

Preface

This collection of lectures in commutative ring theory has grown out of a course I have taught for many years at Colorado College for advanced undergraduates taking a second course in abstract algebra and is intended as an introduction to abstract mathematics. It is abstraction — more than anything else — that characterizes the mathematics of the twentieth century. There is both power and elegance in the axiomatic method, attributes that can and should be appreciated by students early in their mathematical careers and even if they happen to be confronting contemporary abstract mathematics in a serious way for the very first time.

Commutative ring theory arose more than a century ago to treat age-old questions in geometry and number theory; it is therefore, in part, a branch of applied mathematics in the sense that it is applied to other areas of mathematics. Even today it draws nourishment from these two subjects. But commutative ring theory is also very much a part of pure mathematics, and as such it has a life of its own that is quite independent of its origins. It is largely the balance — even the tension — between these two aspects of its personality that makes commutative ring theory such a rich and beautiful subject for study.

While many readers of this book may well have previously studied modern algebra, I will assume no particular knowledge on the part of the reader other than perhaps a passing awareness of what a group is. I believe that, by focusing our attention on a single relatively narrow field of modern abstract mathematics, we can begin at the beginning and take an enthusiastic reader on a trip far into the vibrant world of contemporary mathematics. The itinerary and pace for this journey have been conceived with advanced — and, I repeat, enthusiastic — undergraduates in mind, but I sincerely hope that graduate students beginning their specialization in algebra as well as seasoned mathematicians from other areas of mathematics will also find the journey worthwhile and pleasurable.

The intent, then, is a fairly leisurely and reader-friendly passage. Our goal is to get a feel for the lay of the land, to marvel at some of the vistas, and to poke around a few of the back roads. We may miss a couple of the main highways, and we will certainly resist climbing some of the higher peaks. We may not get as far, or as high, as some would like, but we prefer not to lose anyone along the way. There will always be time — and other guidebooks — for other, more ambitious, trips for those of you who have enjoyed this one.

I have placed a series of problems at the end of each chapter in order to encourage active reading. These exercises provide you with a way of immediately reinforcing new concepts, as well as becoming adept at some of the fundamental techniques of commutative ring theory. I have provided solutions to some of these exercises at the end of the book so that you can compare your work with what could be considered “standard” solutions. Some of the problems are routine exercises designed to build technical skill or reinforce basic new ideas, and advanced readers may well wish to skip most of them. However, many of the problems are in fact extremely important mathematical results in their own right, and will be used freely later on in the text.

I would like to thank first of all the many students at Colorado College who have been subjected to earlier, considerably rougher, versions of this book. In particular, I appreciate the enthusiastic and generous help I have received through the years from David Carlson, T.J. Calvert, Lisa Converse, Courtney Gibbons, Laura Hegerle, Eric Raarup, Karin Reisbeck, Chantelle Szczech-Jones, Mark Sweet, Rahbar Virk, and Trevor Wilson. I would also like to thank two colleagues, Doug Costa and Wojciech Kosek, whose suggestions have improved this book considerably. Mostly, though, I would like to thank Jim Brewer, my thesis advisor at the University of Kansas, from whom I not only first learned this beautiful subject but also received so very much in terms of guidance, wisdom, and friendship.

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May 31, 2006

TOPICS IN COMMUTATIVE RING THEORY

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