Preface

This book deals with the boundary control of wave partial differential equations (PDEs) in one dimension, with a moving actuated boundary, or with finite-dimensional dynamics (modeled as ordinary differential equations; ODEs) at either the unactuated boundary or in the actuation path. These structures are inspired by applications involving cables and strings that move mechanical loads.

Cables moving mechanical loads are most conspicuously employed in elevators (both in buildings, where they may be hundreds of meters long, and in deep mining, where they may be kilometers long), but also in undersea construction, such as laying telecommunication cables along uneven seafloor or building artificial reefs that have an environmentally beneficial purpose for the sea life, and in other applications, including many yet to be revealed in other domains.

In deep-sea drilling, the so-called drill string is not a cable but a kilometers-long thin cylinder, with a drill bit on its unactuated end, and its dynamics of rotation (torsional dynamics) are governed by the wave equation; that is, they are mathematically equivalent to string or cable vibration.

Hence, our book's overarching title for this multitude of physical configurations, load types, and operating purposes — *PDE Control of String-Actuated Motion*.

The actuation of mechanical loads by means of strings (cables) has its advantages over rigid connections. It allows for significant improvements in energy efficiency, weight, size of the operation workspace, operation speed, and maximum payload, compared with rigid-body mechanisms, due to the string's properties of lower weight, resisting relatively large axial loads, and low bending and torsional stiffness.

However, the distributed parameter nature of a string or cable makes the control design of the cable-actuated mechanisms more challenging than the traditional ODE-based control designs for lumped parameter rigid-body mechanisms, giving rise to many new problems in boundary control of PDEs. The theoretical challenges and practical significance have led us to carry out research on many topics involving cables and strings of time-varying length, with moving loads, and with actuator dynamics. This book presents this collection of methodologies, whose meaning is predominantly mathematical, but whose inspiration comes entirely from applications and technology.

Cable elevators and other lifting and depositing tasks, as well as drilling at a high penetration rate, introduce a heretofore unstudied problem of boundary control of wave PDEs with moving boundaries—that is, on time-varying domains. This is the central issue of our book—wave PDE control on one-dimensional domains of time-varying lengths—that is, vibration suppression in strings of time-varying lengths. One should note that, as the cable length varies with time, possibly fast, even if one could be talking about *eigenvalues* and *eigenfunctions* (that rapidly change), the spectral approaches to control design certainly would not be applicable. We approach this challenge using time-domain and Lyapunov-based approaches.

The second key challenge is that, at the moving end of a time-varying string, a load is present, and the motion of this load needs to be controlled with an actuator on the opposite end of the string. The objective is to suppress the vibration in both the string of varying length and in the load at the distal end from the actuator. Using feedback control to add damping artificially, where physical viscous damping might be absent or insufficient, would be easy at the location of (i.e., proximal to) the actuator. However, the actuator being at a boundary makes emulating viscous damping along the entire string, and at the load on the string's distal end, challenging.

This challenge is met using the method of PDE backstepping. Backstepping employs two tools, a Volterra transformation of the infinite-dimensional state, and feedback, to add damping at locations other than the actuator. But, prior to employing the backstepping transformation, we usually employ first a transformation of the state into the Riemann variables, which is a canonical representation for coupled hyperbolic PDEs, to which backstepping is readily applicable.

However, the actuator in most boundary-controlled cable and string systems does not act directly and instantly. The actuator, be it hydraulic or electrical, has its own considerable inertia—namely, its own lumped-parameter dynamics modeled by an ODE. These dynamics themselves have to be overcome using finite-dimensional backstepping (the classical *integrator backstepping*).

Hence, the overall system that arises in string-actuated motion is often a sandwiched ODE-PDE-ODE configuration, with an input acting on only one of the two ODEs at the end of the PDE, and not directly on the PDE.

The reality of applications gives rise to additional effects: nonlinearities, disturbances, unknown parameters, input delays, sampled (or event-triggered) sensing, as well as many more which we deem beyond the page and time limits of this book.

A vast literature exists on control of overhead cranes and gantry cables. A good entry point into this literature, in terms of both the theory and applications, is the tutorial article [23].

A large portion of the research on this topic justifiably focuses on transverse motion and employs the techniques of differential flatness, finite-time motion planning, and finite-horizon optimal control. Many, if not most of these approaches, are concerned with designing open-loop control signals. While some of our work in this book is applicable to the transversal motion of overhead cranes and gantry cables, these problems are not our focus here. Instead, we focus on vibration suppression, in axial and other directions, cables of varying length, Lyapunov stabilization techniques, and handling of uncertainties.

What Does the Book Cover?

