

***Three-Dimensional Geometry
and Topology***

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Frontispiece: The sculpture in the photograph is a marble rendition by Helaman Ferguson of the Klein quartic, a genus-three surface with the greatest possible amount of symmetry (336 isometries). It is not possible to represent all this symmetry in space, but the twenty-four heptagons that tile the surface are in fact intrinsically identical. They come from regular heptagons in the hyperbolic plane, meeting three to a vertex; each such heptagon is made of fourteen triangles of the 2-3-7 tiling of Figures 2.10 and 2.14. If you start on any edge of the surface, proceed along it to a fork and turn right, then turn left at the next fork, and keep alternating in this way, you arrive back where you started after eight turns. For this reason the sculpture, which stands at the Mathematical Sciences Research Institute in Berkeley, is named “The Eightfold Way.”

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