Preface

Topics included are: Basic properties of groups; subgroups, symmetric groups, alternating groups, cyclic groups, cosets; direct products, finitely generated abelian groups, quotient groups and homomorphisms; basic properties of rings; subrings, integral domains, quotient rings and ring homomorphisms, quaternions; ideal theory, isomorphism theorems; unique factorization domains, Euclidean domains and Gaussian integers; polynomial rings, fields, field extensions, algebraic closure, finite fields.

Overview and Approach: In addition to being an important branch of mathematics in its own right, abstract algebra is now an essential tool in number theory, geometry, topology, and, to a lesser extent, analysis. Thus, it is a core requirement for all mathematics majors. Outside of mathematics, abstract algebra also has many applications in cryptography, coding theory, quantum chemistry, and physics.

Abstract algebra is the field of mathematics that studies algebraic structures such as groups, rings, fields, and modules; we will primarily study groups, rings and fields in this course. The power of abstract algebra is embedded in its name: it gives us an arena in which we may study disparate mathematical objects together and abstractly, without considering a particular instance or occurrence. For example, the multiplication of nonzero real numbers, symmetries of a molecule, roots of the unity, actions of a Rubik's cube, and loops on surfaces all form groups. By exploring groups abstractly, we can derive properties and structures that apply to all examples that we currently know or may discover in the future. With this in mind it should come as no surprise that abstract algebra builds a language that is used in nearly every field of mathematics. The applications of abstract algebra within and beyond mathematics are not the only reasons to study abstract algebra. First, learning abstract algebra is one of the best ways to practice working through complex concepts and to develop your abstract reasoning abilities. Second, studying abstract algebra provides a window into what it is like to do research in mathematics. Perhaps most importantly, you will experience the intrinsic beauty of mathematics during this course. While the aesthetic nature of abstract algebra is difficult to describe, it is obvious to any of its practitioners.

About this book: This book is intended as a textbook for a one-term senior undergraduate (or graduate) course in abstract algebra, to prepare students for further readings, for example, Group Theory, Galois Theory. K. Zhao used the earlier drafts of this book as the textbook when he taught MA323 (and MA523) at Wilfrid Laurier University since 2015. L. Li used the earlier drafts as his textbook in his *Abstract Algebra* course since 2018. When they prepared these lecture notes they mainly took [F] as their reference.

The book contains 168 exercises of varying difficulty with sample solutions. Besides standard ones, many of the exercises are very interesting. Some are rather hard. It is not a surprise if the reader cannot solve some of the exercises, particularly for first learners.

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The authors wish all instructors and students who use this book a happy mathematical journey they will undertake into this delightful and beautiful realm of abstract algebra.

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