Chapter 8

Scenarios

8.1 Introduction

In chapters 1–6, the effect of zones' protection is fully known. In chapter 7, the uncertainty that may exist with regard to the survival of the species, both in protected and unprotected zones, is expressed in terms of probabilities. We also show, in chapter 7, how to take into account, in a certain way, the inevitable uncertainties concerning the values of these probabilities. In this new chapter, we consider another way to take into account the uncertainty about the survival of the species in protected and unprotected zones. For this purpose, we consider that a set of scenarios, $\mathrm{Sc} = \{\mathrm{sc}_1, \mathrm{sc}_2, \ldots, \mathrm{sc}_p\}$, are possible (see appendix at the end of the book). A scenario is a set of hypotheses on the evolution of factors that can affect the survival of species in protected or unprotected zones. These assumptions may include direct factors such as land use, climate change, pollution, overexploitation or invasive species, and indirect factors such as economic activity, demographic change, and socio-political contexts. Thus, with each set of protected zones is associated a certain protection of the species under consideration and this protection depends on the scenario. We denote by $\underline{\mathrm{Sc}}$ the set of indices of the possible scenarios. As in the previous chapters, $S = \{s_1, s_2, \ldots, s_m\}$ refers to the set of species, more or less threatened, in which we are interested and $Z = \{z_1, z_2, \ldots, z_n\}$, the set of zones that we can decide whether or not to protect from a given moment, in order to ensure a certain protection to the species in question and thus increase their chance of survival. $\underline{S}$ and $\underline{Z}$ refer to the set of corresponding indices, respectively. With regard to the survival of the species in protected zones, the following two cases are considered: in the first case, it is assumed that, for any scenario sc_ω, we know the zones whose protection ensures the survival of species s_k, and this for all $k \in \underline{S}$, if scenario sc_ω is realized. This set is denoted by Z_k^ω and the corresponding set of indices is denoted by $\underline{Z}_k^\omega$. In other words, to ensure the survival of species s_k if scenario sc_ω is realized, it is necessary and sufficient that at least one zone of Z_k^ω be protected. As we have generally done in the

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previous chapters, we consider here that there is only one level of protection: a zone is protected or not. More precisely, the protection of zone z_i is considered to protect species s_k in the case of scenario sc_ω realization if the population size of species s_k in this zone is greater than or equal to a certain threshold value, depending on the scenario and denoted by v_{ik}^ω. In other words, $Z_k^\omega = \{z_i \in Z : n_{ik} \geq v_{ik}^\omega\}$ where n_{ik} refers to the population size of species s_k – at the beginning of the horizon considered – in zone z_i. Given a reserve R, we refer to $\text{Nb}_1^\omega(R)$ as the number of species protected by this reserve if scenario sc_ω occurs. In the second case, it is assumed that, for any scenario sc_ω, we know the minimal population size of species s_k that must be present in the entire reserve – at the beginning of the period considered – for this species to be protected if scenario sc_ω occurs, and this for all $k \in \underline{S}$. This minimal population size is denoted by θ_k^ω and $\text{Nb}_2^\omega(R)$ is referred to as the number of species protected by reserve R if scenario sc_ω occurs. This chapter focuses on the determination of optimal robust reserves, *i.e.,* the determination of reserves that allow a certain objective to be "best" achieved, knowing that several scenarios are possible.

Example 8.1. The instance described in figure 8.1 is considered and it is assumed that two scenarios are possible: $\text{Sc} = \{\text{sc}_\omega : \omega = 1, 2\}$. We consider the two ways – described above – of calculating the number of species protected by a reserve, R, when scenario sc_ω occurs: $\text{Nb}_1^\omega(R)$ and $\text{Nb}_2^\omega(R)$. With regard to the calculation of $\text{Nb}_1^\omega(R)$, the values of v_{ik}^ω , $i \in \{1,\ldots,20\}$, $k \in \{1,\ldots,15\}$, $\omega \in \{1, 2\}$, are such that the list of species that will survive in each protected zone and in each of the two scenarios is given in figure 8.2. With regard to the calculation of $\text{Nb}_2^\omega(R)$, the values of θ_k^1 and θ_k^2 are given in table 8.1. For example, if reserve R is composed of the 5 zones z_2, z_3, $z_{10,}$ z_{11}, and z_{16}, we obtain $\text{Nb}_1^1(R) = 6$ since the 6 species s_3, s_4, s_6, s_7, s_8, and s_{12} will survive in the case of scenario sc_1, $\text{Nb}_1^2(R) = 7$ since the 7 species s_1, s_3, s_6, s_9, s_{10}, s_{11}, and s_{12} will survive in the case of scenario sc_2, $\text{Nb}_2^1(R) = 6$ since the 6 species s_3, s_4, s_7, s_{10}, s_{11}, and s_{12} will survive in the case of scenario sc_1, and $\text{Nb}_2^2(R) = 6$ since the 6 species s_1, s_3, s_9, s_{10}, s_{11}, and s_{12} will survive in the case of scenario sc_2.

