
Contents

| | |
|---|-----------|
| <i>Preface to the Second Edition</i> | xv |
| <i>Preface to the First Edition</i> | xvii |
| <i>Special Symbols</i> | xxi |
| <i>Conventions, Notation, and Terminology</i> | xxxiii |
| 1. Preliminaries | 1 |
| 1.1 Logic | 1 |
| 1.2 Sets | 2 |
| 1.3 Integers, Real Numbers, and Complex Numbers | 3 |
| 1.4 Functions | 4 |
| 1.5 Relations | 6 |
| 1.6 Graphs | 9 |
| 1.7 Facts on Logic, Sets, Functions, and Relations | 11 |
| 1.8 Facts on Graphs | 15 |
| 1.9 Facts on Binomial Identities and Sums | 16 |
| 1.10 Facts on Convex Functions | 23 |
| 1.11 Facts on Scalar Identities and Inequalities in One Variable | 25 |
| 1.12 Facts on Scalar Identities and Inequalities in Two Variables | 33 |
| 1.13 Facts on Scalar Identities and Inequalities in Three Variables | 42 |
| 1.14 Facts on Scalar Identities and Inequalities in Four Variables | 50 |
| 1.15 Facts on Scalar Identities and Inequalities in Six Variables | 52 |
| 1.16 Facts on Scalar Identities and Inequalities in Eight Variables | 52 |
| 1.17 Facts on Scalar Identities and Inequalities in n Variables | 52 |
| 1.18 Facts on Scalar Identities and Inequalities in $2n$ Variables | 66 |
| 1.19 Facts on Scalar Identities and Inequalities in $3n$ Variables | 74 |
| 1.20 Facts on Scalar Identities and Inequalities in Complex Variables | 74 |
| 1.21 Facts on Trigonometric and Hyperbolic Identities | 81 |
| 1.22 Notes | 84 |
| 2. Basic Matrix Properties | 85 |
| 2.1 Matrix Algebra | 85 |
| 2.2 Transpose and Inner Product | 92 |
| 2.3 Convex Sets, Cones, and Subspaces | 97 |
| 2.4 Range and Null Space | 101 |

| | | |
|-----------|--|------------|
| 2.5 | Rank and Defect | 104 |
| 2.6 | Invertibility | 106 |
| 2.7 | The Determinant | 111 |
| 2.8 | Partitioned Matrices | 115 |
| 2.9 | Facts on Polars, Cones, Dual Cones, Convex Hulls, and Subspaces | 119 |
| 2.10 | Facts on Range, Null Space, Rank, and Defect | 124 |
| 2.11 | Facts on the Range, Rank, Null Space, and Defect of Partitioned Matrices | 130 |
| 2.12 | Facts on the Inner Product, Outer Product, Trace, and Matrix Powers | 136 |
| 2.13 | Facts on the Determinant | 139 |
| 2.14 | Facts on the Determinant of Partitioned Matrices | 144 |
| 2.15 | Facts on Left and Right Inverses | 152 |
| 2.16 | Facts on the Adjugate and Inverses | 153 |
| 2.17 | Facts on the Inverse of Partitioned Matrices | 159 |
| 2.18 | Facts on Commutators | 161 |
| 2.19 | Facts on Complex Matrices | 164 |
| 2.20 | Facts on Geometry | 167 |
| 2.21 | Facts on Majorization | 175 |
| 2.22 | Notes | 178 |
| 3. | Matrix Classes and Transformations | 179 |
| 3.1 | Matrix Classes | 179 |
| 3.2 | Matrices Related to Graphs | 184 |
| 3.3 | Lie Algebras and Groups | 185 |
| 3.4 | Matrix Transformations | 188 |
| 3.5 | Projectors, Idempotent Matrices, and Subspaces | 190 |
| 3.6 | Facts on Group-Invertible and Range-Hermitian Matrices | 191 |
| 3.7 | Facts on Normal, Hermitian, and Skew-Hermitian Matrices | 192 |
| 3.8 | Facts on Commutators | 199 |
| 3.9 | Facts on Linear Interpolation | 200 |
| 3.10 | Facts on the Cross Product | 202 |
| 3.11 | Facts on Unitary and Shifted-Unitary Matrices | 205 |
| 3.12 | Facts on Idempotent Matrices | 215 |
| 3.13 | Facts on Projectors | 223 |
| 3.14 | Facts on Reflectors | 229 |
| 3.15 | Facts on Involutory Matrices | 230 |
| 3.16 | Facts on Tripotent Matrices | 231 |
| 3.17 | Facts on Nilpotent Matrices | 232 |
| 3.18 | Facts on Hankel and Toeplitz Matrices | 234 |
| 3.19 | Facts on Tridiagonal Matrices | 237 |
| 3.20 | Facts on Hamiltonian and Symplectic Matrices | 238 |
| 3.21 | Facts on Matrices Related to Graphs | 240 |
| 3.22 | Facts on Triangular, Irreducible, Cauchy, Dissipative, Contractive, and Centrosymmetric Matrices | 240 |
| 3.23 | Facts on Groups | 242 |
| 3.24 | Facts on Quaternions | 247 |
| 3.25 | Notes | 252 |

