PREFACE

These are notes of lectures given at Princeton University during the fall semester of 1969. The notes present an introduction to p-adic L-functions originated in Kubota-Leopoldt [10] as p-adic analogues of classical L-functions of Dirichlet.

An outline of the contents is as follows. In §1, classical results on Dirichlet's L-functions are briefly reviewed. For some of these, a sketch of a proof is provided in the Appendix. In §2, we define generalized Bernoulli numbers following Leopoldt [12] and discuss some of the fundamental properties of these numbers. In §3, we introduce p-adic L-functions and prove the existence and the uniqueness of such functions; our method is slightly different from that in [10]. §4 consists of preliminary remarks on p-adic logarithms and p-adic regulators. In §5, we prove a formula of Leopoldt for the values of p-adic L-functions at $s = 1$. The formula was announced in [10], but the proof has not yet been published. With his permission, we describe here Leopoldt's original proof of the formula (see [1], [7] for alternate approach). In §6, we explain another method to define p-adic L-functions. Here we follow an idea in [9] motivated by the study of cyclotomic fields. In §7, we discuss some applications of the results obtained in the preceding sections, indicating deep relations which exist between p-adic L-functions and cyclotomic fields. Concluding remarks on problems and future investigations in this area are also mentioned briefly at the end of §7.

Throughout the notes, it is assumed that the reader has basic knowledge of algebraic number theory as presented, for example, in Borevich-Shafarevich [2] or Lang [11]. However, except in few places where certain facts on L-functions and class numbers are referred to, no deeper understanding of that theory may be required to follow the elementary arguments in most of these notes.

As for the notations, some of the symbols used throughout the notes are as follows: $\mathbf{Z}$, $\mathbf{Q}$, $\mathbf{R}$, and $\mathbf{C}$ denote the ring of (rational) integers, the field of rational numbers, the field of real numbers, and the field of complex numbers, respectively. $\mathbf{Z}_p$ and $\mathbf{Q}_p$ will denote the ring of p-adic integers and the field of p-adic numbers, respectively, p being, of course, a prime number. In general, if R is a commutative ring with a unit, $R^\times$ denotes the multiplicative group of all invertible elements in R, and R[[x]] the ring of all formal power series in an indeterminate x with coefficients in R.

I should like to express my thanks here to H. W. Leopoldt for kindly permitting us to include his important unpublished results in §5, and also to R. Greenberg, J. M. Masley, and F. E. Gerth for carefully reading the manuscript and making valuable suggestions for its improvement.

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