In the following sections we examine several problems related to the selection of optimal robust reserves. Such reserves provide the best possible protection for the species under consideration, in the presence of several scenarios and taking into account a given robustness criterion.

8.2 Reserve Protecting All Species Considered Regardless of the Scenario that Occurs

A first question that can be raised is the following: what is the set of zones to be protected, at minimal cost, to protect all species considered, regardless of the scenario that occurs. We first examine the case where the interest in protecting a reserve R, if scenario sc_ω is realized, is assessed by $\text{Nb}_1^\omega(R)$ then the case where this interest is assessed by $\text{Nb}_2^\omega(R)$.

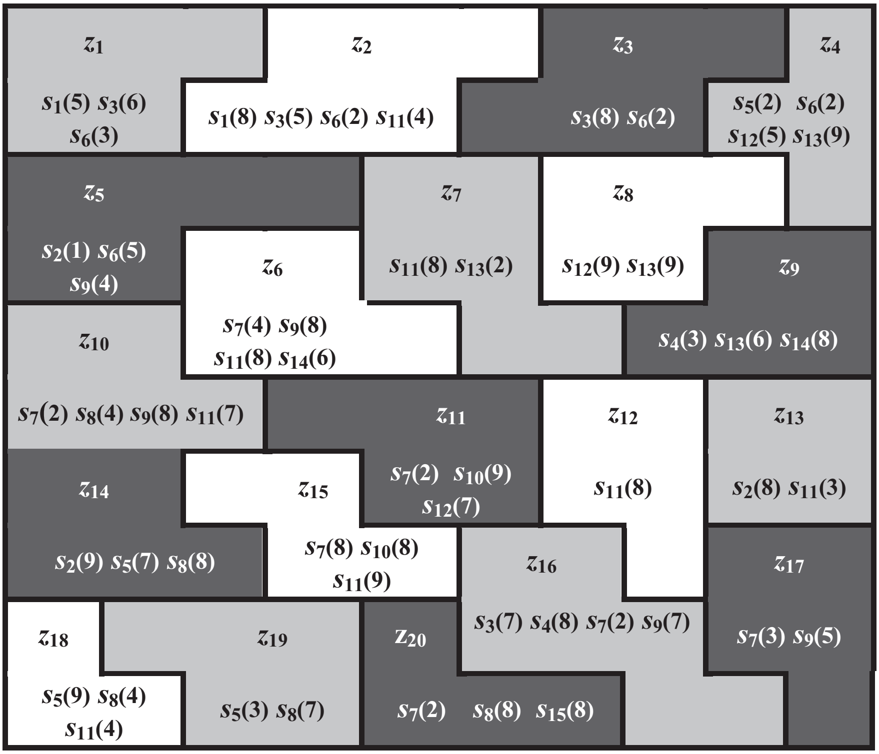

FIG. 8.1 – The 20 zones z_1, z_2,…, z_{20} are candidates for protection and the 15 species s_1, s_2, …, s_{15} living in these zones are concerned. For each zone, the species present and their population size – in brackets – are indicated. The cost of protecting the white zones is equal to 1 unit, the cost of protecting the light grey zones is equal to 2 units and the cost of protecting the dark grey zones is equal to 4 units. For example, s_7, s_9, s_{11}, and s_{14} are present in zone z_6, their population size is equal to 4, 8, 8, and 6 units, respectively, and the cost of protecting this zone is equal to 1 unit.

8.2.1 Case Where the Number of Species Protected by a Reserve, R, if Scenario sc_ω is Realized, is Assessed by $Nb_1^\omega(R)$; in this Case the Protection of Each Zone Allows to Protect a Given Set of Species Depending on the Scenario

The problem can be formulated as a linear program in Boolean variables by associating to each zone z_i, as in the previous programs, a Boolean variable x_i that takes the value 1 if and only if zone z_i is selected for protection. This results in program $P_{8.1}$ which is known, in the field of operational research, as the set-covering problem.