| | |
|---|------------|
| 4. Polynomial Matrices and Rational Transfer Functions | 253 |
| 4.1 Polynomials | 253 |
| 4.2 Polynomial Matrices | 256 |
| 4.3 The Smith Decomposition and Similarity Invariants | 258 |
| 4.4 Eigenvalues | 261 |
| 4.5 Eigenvectors | 267 |
| 4.6 The Minimal Polynomial | 269 |
| 4.7 Rational Transfer Functions and the Smith-McMillan Decomposition | 271 |
| 4.8 Facts on Polynomials and Rational Functions | 276 |
| 4.9 Facts on the Characteristic and Minimal Polynomials | 282 |
| 4.10 Facts on the Spectrum | 288 |
| 4.11 Facts on Graphs and Nonnegative Matrices | 297 |
| 4.12 Notes | 307 |
| 5. Matrix Decompositions | 309 |
| 5.1 Smith Form | 309 |
| 5.2 Multicompanion Form | 309 |
| 5.3 Hypercompanion Form and Jordan Form | 314 |
| 5.4 Schur Decomposition | 318 |
| 5.5 Eigenstructure Properties | 321 |
| 5.6 Singular Value Decomposition | 328 |
| 5.7 Pencils and the Kronecker Canonical Form | 330 |
| 5.8 Facts on the Inertia | 334 |
| 5.9 Facts on Matrix Transformations for One Matrix | 338 |
| 5.10 Facts on Matrix Transformations for Two or More Matrices | 345 |
| 5.11 Facts on Eigenvalues and Singular Values for One Matrix | 350 |
| 5.12 Facts on Eigenvalues and Singular Values for Two or More Matrices | 362 |
| 5.13 Facts on Matrix Pencils | 369 |
| 5.14 Facts on Matrix Eigenstructure | 369 |
| 5.15 Facts on Matrix Factorizations | 377 |
| 5.16 Facts on Companion, Vandermonde, Circulant, and Hadamard Matrices | 385 |
| 5.17 Facts on Simultaneous Transformations | 391 |
| 5.18 Facts on the Polar Decomposition | 393 |
| 5.19 Facts on Additive Decompositions | 394 |
| 5.20 Notes | 396 |
| 6. Generalized Inverses | 397 |
| 6.1 Moore-Penrose Generalized Inverse | 397 |
| 6.2 Drazin Generalized Inverse | 401 |
| 6.3 Facts on the Moore-Penrose Generalized Inverse for One Matrix | 404 |
| 6.4 Facts on the Moore-Penrose Generalized Inverse for Two or More Matrices | 411 |
| 6.5 Facts on the Moore-Penrose Generalized Inverse for Partitioned Matrices | 422 |
| 6.6 Facts on the Drazin and Group Generalized Inverses | 431 |
| 6.7 Notes | 438 |

| | |
|---|------------|
| 7. Kronecker and Schur Algebra | 439 |
| 7.1 Kronecker Product | 439 |
| 7.2 Kronecker Sum and Linear Matrix Equations | 443 |
| 7.3 Schur Product | 444 |
| 7.4 Facts on the Kronecker Product | 445 |
| 7.5 Facts on the Kronecker Sum | 450 |
| 7.6 Facts on the Schur Product | 454 |
| 7.7 Notes | 458 |
| 8. Positive-Semidefinite Matrices | 459 |
| 8.1 Positive-Semidefinite and Positive-Definite Orderings | 459 |
| 8.2 Submatrices | 461 |
| 8.3 Simultaneous Diagonalization | 465 |
| 8.4 Eigenvalue Inequalities | 467 |
| 8.5 Exponential, Square Root, and Logarithm of Hermitian Matrices | 473 |
| 8.6 Matrix Inequalities | 474 |
| 8.7 Facts on Range and Rank | 486 |
| 8.8 Facts on Structured Positive-Semidefinite Matrices | 488 |
| 8.9 Facts on Identities and Inequalities for One Matrix | 495 |
| 8.10 Facts on Identities and Inequalities for Two or More Matrices | 501 |
| 8.11 Facts on Identities and Inequalities for Partitioned Matrices | 514 |
| 8.12 Facts on the Trace | 523 |
| 8.13 Facts on the Determinant | 533 |
| 8.14 Facts on Convex Sets and Convex Functions | 543 |
| 8.15 Facts on Quadratic Forms | 550 |
| 8.16 Facts on the Gaussian Density | 556 |
| 8.17 Facts on Simultaneous Diagonalization | 558 |
| 8.18 Facts on Eigenvalues and Singular Values for One Matrix | 559 |
| 8.19 Facts on Eigenvalues and Singular Values for Two or More Matrices | 564 |
| 8.20 Facts on Alternative Partial Orderings | 574 |
| 8.21 Facts on Generalized Inverses | 577 |
| 8.22 Facts on the Kronecker and Schur Products | 584 |
| 8.23 Notes | 595 |
| 9. Norms | 597 |
| 9.1 Vector Norms | 597 |
| 9.2 Matrix Norms | 601 |
| 9.3 Compatible Norms | 604 |
| 9.4 Induced Norms | 607 |
| 9.5 Induced Lower Bound | 613 |
| 9.6 Singular Value Inequalities | 615 |
| 9.7 Facts on Vector Norms | 618 |
| 9.8 Facts on Matrix Norms for One Matrix | 627 |
| 9.9 Facts on Matrix Norms for Two or More Matrices | 636 |
| 9.10 Facts on Matrix Norms for Partitioned Matrices | 649 |
| 9.11 Facts on Matrix Norms and Eigenvalues for One Matrix | 653 |
| 9.12 Facts on Matrix Norms and Eigenvalues for Two or More Matrices | 656 |
| 9.13 Facts on Matrix Norms and Singular Values for One Matrix | 659 |

