

FEARLESS SYMMETRY
FEARLESS SYMMETRY

FEARLESS LEVEES

Exposing the Hidden

With a new preface

Avner Ash *and*

PRINCETON UNIVERSITY PRESS

SYMMETRY

ΣΛΠΠΕΛΒΛ

Patterns of Numbers

by the authors

Robert Gross

PRINCETON AND OXFORD

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About the cover image, courtesy of Bahman Kalantari: Polynomiography is the art and science of visualization in approximation of the zeros of polynomial equations using iteration functions. Although its theoretical foundation can be traced to the well-known Fundamental Theorem of Algebra, polynomiography offers a new and exciting view into the world of polynomials as well as the mysteries of this beautiful theorem itself. Not only is polynomiography interesting and useful from the scientific and educational points of view, but it turns the ancient root-finding problem into a serious medium for creating artwork of great variety and diversity through a combination of human creativity and computer power. Each polynomial gives rise to an infinite number of 2D images, each called a *polynomiograph*. Each natural number can be identified as a polynomial. Hence, for each polynomial and each natural number there is an infinite number of polynomiographs waiting to be discovered. The particular image on the cover is based on a polynomiograph produced by one of the techniques in polynomiography, referred to as *Voronoi coloring*. The title of the image is *Acrobats*. For more information, visit www.polynomiography.com.

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FOR OUR PARENTS
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Tyger, Tyger, burning bright,
In the forests of the night,
What immortal hand or eye
Could frame thy fearful symmetry?

—*William Blake*



In seed time learn, in harvest teach, in winter enjoy.

—*William Blake, "Proverbs of Hell"*

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