PREFACE

Linear algebra is one of the most widely used mathematical theories and has applications in virtually every area of mathematics, including multivariate calculus, differential equations, and probability theory. The purpose of this book is to bridge the gap between the abstract theoretical aspects and the computational applications of linear algebra. The aim of this book is two-fold: to introduce the fundamental concepts of linear algebra and to apply the theorems in computation-oriented applications. There are many good introductory texts on linear algebra, and the intention of this book is to be a supplement to those texts, or to serve as a text for senior undergraduate students or first year graduate students, whose interests are computational mathematics, science, engineering, and computer science. The presentation of the material combines definitions and proofs with an emphasis on computational applications. We provide examples that illustrate the use of software packages such as *Mathematica*, *Maple*, and *Sage*.

This book has evolved from our experience over several years of teaching linear algebra to mixed audiences of upper division mathematics majors, beginning graduate students, and students from the fields of science and engineering that rely heavily on mathematical methods. Our goal in writing this book has been to develop a text that addresses the exceptional diversity of the audience, and introduce some of the most essential topics about the subject of linear algebra to these groups. To accomplish our goal, we have selected material that covers both the theory and applications, while emphasizing topics useful in other disciplines.

Throughout the text, we present a brief introduction to some aspects of abstract algebra that relate directly to linear algebra, such as groups, rings, modules, fields and polynomials over fields. In particular, the last section of this book is dedicated to the matrix decomposition over principle ideal domains, because this structure theorem is a generalization of the fundamental theorem of finitely-generated abelian groups, and this result provides a simple framework to understand various canonical form results for square matrices over fields.

We use the material from the book to teach our own elective linear algebra course, and some of the solutions to the exercises are provided by our students. It is our hope that this book will help a general reader appreciate the abstract mathematics behind the real applications.

By design, each chapter consists of two parts: the theoretical background and the applications, which make the book suitable for a one semester course in linear algebra that can be used in a variety of contexts. For an audience composed primarily of mathematics undergraduate majors, the material on the theories of abstract vector spaces, linear transformations, linear operators, orthogonal bases, and decomposition over rings can be covered in depth. For an applied mathematics course with students from the fields of science and engineering that rely heavily on mathematical methods, the material on applications of these areas such as linear codes, affine or projective transformations, geometry of transformations, matrix in graph theory, image processing, and QR decomposition, can be treated with more emphasis. In the applications, we allow ourselves to present a number of results from a wide range of sources, and sometimes without detailed proofs. The applications portion of the chapter is suitable for a reader who knows some linear algebra and a particular related area such as coding theory, geometric modeling, or graph theory. Some of the applications can serve as a guide to some interesting research topics.

The prerequisite for this book is a standard first year undergraduate course in linear algebra. In Chapter 1 and Chapter 2 we start with a quick review of the fundamental concepts of vector spaces and linear transformations. To better understand the behavior of a linear transformation, we discuss the eigenvectors in Chapter 3, where the eigenvectors act as the "axes" along which linear transformations behave simply as stretching, compressing, or flipping, and hopefully make understanding of linear transformations easier. Because one can perform some operations on vectors by performing the same operations on the basis, we study orthogonal bases in Chapter 4. In particular, we study linear transformations relative to orthonormal bases that faithfully preserve the linear properties and the metric properties. Finally, in Chapter 5, we focus on the matrix decomposition over real or complex numbers and over principle ideal domains.

This book should be thought of as an introduction to more advanced texts and research topics. The novelty of this book, we hope, is that the material presented here is a unique combination of the essential theory of linear algebra and computational methods in a variety of applications.