

# PREFACE

This is the second edition of the published textbook: *Tensor Analysis for Engineers*. In this edition, we expand the content on the rigid body rotation and Cartesian tensors by including Euler angles and quaternions methods. In addition to the rotation matrix method, presented in the first edition and included in this edition, we collect all three methods in this volume of the textbook. In this edition, the quaternions and their algebraic calculation rules are presented. We also discuss the active and passive rotations and present several worked-out examples using the Euler angles and quaternions methods applications and their interrelations. The problem of gimbal lock is also analyzed and presented with detailed worked out examples. Additional references have been included in the second edition.

In engineering and science, physical quantities are often represented by mathematical functions, namely *tensors*. Examples include temperature, pressure, force, mechanical stress, electric/magnetic fields, velocity, enthalpy, entropy, etc. In turn, tensors are categorized based on their rank, i.e. rank zero, one, and so forth. The so-called scalar quantities (e.g., temperature) are tensors of rank zero. Likewise, velocity and force are tensors of rank one and mechanical stress and gradient of velocity are tensors of rank two. In Euclidean space, which could be of dimension  $N = 3, 4, \dots$ , we can define several coordinate systems for our calculation and measurement of physical quantities. For example, in a 3D space, we can have Cartesian, cylindrical, and spherical coordinate systems. In general, we prefer defining a coordinate system whose coordinate surfaces (where one of the coordinate variables is invariant or remains constant) match to the physical problem geometry at hand. This enables us to easily define the boundary conditions of the physical problem to the related governing equations, written in terms of the selected coordinate system. This action requires transformation of the tensor quantities and their related derivatives (e.g., gradient,

curl, divergence) from Cartesian to the selected coordinate system or vice versa. The topic of *tensor analysis* (also referred to as “tensor calculus” or “Ricci’s calculus” since originally developed by Ricci, 1835–1925, [1], [2]), is mainly engaged with the definition of tensor-like quantities and their transformation among coordinate systems and others. The topic provides a set of mathematical tools which enables users to perform transformation and calculations of tensors for any well-defined coordinate systems in a systematic way—it is a “machine.” The merit of tensor analysis is to provide a systematic mathematical formulation to derive the general form of the governing equations for arbitrary coordinate systems.

In this book, we aim to provide engineers and applied scientists the tools and techniques of tensor analysis for applications in practical problem solving and analysis activities. The geometry is limited to the Euclidean space/geometry, where the Pythagorean Theorem applies, with well-defined Cartesian coordinate systems as the reference. We discuss quantities defined in curvilinear coordinate systems, like cylindrical, spherical, parabolic, etc., and present several examples and coordinates sketches with related calculations. In addition, we listed several worked-out examples for helping the readers with mastering the topics provided in the prior sections. A list of exercises is provided for further practice for readers.

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September 30, 2020*