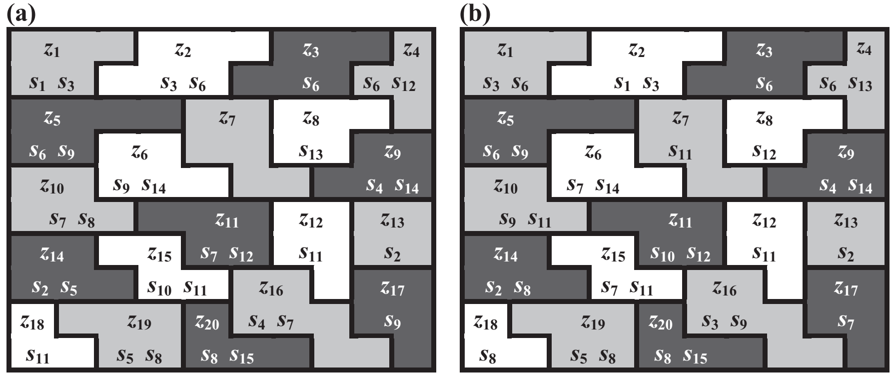

FIG. 8.2 – A region divided into twenty zones. Two scenarios are possible. (a) Species protected by the protection of a zone in the case of scenario sc_1. (b) Species protected by the protection of a zone in the case of scenario sc_2. For example, the protection of zone z_{10} ensures the protection of species s_7 and s_8, if scenario sc_1 occurs, and that of species s_9 and s_{11}, if scenario sc_2 occurs.

TAB. 8.1 – Values of θ_k^1 and θ_k^2. For example, species s_3 will survive in the selected reserve if its total population size in this reserve is greater than or equal to 20, in the case of scenario sc_1, and 17, in the case of scenario sc_2.

k	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
θ_k^1	9	12	20	8	14	12	6	13	16	7	10	5	17	10	7
θ_k^2	8	16	17	10	18	9	12	12	13	9	9	6	22	12	6

$$
\mathrm{P}_{8.1}: \begin{cases} \min \sum_{i \in \underline{Z}} c_i x_i \\ \text{s.t.} \left| \begin{array}{lll} \sum_{i \in \underline{Z}_k^\omega} x_i \geq 1 & k \in \underline{S},\ \omega \in \underline{\mathrm{Sc}} & (8.1.1) \\ x_i \in \{0,1\} & i \in \underline{Z} & (8.1.2) \end{array} \right. \end{cases}
$$

The economic function expresses the total cost of protecting the selected zones. Constraints 8.1.1 express that, for any species s_k and scenario sc_ω, at least one zone of Z_k^ω must be selected. It should be noted that wanting to protect all species considered regardless of the scenario that occurs is a very conservative but often unrealistic objective. Indeed, the optimal solution will generally consist in protecting a large number of zones – possibly all of them – to be guarded against the consequences of the different scenarios.

Example 8.2. Let us take again the instance built from figure 8.1 and described by figure 8.2. In this example, the cheapest strategy to protect all species, regardless of the scenario that occurs – among the 2 possible scenarios – is to protect the 12 zones z_1, z_2, z_4, z_6, z_8, z_9, z_{11}, z_{13}, z_{15}, z_{16}, z_{19}, and z_{20}, which costs 26 units (figure 8.3).

