## **CONTENTS**

Preface		xi
About the Auth	or	xiii
Chapter 1:	Introduction	1
	1.1 Index Notation—The Einstein Summation Convention	2
Chapter 2:	<b>Coordinate Systems Definition</b>	3
Chapter 3:	<b>Basis Vectors and Scale Factors</b>	
Chapter 4:	<b>Contravariant Components and Transformations</b>	9
Chapter 5:	<b>Covariant Components and Transformations</b>	13
Chapter 6:	Physical Components and Transformations	15
Chapter 7:	Tensors—Mixed and Metric	17
Chapter 8:	<b>Metric Tensor Operation on Tensor Indices</b>	21
	8.1 Example: Cylindrical Coordinate Systems	23
	8.2 Example: Spherical Coordinate Systems	25
Chapter 9:	<b>Dot and Cross Products of Tensors</b>	29
	9.1 Determinant of an $N \times N$ Matrix Using Permutation Symbols	34
Chapter 10:	Gradient Vector Operator—Christoffel Symbols	35
	10.1 Covariant Derivatives of Vectors—Christoffel Symbols of the $2^{nd}$ Kind	35
	10.2 Contravariant Derivatives of Vectors	39
	10.3 Covariant Derivatives of a Mixed Tensor	40
	$10.4$ Christoffel Symbol Relations and Properties— $1^{\rm st}$ and $2^{\rm nd}$ Kinds	41

Chapter 11:	Derivative Forms—Curl, Divergence, Laplacian		51
	11.1	Curl Operations on Tensors	51
	11.2	Physical Components of the Curl of	
		Tensors—3D Orthogonal Systems	54
	11.3	Divergence Operation on Tensors	55
	11.4	Laplacian Operations on Tensors	57
	11.5	Biharmonic Operations on Tensors	58
	11.6	Physical Components of the Laplacian of a Vector—3D Orthogonal Systems	59
Chapter 12:	Cart	esian Tensor Transformation—Rotations	65
	12.1	Rotation Matrix	67
	12.2	Equivalent Single Rotation: Eigenvalues and Eigenvectors	67
Chapter 13:	Coordinate Independent Governing Equations		<b>75</b>
	13.1	The Acceleration Vector—Contravariant Components	76
	13.2	The Acceleration Vector—Physical Components	78
	13.3	The Acceleration Vector in Orthogonal Systems—Physical Components	79
	13.4	Substantial Time Derivatives of Tensors	82
	13.5	Conservation Equations—Coordinate Independent Forms	85
Chapter 14:	Colle	ection of Relations for Selected Coordinate	
•	Syste	ems	89
	14.1	Cartesian Coordinate System	89
	14.2	Cylindrical Coordinate Systems	91
	14.3	Spherical Coordinate Systems	93
	14.4	Parabolic Coordinate Systems	96
	14.5	Orthogonal Curvilinear Coordinate Systems	98
Chapter 15:	_	d Body Rotation: Euler Angles, Quaternions,	
	and 1	Rotation Matrix	103
	15.1	Active and Passive Rotations	104
	15.2	Euler Angles	107

	15.3	Categorizing Euler Angles	113
	15.4	Gimbal Lock-Euler Angles Limitation	121
	15.5	Quaternions-Applications for Rigid Body Rotation	123
	15.6	From a Given Quaternion to Rotation Matrix	135
	15.7	From a Given Rotation Matrix to Quaternion	138
	15.8	From Euler Angles to a Quaternion	139
	15.9	Putting It All Together	140
Chapter 16:	Work	ted-out Examples	143
_	16.1	Example: Einstein Summation Conventions	143
	16.2	Example: Conversion from Vector to Index Notations	144
	16.3	Example: Oblique Rectilinear Coordinate Systems	145
	16.4	Example: Quantities Related to Parabolic Coordinate System	149
	16.5	Example: Quantities Related to Bi-Polar Coordinate Systems	152
	16.6	Example: Application of Contravariant Metric Tensors	155
	16.7	Example: Dot and Cross Products in Cylindrical and Spherical Coordinates	155
	16.8	Example: Relation between Jacobian and Metric Tensor Determinants	157
	16.9	Example: Determinant of Metric Tensors Using Displacement Vectors	158
	16.10	Example: Determinant of a $4 \times 4$ Matrix Using Permutation Symbols	159
	16.11	Example: Time Derivatives of the Jacobian	159
		Example: Covariant Derivatives of a Constant Vector	160
	16.13	Example: Covariant Derivatives of Physical	
		Components of a Vector	161
	16.14	Example: Continuity Equations in Several Coordinate Systems	161
	16.15	Example: 4D Spherical Coordinate Systems	162

References Index		177 179
Chapter 17:	Exercise Problems	171
	16.20 Example: Successive Rotations Using Quaternions Method	170
	16.19 Example: Passive Rotation Using Single-Axis and Quaternions Methods	168
	16.18 Example: Active Rotation Using Single-Axis and Quaternions Methods	166
	16.17 Example: Covariant Derivatives of Metric Tensors	166
	16.16 Example: Complex Double Dot-Cross Product Expressions	165