

CONTENTS

CHAPTER I

FEEDBACK CONTROL AND THE CALCULUS OF VARIATIONS

1.1	Introduction	13
1.2	Mathematical description of a physical system	13
1.3	Parenthetical	15
1.4	Hereditary influences	16
1.5	Criteria of performance	17
1.6	Terminal control	18
1.7	Control process	18
1.8	Feedback control	19
1.9	An alternate concept	20
1.10	Feedback control as a variational problem	21
1.11	The scalar variational problem	22
1.12	Discussion	24
1.13	Relative minimum versus absolute minimum	24
1.14	Nonlinear differential equations	26
1.15	Two-point boundary value problems	27
1.16	An example of multiplicity of solution	29
1.17	Non-analytic criteria	30
1.18	Terminal control and implicit variational problems	31
1.19	Constraints	32
1.20	Linearity	34
1.21	Summing up	35
	Bibliography and discussion	36

CHAPTER II

DYNAMICAL SYSTEMS AND TRANSFORMATIONS

2.1	Introduction	41
2.2	Functions of initial values	41
2.3	The principle of causality	42
2.4	The basic functional equation	42
2.5	Continuous version	43
2.6	The functional equations satisfied by the elementary functions	43
2.7	The matrix exponential	44

CONTENTS

2.8	Transformations and iteration	44
2.9	Carleman's linearization	45
2.10	Functional equations and maximum range	46
2.11	Vertical motion—I	46
2.12	Vertical motion—II	47
2.13	Maximum altitude	47
2.14	Maximum range	48
2.15	Multistage processes and differential equations	48
	Bibliography and discussion	48

CHAPTER III

MULTISTAGE DECISION PROCESSES AND DYNAMIC PROGRAMMING

3.1	Introduction	51
3.2	Multistage decision processes	51
3.3	Discrete deterministic multistage decision processes	52
3.4	Formulation as a conventional maximization problem	53
3.5	Markovian-type processes	54
3.6	Dynamic programming approach	55
3.7	A recurrence relation	56
3.8	The principle of optimality	56
3.9	Derivation of recurrence relation	57
3.10	"Terminal" control	57
3.11	Continuous deterministic processes	58
3.12	Discussion	59
	Bibliography and comments	59

CHAPTER IV

DYNAMIC PROGRAMMING AND THE CALCULUS OF VARIATIONS

4.1	Introduction	61
4.2	The calculus of variations as a multistage decision process . .	62
4.3	Geometric interpretation	62
4.4	Functional equations	63
4.5	Limiting partial differential equations	64
4.6	The Euler equations and characteristics	65
4.7	Discussion	66
4.8	Direct derivation of Euler equation	66
4.9	Discussion	67
4.10	Discrete processes	67

CONTENTS

4.11	Functional equations	68
4.12	Minimum of maximum deviation	69
4.13	Constraints	70
4.14	Structure of optimal policy	70
4.15	Bang-bang control	72
4.16	Optimal trajectory	73
4.17	The brachistochrone	74
4.18	Numerical computation of solutions of differential equations .	76
4.19	Sequential computation	77
4.20	An example	78
	Bibliography and comments	79

CHAPTER V

COMPUTATIONAL ASPECTS OF DYNAMIC PROGRAMMING

5.1	Introduction	85
5.2	The computational process—I	86
5.3	The computational process—II	87
5.4	The computational process—III	88
5.5	Expanding grid	88
5.6	The computational process—IV	88
5.7	Obtaining the solution from the numerical results	89
5.8	Why is dynamic programming better than straightforward enumeration	90
5.9	Advantages of dynamic programming approach	90
5.10	Absolute maximum versus relative maximum	91
5.11	Initial value versus two-point boundary value problems	91
5.12	Constraints	91
5.13	Non-analyticity	92
5.14	Implicit variational problems	92
5.15	Approximation in policy space	93
5.16	The curse of dimensionality	94
5.17	Sequential search	95
5.18	Sensitivity analysis	95
5.19	Numerical solution of partial differential equations	95
5.20	A simple nonlinear hyperbolic equation	96
5.21	The equation $f_T = g_1 + g_2 f_e + g_3 f_e^2$	97
	Bibliography and comments	98

CHAPTER VI

THE LAGRANGE MULTIPLIER

6.1	Introduction	100
6.2	Integral constraints	101

CONTENTS

6.3	Lagrange multiplier	102
6.4	Discussion	103
6.5	Several constraints	103
6.6	Discussion	104
6.7	Motivation for the Lagrange multiplier	105
6.8	Geometric motivation	106
6.9	Equivalence of solution	108
6.10	Discussion	109
	Bibliography and discussion	110

CHAPTER VII

TWO-POINT BOUNDARY VALUE PROBLEMS

7.1	Introduction	111
7.2	Two-point boundary value problems	112
7.3	Application of dynamic programming techniques	113
7.4	Fixed terminal state	113
7.5	Fixed terminal state and constraint	115
7.6	Fixed terminal set	115
7.7	Internal conditions	116
7.8	Characteristic value problems	116
	Bibliography and comments	117

CHAPTER VIII

SEQUENTIAL MACHINES AND THE SYNTHESIS OF LOGICAL SYSTEMS

8.1	Introduction	119
8.2	Sequential machines	119
8.3	Information pattern	120
8.4	Ambiguity	121
8.5	Functional equations	121
8.6	Limiting case	122
8.7	Discussion	122
8.8	Minimum time	123
8.9	The coin-weighing problem	123
8.10	Synthesis of logical systems	124
8.11	Description of problem	124
8.12	Discussion	125
8.13	Introduction of a norm	125
8.14	Dynamic programming approach	125
8.15	Minimum number of stages	125