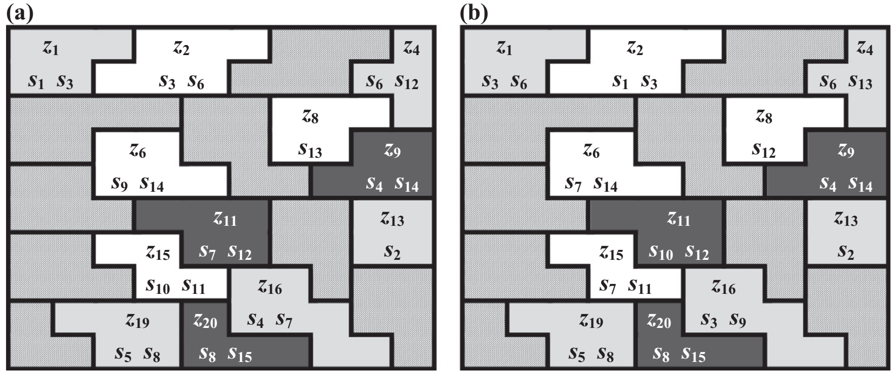

FIG. 8.3 – The least costly solution to protect all species, regardless of the scenario that occurs – among the 2 possible scenarios – is to protect the 12 non-hatched zones: z_1, z_2, z_4, z_6, z_8, z_9, z_{11}, z_{13}, z_{15}, z_{16}, z_{19}, and z_{20}. This protection costs 26 units while the protection of all zones costs 48 units. (a) All species are protected if scenario sc_1 occurs. (b) All species are protected if scenario sc_2 occurs.

As we have already discussed in the case of a single scenario (chapter 1), it can be considered that to be protected species s_k must be protected in at least b_k zones. The formulation of this variant of the problem is obtained by replacing in $P_{8.1}$ the constraints $\sum_{i \in \underline{Z}_k^\omega} x_i \geq 1,\ k \in \underline{S}, \omega \in \underline{\mathrm{Sc}}$, by the constraints $\sum_{i \in \underline{Z}_k^\omega} x_i \geq b_k$, $k \in \underline{S}, \omega \in \underline{\mathrm{Sc}}$.

8.2.2 Case Where the Number of Species Protected by a Reserve, R, if Scenario sc_ω Occurs, is Assessed by $Nb_2^\omega(R)$; in this Case, a Species is Protected by R if its Total Population Size in R Exceeds a Certain Value Depending on the Scenario

The problem of determining the minimal cost reserve, making it possible to protect all species considered, whatever the scenario that occurs, can be formulated as the linear program in Boolean variables obtained by replacing in $P_{8.1}$ the constraints $\sum_{i \in \underline{Z}_k^\omega} x_i \geq 1$, $k \in \underline{S}, \omega \in \underline{\mathrm{Sc}}$, by the constraints $\sum_{i \in \underline{Z}} n_{ik} x_i \geq \theta_k^\omega$, $k \in \underline{S}, \omega \in \underline{\mathrm{Sc}}$.

8.3 Reserve Protecting as Many Species – of a Given Set – as Possible Under a Budgetary Constraint and in the Worst-Case Scenario

A second problem that may naturally arise is to determine the zones to be protected, taking into account an available budget, B, in order to protect as many species as possible in the worst-case scenario. The worst-case scenario is related to a set of protected zones. This is the scenario for which the number of protected species is minimal, taking into account the zones selected for protection. This problem, related to species richness, can be written $\max_{R \subseteq Z, C(R) \leq B} [\min_{\omega \in \underline{\mathrm{Sc}}} \mathrm{Nb}_f^{\omega}(R)]$ where $\mathrm{Nb}_f^{\omega}(R)$ refers to the number of protected species – calculated in two different ways depending on the value of f – when the set of zones R is protected and scenario sc_ω is realized. $C(R)$ refers to the cost of reserve R. These problems can be formulated as linear programs in Boolean variables. For this purpose, as in all previous programs, with each zone z_i is associated a Boolean decision variable, x_i. With each possible pair (species, scenario) is also associated a "working" Boolean variable, y_k^{ω}, which, by convention, takes the value 1 if and only if the zones selected to be protected allow species s_k to be protected in the event that scenario sc_ω is realized.

8.3.1 Case Where the Interest of Protecting a Reserve, R, if Scenario sc_ω is Realized, is Assessed by $Nb_1^{\omega}(R)$

In this case, the problem can be formulated as program $\mathrm{P}_{8.2}$.

$$
\mathrm{P}_{8.2}: \begin{cases} \max \ \alpha \\ \text{s.t.} \left| \begin{array}{llll|lll} \alpha \leq \sum\limits_{k \in \underline{S}} y_k^{\omega} & \omega \in \underline{\mathrm{Sc}} & (8.2.1) & & x_i \in \{0,1\} & i \in \underline{Z} & (8.2.4) \\ y_k^{\omega} \leq \sum\limits_{i \in \underline{Z}_k^{\omega}} x_i & k \in \underline{S}, \omega \in \underline{\mathrm{Sc}} & (8.2.2) & & y_k^{\omega} \in \{0,1\} & k \in \underline{S}, \omega \in \underline{\mathrm{Sc}} & (8.2.5) \\ \sum\limits_{i \in \underline{Z}} c_i x_i \leq B & & (8.2.3) & & & & \end{array} \right. \end{cases}
$$

