued function of pairs α, β such that $\alpha\beta$ is in S for α, β in S. Adopting this as our starting point, we superimpose an equivalence relation R on S in order to investigate how a desired restricted type of equivalence relation arises. Namely, denoting by Σ the set of equivalence classes $C_\alpha \bmod R$, we raise the following question: Can an operation $\circ$ be defined in Σ based upon the composition operation in S?

We proceed along the lines of what might be a first attempt to investigate this question by defining:

$$C_\alpha \circ C_\beta = C_{\alpha\beta}$$

The above apparently makes the product dependent upon the choice of class representatives. This deficiency can be amended by requiring that, if $C_{\alpha'} = C_\alpha$ and $C_{\beta'} = C_\beta$, then $C_{\alpha'}C_{\beta'} = C_\alpha C_\beta$. This amounts to the assertion, if $\alpha' R\alpha$ and $\beta' R\beta$, then $(\alpha'\beta')R(\alpha\beta)$. Equivalently stated, we obtain the condition: $\alpha' R\alpha$ implies that $(\alpha' x)R(\alpha x)$ and $(x\alpha')R(x\alpha)$ for all x. We call regular those equivalence relations which satisfy the condition above. The latter constitutes a necessary and sufficient condition upon R in order that

$$C_\alpha \circ C_\beta = C_{\alpha\beta}$$

stands for a well-defined operation. Then, we can easily deduce that the correspondence φ of S onto Σ defined by: $\varphi(x) = C_\alpha$ if and only if $x \in C_\alpha$ is an algebraic homomorphism, called the natural homomorphism. Essentially, from a reciprocal standpoint, Σ should be a homomorphic image of S under a correspondence, mapping all elements of S belonging to an equivalence class onto an element of Σ. But the existence of such a homomorphism immediately implies the existence of one mapping the class containing α upon C_α, and the homomorphism property then requires that $C_\alpha \circ C_\beta = C_{\alpha\beta}$ holds true.

The central idea explained previously can be now easily applied to structures of some *algebraic genus*, for example, to groups. In this case, we consider a group S together with a regular equivalence relation R. Then, defining an operation in $\Sigma = \{C_\alpha, C_\beta, \ldots\}$ according to the composition rule $C_\alpha \circ C_\beta = C_{\alpha\beta}$, we obtain a homomorphic image of S. Since a homomorphic image of a group is necessarily a group, we deduce that Σ is actually a group whose identity element is C_e, where e is the identity element of the group S.

The above construction shows that the process of shrinking a group S with the aid of a regular equivalence R produces a homomorphic image Σ of S being also a group, thereby preserving the structural specification of its algebraic genus. Conversely, given a homomorphic image Σ of S, there is defined a partition, and therefore, an equivalence relation R on S. Moreover, the homomorphism property implies that R is a regular equivalence relation.

In a nutshell, we conclude that in the case of groups, the problem of finding all homomorphic images of S reduces to that of finding all regular equivalence relations over S. For this purpose, we make use of the coset decomposition of a group S with respect to a subgroup H. More precisely, we define, $\alpha R\beta$ if and only if $\alpha = h\beta$, where, $h \in H$.

We can easily show that R is actually an equivalence relation, such that the equivalence class $C_\alpha = H\alpha$, called the *right coset* of H. Moreover, since $\alpha' R\alpha$ implies that $(\alpha' x)R(\alpha x)$, the equivalence relation R is right regular.

But conversely, starting with a right regular equivalence R in S we find that C_e is a subgroup and $C_\alpha = C_e\alpha$, since $\beta R\alpha$ implies that $(\beta\alpha^{-1})Re$; Hence $\beta\alpha^{-1} \in C_e$, or, $\beta \in C_e\alpha$ and conversely.

Thus, the problem of finding the various right regular equivalence relations in S is reduced to the problem of determination of the right coset decompositions of S with respect to its subgroups.