The book comprises three parts. The first part is devoted to various control applications, as drivers for control design and theoretical study. Control problems for mining cable elevators are introduced in chapters 2–5, focusing on single-cable elevators in chapter 2, dual-cable elevators in chapter 3, and airflow disturbances and the influence of flexible guideways in chapter 4 and chapter 5, respectively. In addition to the mining cable elevator, the deep-sea construction vessel for undersea moving is also a cable-actuated manipulator to move mechanical loads. Its basic control design is introduced in chapter 6, and additional real-world effects—that is,

Table 1. The motion types

Chapter	Axial	Transversal	Torsional
2	√		
3	√		
4	√		
5		√	
6	√	√	
7		√	
8			√
10		√	
11	√		
13		√	
15	√		

sensor signal delays occurring in large-distance signal transmission through acoustic devices and the requirement of reducing changes in the actuator signal considering the massive ship-mounted crane—are dealt with in chapter 7. Apart from the cable-actuated mining elevator and the deep-sea construction vessel, another distinct but kindred application, deep-sea drilling, is tackled in chapter 8.

Inspired by but going beyond the applications in part I, generalized control problems are dealt with in part II—that is, boundary control of sandwich hyperbolic PDEs in particular. Control of the sandwich systems is covered in chapters 9–12, with the basic control design presented in chapter 9 and then extended to a variety of more challenging problems, including control of the sandwich systems with sensor delay in chapter 10, with event-triggered design in chapter 11, and with nonlinearities in chapter 12. The general results in part II are justified by applications in part I.

The last of the three parts presents triggered-type adaptive control of hyperbolic PDEs. Three triggered adaptive control schemes (event-triggered control, regulation-triggered parameter estimation, and a combination of both) for hyperbolic PDE-ODE systems are developed in chapters 13–15, respectively. The triggered-type adaptive control results in part III are also verified in the applications in part I.

The book deals with all of the three possible motions of strings or cables: longitudinal/axial/stretching, lateral/transversal/bending, and angular/rotational/torsional. However, it is only in one chapter that we deal with more than one of these three motions. In chapter 6 we deal with coupled longitudinal-lateral vibrations. Table 1 gives an overview of the motion types that each chapter covers. The suppression of axial vibrations dominates our exposition, with transversal vibrations a close second.

Table 2. Configurations

Chapter	ODE at distal end of PDE	ODE at proximal end of PDE
2	$\surd$	
3	$\surd$	
4	$\surd$	
5	$\surd$	
6	$\surd$	
7	$\surd$	
8	$\surd$	
9	$\surd$	$\surd$
10	$\surd$	$\surd$
11	$\surd$	$\surd$
12	$\surd$	$\surd$
13	$\surd$	
14	$\surd$	
15	$\surd$	

All of the problems considered in the book incorporate at least one PDE and one ODE. Some consider a second ODE as well. The configurations considered are given in table 2. Those configurations that include ODEs at both distal and proximal ends in chapters 9–12 (namely, about a third of the book) are sandwich systems. In addition, chapter 10 contains a sensor delay at the distal end.

We consider both PDE-ODE systems that are fully known and those that contain unknown or unmeasured quantities—such as unmeasured states, unmeasured disturbances, and unknown parameters. In table 3 we overview the contents of the book based on the unmeasured and unknown effects. Virtually all of our exposition is for cables and (drill) strings that are not instrumented with distributed sensing, as is consistent with reality. Disturbances and unknown parameters occupy a large share of the book and create some of the most significant challenges for design and analysis.

When it comes to adaptive control, a topic dealt with comprehensively for coupled hyperbolic PDEs in [9] (and for parabolic PDEs a decade earlier in [166]) is tackled in chapters 5, 8, 13, 14, and 15 of this book, as indicated in table 3. Chapters 5 and 8 employ a classical continuous-in-time Lyapunov-based approach. On the other hand, chapters 13–15 employ novel event-triggered approaches. Chapters 13 and 14 are very different in what event triggering is employed for. An adaptive controller consists of two components: the control law and the parameter estimator (update law). Both of these components can employ piecewise-constant values—the control input and the parameter estimate. And the changes in the piecewise-constant values in both of these components can be triggered in various ways. In

Table 3. Unmeasured and unknown effects

Chapter	Unmeasured PDE state	Unmeasured ODE state	Unmeasured disturbance	Unknown parameters
2	√			
3	√			
4	√		√	
5	√		√	√
6	√	√		
7	√	√		
8	√		√	√
10	√	√		
11	√	√		
12	√	√		
13	√			√
14				√
15				√

chapter 13 we employ continuous parameter updates and event-triggered control inputs. Conversely, in chapter 14 we employ continuous control inputs (except for finite time instants) and event-triggered parameter updates. Because the parameter estimator is the more delicate of the two components of an adaptive control since it is generally not endowed with convergence guarantees, it is chapter 14 that is considerably more challenging of the two chapters. In chapter 15, simultaneous triggering is employed for the parameter update law and the control law, the result of which is that both the parameter estimates and the control input employ piecewise-constant values.