CONTENTS

8.16	Medical diagnosis	126
	Bibliography and discussion	126

CHAPTER IX

UNCERTAINTY AND RANDOM PROCESSES

9.1	Introduction	129
9.2	Uncertainty	130
9.3	Sour grapes or truth?	131
9.4	Probability	132
9.5	Enumeration of equally likely possibilities	132
9.6	The frequency approach	134
9.7	Ergodic theory	136
9.8	Random variables	136
9.9	Continuous stochastic variable	137
9.10	Generation of random variables	138
9.11	Stochastic process	138
9.12	Linear stochastic sequences	138
9.13	Causality and the Markovian property	139
9.14	Chapman-Kolmogoroff equations	140
9.15	The forward equations	141
9.16	Diffusion equations	142
9.17	Expected values	142
9.18	Functional equations	144
9.19	An application	145
9.20	Expected range and altitude	145
	Bibliography and comments	146

CHAPTER X

STOCHASTIC CONTROL PROCESSES

10.1	Introduction	152
10.2	Discrete stochastic multistage decision processes	152
10.3	The optimization problem	153
10.4	What constitutes an optimal policy?	154
10.5	Two particular stochastic control processes	155
10.6	Functional equations	155
10.7	Discussion	156
10.8	Terminal control	156
10.9	Implicit criteria	157
10.10	A two-dimensional process with implicit criterion	157
	Bibliography and discussion	158

CONTENTS

CHAPTER XI

MARKOVIAN DECISION PROCESSES

11.1	Introduction	160
11.2	Limiting behavior of Markov processes	160
11.3	Markovian decision processes—I	161
11.4	Markovian decision processes—II	162
11.5	Steady-state behavior	162
11.6	The steady-state equation	163
11.7	Howard's iteration scheme	164
11.8	Linear programming and sequential decision processes	164
	Bibliography and comments	165

CHAPTER XII

QUASILINEARIZATION

12.1	Introduction	167
12.2	Continuous Markovian decision processes	168
12.3	Approximation in policy space	168
12.4	Systems	170
12.5	The Riccati equation	170
12.6	Extensions	171
12.7	Monotone approximation in the calculus of variations	171
12.8	Computational aspects	172
12.9	Two-point boundary-value problems	173
12.10	Partial differential equations	174
	Bibliography and comments	175

CHAPTER XIII

STOCHASTIC LEARNING MODELS

13.1	Introduction	176
13.2	A stochastic learning model	176
13.3	Functional equations	177
13.4	Analytic and computational aspects	177
13.5	A stochastic learning model—II	177
13.6	Inverse problem	178
	Bibliography and comments	178

CHAPTER XIV

THE THEORY OF GAMES AND PURSUIT PROCESSES

14.1	Introduction	180
14.2	A two-person process	181

CONTENTS

14.3	Multistage process	181
14.4	Discussion	182
14.5	Borel-von Neumann theory of games	182
14.6	The min-max theorem of von Neumann	184
14.7	Discussion	184
14.8	Computational aspects	185
14.9	Card games	185
14.10	Games of survival	185
14.11	Control processes as games against nature	186
14.12	Pursuit processes—minimum time to capture	187
14.13	Pursuit processes—minimum miss distance	189
14.14	Pursuit processes—minimum miss distance within a given time	189
14.15	Discussion	190
	Bibliography and discussion	190

CHAPTER XV

ADAPTIVE PROCESSES

15.1	Introduction	194
15.2	Uncertainty revisited	195
15.3	Reprise	198
15.4	Unknown—I	199
15.5	Unknown—II	200
15.6	Unknown—III	200
15.7	Adaptive processes	201
	Bibliography and comments	201

CHAPTER XVI

ADAPTIVE CONTROL PROCESSES

16.1	Introduction	203
16.2	Information pattern	205
16.3	Basic assumptions	207
16.4	Mathematical formulation	208
16.5	Functional equations	208
16.6	<i>From information patterns to distribution functions</i>	209
16.7	Feedback control	209
16.8	Functional equations	210
16.9	Further structural assumptions	210
16.10	Reduction from functionals to functions	211
16.11	<i>An illustrative example—deterministic version</i>	211
16.12	Stochastic version	212
16.13	Adaptive version	213
16.14	Sufficient statistics	215

CONTENTS

16.15 The two-armed bandit problem	215
Bibliography and discussion	216

CHAPTER XVII

SOME ASPECTS OF COMMUNICATION THEORY

17.1 Introduction	219
17.2 A model of a communication process	220
17.3 Utility a function of use	221
17.4 A stochastic allocation process	221
17.5 More general processes	222
17.6 The efficient gambler	222
17.7 Dynamic programming approach	223
17.8 Utility of a communication channel	224
17.9 Time-dependent case	224
17.10 Correlation	225
17.11 M-signal channels	225
17.12 Continuum of signals	227
17.13 Random duration	228
17.14 Adaptive processes	228
Bibliography and comments	230

CHAPTER XVIII

SUCCESSIVE APPROXIMATION

18.1 Introduction	232
18.2 The classical method of successive approximations	233
18.3 Application to dynamic programming	234
18.4 Approximation in policy space	235
18.5 Quasilinearization	236
18.6 Application of the preceding ideas	236
18.7 Successive approximations in the calculus of variations	237
18.8 Preliminaries on differential equations	239
18.9 A terminal control process	240
18.10 Differential-difference equations and retarded control	241
18.11 Quadratic criteria	241
18.12 Successive approximations once more	243
18.13 Successive approximations—II	244
18.14 Functional approximation	244
18.15 Simulation techniques	246
18.16 Quasi-optimal policies	246
18.17 Non-zero sum games	247
18.18 Discussion	248
Bibliography and discussion	249