Precisely analogous considerations establish that the various left regular equivalence relations in S are completely determined by the left coset decompositions of S with respect to its subgroups. Thus, we conclude that, if R is a regular equivalence relation, then, on the one side, it defines a left coset decomposition with respect to the subgroup H of all elements x such that xRe, and on the other side, it defines a right coset decomposition with respect to the same subgroup. Hence R stems from a subgroup for which the left cosets are identical with its right cosets. Such a subgroup N is called a normal subgroup of S, satisfying: $xN = Nx$ for all x in S.

Thus, a regular equivalence relation R in S stems from a normal subgroup N of S, viz., a subgroup remaining invariant under logical conjugation, meaning that $N = xNx^{-1}$ for all x in S. Conversely, a normal subgroup of S defines a regular equivalence relation on S. Now, if N is a normal subgroup of S, then its cosets $C_x = xN$ form a group with the following composition rule of closure:

$$\alpha N \circ \beta N = (\alpha\beta)N,$$

or equivalently, $C_\alpha \circ C_\beta = C_{\alpha\beta}$ holds. The resulting quotient $\Sigma = S/N$ is a group homomorphic to S and constitutes that group, which collapses the normal subgroup N of S to the identity element of Σ.

Conversely, every homomorphic image of S can be duplicated by, hence it becomes isomorphic to, such a quotient group.

The completely analogous analysis for the case of rings yields the corresponding homomorphism theorem with the same efficiency. Thus, we have deduced the modeling of the notion of a homeotic perspective between structures of the same algebraic genus, by the concept of regular equivalence relations. Consequently, the implementation of self-referential metaphoras within the context of objects belonging to some algebraic structural genus, becomes possible if we formalize the notion of a homeotic perspective precisely as a regular equivalence kernel of a comparison morphism (homomorphism) between structures of the same algebraic genus.

More concretely, in the case of groups, we have the following: Let S and T be groups and let ϕ be a group homomorphism from S to T. If e_T is the identity element of T, then the kernel of ϕ is the subset of S consisting of all those elements of S which are being mapped by ϕ to the element e_T:

$$Ker(\phi) = \{x \in S : \phi(x) = e_T\}$$

Since a group homomorphism preserves identity elements, the identity element e_S of S must belong to $Ker(\phi)$. By the preceding analysis, it turns out that $Ker(\phi)$ is actually a normal subgroup of S. Thus, we can form the quotient group $S / Ker(\phi)$, which is naturally isomorphic to $Im(\phi)$, viz. the image of ϕ (which is a subgroup of T).

Analogously, in the case of rings with a unit element we have the following: Let S and T be rings and let ϕ be a ring homomorphism from S to T. If 0_T is the zero element of T, then the kernel of ϕ is the subset of S consisting of all those elements of S which are being mapped by ϕ to the element 0_T:

$$Ker(\phi) = \{x \in S : \phi(x) = 0_T\}$$

Since a ring homomorphism preserves zero elements, the zero element 0_S of S must belong to the kernel. It turns out that, although $Ker(\phi)$ is generally not a subring of S, since it may not contain the multiplicative identity, it is nevertheless a two-sided ideal of S. Thus, we can form the quotient ring $S / Ker(\phi)$, which is naturally isomorphic to $Im(\phi)$, viz. the image of ϕ (which is a subring of T).

The effective generation of self-referential structural metaphoras via logical conjugation in the context of some algebraic genus, implicated by the action of regular equivalence relations within this genus, provides a powerful methodological device for the resolution of a wide range of problems. Moreover, a self-referential structural metaphora may be combined with another type of metaphora, for instance a genus to species metaphora.