| | | |
|------------|--|------------|
| 9.14 | Facts on Matrix Norms and Singular Values for Two or More Matrices | 665 |
| 9.15 | Facts on Linear Equations and Least Squares | 676 |
| 9.16 | Notes | 680 |
| 10. | Functions of Matrices and Their Derivatives | 681 |
| 10.1 | Open Sets and Closed Sets | 681 |
| 10.2 | Limits | 682 |
| 10.3 | Continuity | 684 |
| 10.4 | Derivatives | 685 |
| 10.5 | Functions of a Matrix | 688 |
| 10.6 | Matrix Square Root and Matrix Sign Functions | 690 |
| 10.7 | Matrix Derivatives | 690 |
| 10.8 | Facts on One Set | 693 |
| 10.9 | Facts on Two or More Sets | 695 |
| 10.10 | Facts on Matrix Functions | 698 |
| 10.11 | Facts on Functions | 699 |
| 10.12 | Facts on Derivatives | 701 |
| 10.13 | Facts on Infinite Series | 704 |
| 10.14 | Notes | 705 |
| 11. | The Matrix Exponential and Stability Theory | 707 |
| 11.1 | Definition of the Matrix Exponential | 707 |
| 11.2 | Structure of the Matrix Exponential | 710 |
| 11.3 | Explicit Expressions | 715 |
| 11.4 | Matrix Logarithms | 718 |
| 11.5 | Principal Logarithm | 720 |
| 11.6 | Lie Groups | 722 |
| 11.7 | Lyapunov Stability Theory | 725 |
| 11.8 | Linear Stability Theory | 726 |
| 11.9 | The Lyapunov Equation | 730 |
| 11.10 | Discrete-Time Stability Theory | 734 |
| 11.11 | Facts on Matrix Exponential Formulas | 736 |
| 11.12 | Facts on the Matrix Sine and Cosine | 742 |
| 11.13 | Facts on the Matrix Exponential for One Matrix | 743 |
| 11.14 | Facts on the Matrix Exponential for Two or More Matrices | 746 |
| 11.15 | Facts on the Matrix Exponential and Eigenvalues, Singular Values, and Norms for One Matrix | 756 |
| 11.16 | Facts on the Matrix Exponential and Eigenvalues, Singular Values, and Norms for Two or More Matrices | 759 |
| 11.17 | Facts on Stable Polynomials | 763 |
| 11.18 | Facts on Stable Matrices | 766 |
| 11.19 | Facts on Almost Nonnegative Matrices | 774 |
| 11.20 | Facts on Discrete-Time-Stable Polynomials | 777 |
| 11.21 | Facts on Discrete-Time-Stable Matrices | 782 |
| 11.22 | Facts on Lie Groups | 786 |
| 11.23 | Facts on Subspace Decomposition | 786 |
| 11.24 | Notes | 793 |

| | |
|---|------------|
| 12. Linear Systems and Control Theory | 795 |
| 12.1 State Space and Transfer Function Models | 795 |
| 12.2 Laplace Transform Analysis | 798 |
| 12.3 The Unobservable Subspace and Observability | 800 |
| 12.4 Observable Asymptotic Stability | 805 |
| 12.5 Detectability | 807 |
| 12.6 The Controllable Subspace and Controllability | 808 |
| 12.7 Controllable Asymptotic Stability | 816 |
| 12.8 Stabilizability | 820 |
| 12.9 Realization Theory | 822 |
| 12.10 Zeros | 830 |
| 12.11 H_2 System Norm | 838 |
| 12.12 Harmonic Steady-State Response | 841 |
| 12.13 System Interconnections | 842 |
| 12.14 Standard Control Problem | 845 |
| 12.15 Linear-Quadratic Control | 847 |
| 12.16 Solutions of the Riccati Equation | 850 |
| 12.17 The Stabilizing Solution of the Riccati Equation | 855 |
| 12.18 The Maximal Solution of the Riccati Equation | 859 |
| 12.19 Positive-Semidefinite and Positive-Definite Solutions of the Riccati Equation | 862 |
| 12.20 Facts on Stability, Observability, and Controllability | 863 |
| 12.21 Facts on the Lyapunov Equation and Inertia | 866 |
| 12.22 Facts on Realizations and the H_2 System Norm | 872 |
| 12.23 Facts on the Riccati Equation | 875 |
| 12.24 Notes | 879 |
| Bibliography | 881 |
| Author Index | 967 |
| Index | 979 |