Preface

Convexity has been increasingly important in recent years in the study of extremum problems in many areas of applied mathematics. The purpose of this book is to provide an exposition of the theory of convex sets and functions in which applications to extremum problems play the central role.

Systems of inequalities, the minimum or maximum of a convex function over a convex set, Lagrange multipliers, and minimax theorems are among the topics treated, as well as basic results about the structure of convex sets and the continuity and differentiability of convex functions and saddle-functions. Duality is emphasized throughout, particularly in the form of Fenchel's conjugacy correspondence for convex functions.

Much new material is presented. For example, a generalization of linear algebra is developed in which "convex bifunctions" are the analogues of linear transformations, and "inner products" of convex sets and functions are defined in terms of the extremal values in Fenchel's Duality Theorem. Each convex bifunction is associated with a generalized convex program, and an adjoint operation for bifunctions that leads to a theory of dual programs is introduced. The classical correspondence between linear transformations and bilinear functionals is extended to a correspondence between convex bifunctions and saddle-functions, and this is used as the main tool in the analysis of saddle-functions and minimax problems.

Certain topics which might properly be regarded as part of "convex analysis," such as fixed-point theorems, have been omitted, not because they lack charm or applications, but because they would have required technical developments somewhat outside the mainstream of the rest of the book.

In view of the fact that economists, engineers, and others besides pure mathematicians have become interested in convex analysis, an attempt has been made to keep the exposition on a relatively elementary technical level, and details have been supplied which, in a work aimed only at a mathematical in-group, might merely have been alluded to as "exercises." Everything has been limited to R^n, the space of all n-tuples of real numbers, even though many of the results can easily be formulated in a broader setting of functional analysis. References to generalizations and extensions are collected along with historical and bibliographical comments in a special section at the end of the book, preceding the bibliography itself.

As far as technical prerequisites are concerned, the reader should be able to get by, for the most part, with a sound knowledge of linear algebra and elementary real analysis (convergent sequences, continuous functions, open and closed sets, compactness, etc.) as pertains to the space R^n. Nevertheless, while no actual familiarity with any deeper branch of abstract mathematics is required, the style does presuppose a certain "mathematical maturity" on the part of the reader.

A section of remarks at the beginning of the book describes the contents of each part and outlines a selection of material which would be appropriate for an introduction to the subject.

This book grew out of lecture notes from a course I gave at Princeton University in the spring of 1966. In a larger sense, however, it grew out of lecture notes from a similar course given at Princeton fifteen years earlier by Professor Werner Fenchel of the University of Copenhagen. Fenchel's notes were never published, but they were distributed in mimeographed form, and they have served many researchers long and well as the main, and virtually the only, reference for much of the theory of convex functions. They have profoundly influenced my own thinking, as evidenced, to cite just one aspect, by the way conjugate convex functions dominate much of this book. It is highly fitting, therefore, that this book be dedicated to Fenchel, as honorary co-author.

I would like to express my deep thanks to Professor A. W. Tucker of Princeton University, whose encouragement and support has been a mainstay since student days. It was Tucker in fact who suggested the title of this book. Further thanks are due to Dr. Torrence D. Parsons, Dr. Norman Z. Shapiro, and Mr. Lynn McLinden, who looked over the manuscript and gave some very helpful suggestions. I am also grateful to my students at Princeton and the University of Washington, whose comments on the material as it was taught led to many improvements of the presentation, and to Mrs. Janet Parker for her patient and very competent secretarial assistance.

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R. T. R.