As an example, we may consider the case of the genus of (finite) groups. We have already seen previously that a regular equivalence relation on a group corresponds to a normal subgroup of this group, interpreted as an internal homeotic perspective. More precisely, this homeotic perspective constitutes the regular equivalence kernel of the homomorphism from the group to its corresponding quotient group. A natural problem arising in this context refers to the possibility of decomposition of a group into a finite series of non-further decomposable groups (simple groups) using the method of division with respect to internal homeotic perspectives, namely, with respect to normal subgroups. This is the problem of solvability of a group-theoretic structure, which has been first posed in the context of Galois theory. If solvability is attainable, then the initial group can be thought of as being decomposed into a finite series of irreducible group layers (factor groups) adjoined to each other in a proper way.

This problem can be successfully tackled by means of the conjugation strategy, if we combine the previously explained self-referential structural metaphora with a genus-to-species metaphora between the genus of multiplicative groups and the species of the integers. In the context of the latter metaphora, if a group corresponds to an integer, then a normal subgroup corresponds to a divisor of this integer and the associated quotient group corresponds to the quotient of the integer by the divisor. Furthermore, a non-further decomposable group (simple group) corresponds to a prime integer number, and finally, the notion of decomposition of a group into a finite series of simple groups using the method of division with respect to normal subgroups corresponds to the Euclidean algorithm for divisibility of the integers.

1.11 LOGICAL CONJUGATION VIA A *GNOMON*: *HOMEOTIC* CRITERION OF IDENTITY

It is instructive to emphasize that the appropriate operational implementation of all different manifestations of the logical conjugation strategy rests only on two prerequisites:

First, the ability to induce a meaningful stratification into different hypostatic domains or levels which can be connected by means of encoding and decoding bridges. In the general case, we may think of these levels as structural ones. The stratification may even involve substructures of an initially given structure, delineated according to a specific characteristic and adjoined to the initial structure, as separate levels. The latter is particularly suited to the resolution of self-referential problems through a cyclical conjugation process by means of the reciprocal and reflexive techniques of descending and ascending.

Second, the ability to establish a congruence, or a homeotic relation among the stratified levels. It is precisely the ingenuity of a homeotic criterion that provides the seed for the successful implementation of the logical conjugation strategy. Put differently, an effective analogia or metaphora subsumed by logical conjugation requires an appropriate criterion of homeotics among stratified levels in order to operate. We point out that the notion of metaphora literally means transference or transportation. Thus, logical conjugation can be conceived as a logical transportation process involving at least two separate levels according to a specific homeotic relation among these levels. We also note that metaphora may refer to transportation of information or structure or matter or energy or whatever else this notion can refer to, whereas the logical conjugation strategy via which it takes place is indifferent to its particular qualifications. This provides the sought for universality in the application of logical conjugation to different fields.

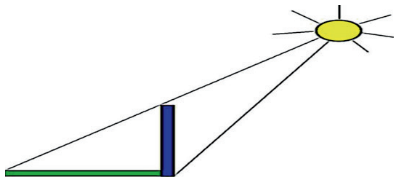
From the above, we deduce that what is crucial for the logical conjugation method is the establishment of some appropriate homeotic criterion operating among the stratified hypostatic levels. Then, based on this homeotic criterion it becomes more tractable to devise appropriate encoding and decoding bridges reciprocally connecting all different levels and effectuating a metaphora. It is interesting to note that from the present viewpoint the notion of homeotics bears a logical function although it is usually introduced and implemented by topological means. At least, it is important to stress that a homeotic criterion is independent of local metrical spatiotemporal notions of distance. For this reason, it can operate non-locally or among different scales. The ubiquity of a homeotic criterion is that it establishes some particular measure of invariance among the stratified levels. This measure can be expressed as an arithmetic invariant, like a ratio or a fraction, or even in structural terms like a group or groupoid. The essential thing is that inter-level connectivity and congruence obtained by metaphora, requires a homeotic criterion in order to be expressed via the logical conjugation strategy, just as the homeotic criterion rests conversely on such inter-level connectivity.