For the researcher in coupled hyperbolic PDE systems who is interested in going beyond the basic 2×2 case, which is superbly covered in [16] and [9], there are interesting designs for 4×4 cases in this book—for example, in chapter 3 and chapter 6. Chapter 3 deals with axial oscillations in a pair of cables connected by a payload at the distal boundary. So the 4×4 system in chapter 3 is a set of two 2×2 pairs that are coupled not along the domain but at the boundary. In contrast, chapter 6 deals with a single cable but with axial and lateral oscillations that bring domain-wide coupling into the plant and, therefore, a fully coupled 4×4 hyperbolic system. While in multiphase flows, in both oil drilling and production, as well as in congested multiclass traffic flow, a larger number of first-order hyperbolic PDEs arise in the direction toward the actuated end than away from it, this interesting occurrence of underactuated heterodirectional hyperbolic PDEs does not arise with cables.

What Niche in the Literature Does This Book Fill?

The main inspiration for this book comes from the cascade PDE-ODE configurations in [116]. The book [111] develops the cascade ideas from [116] in the parabolic PDE realm for applications in additive manufacturing.

The present book expands the reach of [116] into cable-operated systems and on time-varying domains. But this book's closest cousin may be [77], a major volume spawned from the classic [132]. While [77, 132] employ collocated static feedbacks for wave PDEs on static domains, and in the absence of ODEs, our focus is broader in terms of plant structure (varying domain, ODE included), methodology (backstepping controller and observer designs), and the emphasis on applications. We do not, however, deal with beam systems.

For Whom Is This Book

This book should be valuable to researchers working on control and dynamic systems—engineers, graduate students, and PDE system specialists in academia.

Mathematicians with interest in control of distributed parameter systems will find the book stimulating, because it tackles and opens a door for control of sandwich PDE systems, which present many stimulating challenges and opportunities for further research on the stabilization of ever-expanding classes of unstable sandwich PDE systems.

Engineers in mechanical, aerospace, and civil/structural engineering, focusing on vibration or motion control, especially for flexible structures or manipulators, will learn some new and useful methodologies for designing controller/observer algorithms, and addressing some problems they have no doubt faced in practice: time delay, disturbances, uncertainties, and so on.

The background required to read this book includes little beyond the basics of function spaces and Lyapunov theory for ODEs. We hope that the reader will regard the book not as a collection of problems that have been solved but as a collection of tools and techniques that are applicable to open problems, particularly in interconnected systems of ODEs and PDEs, and to physical problems modeled by PDE-ODE coupled systems.

ACKNOWLEDGMENTS

We would like to thank Shumon Koga for his contribution to chapter 2, Yangjun Pi for his contributions to chapters 2, 3, and 4, Shuxia Tang for her contributions to chapters 4, 5, and 8, and Iasson Karafyllis for his contribution to chapter 14.

We are grateful to Rafael Vazquez, Jean-Michel Coron, Ole Morten Aamo, Florent Di Meglio, Federico Bribiesca Argomedo, Long Hu, Joachim Deutscher, Henrik Anfinsen, Delphine Bresch-Pietri, and their coauthors for their inspiring early work on boundary control of the 2×2 linear hyperbolic system. The pioneering work of Wei Guo, Hongyinping Feng, and Bao-Zhu Guo on extending the active disturbance rejection control (ADRC), introduced by Jingqing Han for finite dimension systems, to infinite dimension systems inspired chapters 4 and 8. We also would like to thank Nicolas Espitia, Christophe Prieur, and their coauthors for the original work on event-triggered designs in hyperbolic PDEs, from which we benefited in developing chapters 7, 11, and 13. We also thank CITIC Heavy Industries Co. Ltd.

for providing practical motivations and expertise on the mining cable elevator in chapters 2–5.

Ji Wang would like to give a special thanks to his family for their unwavering support and love. Miroslav Krstic thanks the National Science Foundation, the Air Force Office of Scientific Research, the Los Alamos National Laboratory, and Dr. Dan Alspach for research support, as well as his colleagues at General Atomics for enriching discussions on cable-actuated control systems.

La Jolla, California — *Ji Wang*
July 2021 — *Miroslav Krstic*