In standard mathematical terminology, what we call a homeotic criterion appears in a variety of different formulations, which are unified conceptually from our perspective. This unification is facilitated by means of logical conjugation and its net effect, which is metaphora according to some qualification, and ultimately serves as an effective means of coping with complexity and self-reference. For instance, a homeotic criterion may be expressed in the simplest possible manifestation as a relation of homeothesis or proportionality of integer magnitudes as in the original Thalesian conception. It may also be expressed as a relation of similarity between two square matrices, where the homeotic criterion is the representation of the same linear transformation with respect to two different bases of a vector space. In this case, the logical conjugation strategy resolves the problem of diagonalization via the method of eigenvalues.

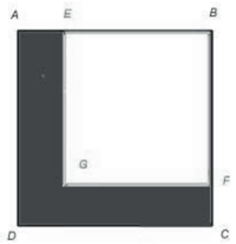
In the field of differential topology and differential geometry a homeotic criterion is provided by the notion of a *local homeomorphism* or *local diffeomorphism* correspondingly. We may note parenthetically that from the perspective of logical conjugation the notions of topological or differential manifolds defined by descending to simpler spaces like the Euclidean ones and then ascending back via the method of gluing from the local to the global level, are solely needed for the formulation of the metaphora process of differentiation, called *covariant parallel transport*, and giving rise to the invariants of curvature. Finally, a homeotic criterion may be literally expressed in standard algebraic topological terms, namely by means of homology and cohomology theory. In broad terms, *homology theory* establishes invariant measures of topological similarity in terms of a series of groups stratified into different scales or dimensions. The topological similarity is defined by means of classifying chains of connectivity into two classes, called *cycles* and *boundaries* correspondingly. More precisely, two cycles are homologically equivalent if they differ by a boundary. The dual theory, called *cohomology theory*, is based correspondingly on the notion of cochains of connectivity, which are classified respectively into *cocycles* and *coboundaries*. In this case, two cocycles are cohomologically equivalent if they differ by a coboundary. For example, in the case of *de Rham* cohomology theory, the cocycles are represented as *closed differential forms* and the coboundaries as *exact differential forms*.

A natural question arising in this context is the following: Notwithstanding the technicalities involved, for example in the setting of homology and cohomology theories of various forms, is there a guiding concept that lends itself to a proper and efficient depiction of a homeotic criterion? In other words, what is the common thread between the homothesis equivalence relation and the more sophisticated algebraic-topological homology equivalence relation which renders them both amenable to the logical conjugation strategy?

We argue that the common conceptual thread for establishing a proper homeotic criterion is provided by the use of a gnomon. The intuitive idea of a gnomon also makes more easily conceptualized the quite abstract notion of a homeotic algebraic kernel. The best definition of the notion of a gnomon has been given by the great mathematician *Heron* of Alexandria in the following terms: A gnomon is that entity which, if it is adjoined to some originally given entity, results in a new augmented entity becoming homeotic, or partially congruent, or even similar, to the original one. In order to understand the depth of this simple-seeming definition of a gnomon it is necessary to start from its initial conception in the context of the Thalesian theory of homeothesis. In this context, the gnomon is, literally speaking, the part of the sundial that casts the shadow.

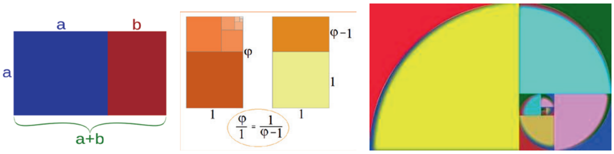


We can easily see that it is exactly the adjunction of the gnomon to the pyramid, which induces a homeothetic equivalence relation between the level of objects and the level of their shadows with reference to their magnitudes at the same time of the day, and consequently makes logical conjugation operative for the determination of the non-directly accessible magnitude of the height of the pyramid in terms of proportion. In its simplest possible form the general process of adjoining a gnomon in order to obtain a relation of homeothesis may be visualized as follows:



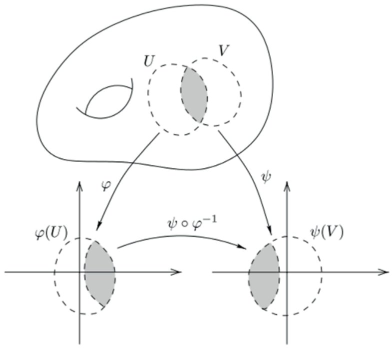
Formally, the relation of homeothesis is an equivalence relation, and thus induces a partition spectrum consisting of equivalence classes standing for the blocks or cells of this partition. The quotient structure obtained by factoring out this equivalence relation incorporates a new homeotic criterion of identity in comparison to the initial one, which is precisely characterized in terms of the chosen gnomon of homeothesis. In other words, the notion of logical identity is relativized with respect to the gnomon, such that the unit element of the quotient structure expresses equivalence modulo the gnomon.

In the case of homeothesis or proportionality of magnitudes, the metaphorical aspect of logical conjugation may be easily visualized in terms of a recursive or periodic application of a gnomon. This leads naturally to the dynamical notions of gnomonic growth or unfolding and reciprocally gnomonic subdivision or folding by means of logical conjugation. A particular well known example is provided by the function of the golden mean gnomon, depicted graphically as follows:

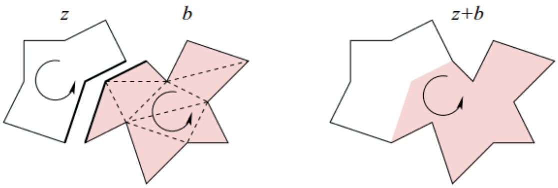


The conclusion obtained from the analysis of the notion of a homeothetic gnomon can be extrapolated to more complex situations, where a more general homeotic criterion is required for the effective application of logical conjugation. The abstraction consists in thinking of a gnomon as a means to indicate, or discern, or distinguish, or to set a boundary. The function of a gnomon is again to induce a certain type of modularity incorporating a homeotic criterion of identity.

For instance, in the case of a manifold, the gnomon is a local Euclidean space and the homeotic criterion is subsumed by the notion of a local homeomorphism. The modularity type is expressed by the gluing conditions of local Euclidean patches adjoined homeotically to a globally intractable space endowing it with the structure of a manifold. The logical conjugation strategy is used as a means to resolve a difficult problem for manifolds in terms of simpler problems, which can be solved at the level of local Euclidean patches and their amalgamations. Equivalently put, this logical method conjugates a complex problem at the manifold level to a simpler problem at the local Euclidean level where it can be directly resolved. The efficiency of logical conjugation rests on the fact that we are able to descend and ascend between these levels due to the homeotic criterion enforced by the associated gnomon.



Finally, it is worth explaining the notion of gnomon employed in standard homology theory, as it is conceptualized in algebraic topology. In this case, the role of a gnomon is played by the notion of a boundary. We recall that chains of connectivity in homology theory are classified in terms of cycles and boundaries. Intuitively, a boundary at some dimension is a bounding chain of a higher dimensional topological form, whereas a cycle stands for a non-bounding chain. Visually, non-bounding chains may be thought of in terms of holes or punctures or higher dimensional cavities, whereas boundaries may be thought of in terms of filled, and thus bounding chains. The basic idea of a boundary as a gnomon, establishing the homeotic criterion of homology, such that logical conjugation can operate, is that adjoining a boundary to a cycle gives a topologically similar or homologous cycle. Thus, two cycles differing by a boundary belong to the same homology equivalence class as depicted visually below.



In this sense, homology equivalence classes, which are actually abelian groups due to the algebraic operations involved in composing chains and orienting boundaries, enfold the invariant information of holes and cavities of topological forms. We emphasize again that these group invariants are obtained solely by the logical conjugation strategy on the basis of the homological criterion of identity set up by the notion of a gnomonic boundary.