The objective of $\mathrm{P}_{8.2}$ is to maximize variable α. Because of constraints 8.2.1, this variable α takes the value $\min_{\omega \in \underline{\mathrm{Sc}}} \left\{ \sum_{k \in \underline{S}} y_k^{\omega} \right\}$ at the optimum of $\mathrm{P}_{8.2}$ since there is no other constraint on this variable, which corresponds to the number of protected species in the event that the worst-case scenario occurs – for a fixed set of protected zones. According to constraints 8.2.2, variable y_k^{ω}, which is a Boolean variable, takes, at the optimum of $\mathrm{P}_{8.2}$, the value 0 if $\sum_{i \in \underline{Z}_k^{\omega}} x_i = 0$, *i.e.*, if no zone of Z_k^{ω} is selected, and the value 1 if $\sum_{i \in \underline{Z}_k^{\omega}} x_i \geq 1$, *i.e.*, if at least one zone of Z_k^{ω} is selected. Variable y_k^{ω}, therefore, takes the value 1 if and only if the zones selected for protection allow species s_k to be protected, in the event that scenario sc_ω occurs. Constraints 8.2.4 and 8.2.5 specify the Boolean nature of all variables.

Example 8.3. Let us take again the instance described in figures 8.1 and 8.2 and assume that the budget available for the protection of the zones is equal to 10 units. By protecting the 7 zones z_2, z_4, z_6, z_8, z_{15}, z_{16}, and z_{19} we are sure that, whatever the scenario, at least 11 species will be protected. Indeed, if scenario sc_1 is realized, the 12 species s_3, s_4, s_5, s_6, s_7, s_8, s_9, s_{10}, s_{11}, s_{12}, s_{13}, and s_{14} will be protected, and if scenario sc_2 is realized, the 11 species s_1, s_3, s_5, s_6, s_7, s_8, s_9, s_{11}, s_{12}, s_{13}, and s_{14} will be protected (figure 8.4).

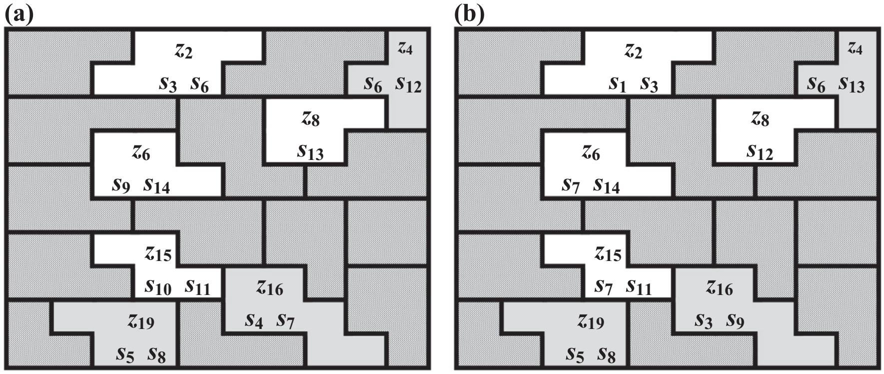

FIG. 8.4 – If the budget available for zone protection is 10 units, the optimal solution for the instance described in figures 8.1 and 8.2 consists in protecting the 7 non-hatched zones z_2, z_4, z_6, z_8, z_{15}, z_{16}, and z_{19}, which costs 10 units. (a) 12 species are protected in scenario sc_1. (b) 11 species are protected in scenario sc_2.

8.3.2 *Case Where the Interest of Protecting a Reserve, R, if Scenario sc_ω is Realized, is Assessed by $Nb_2^\omega(R)$*

In this case, the problem can be formulated as the mathematical program obtained by replacing in $\mathrm{P}_{8.2}$ the constraints $y_k^\omega \leq \sum_{i \in \underline{Z}_k^\omega} x_i,\ k \in \underline{S}, \omega \in \underline{\mathrm{Sc}}$, by the constraints $\theta_k^\omega y_k^\omega \leq \sum_{i \in \underline{Z}} n_{ik} x_i,\ k \in \underline{S}, \omega \in \underline{\mathrm{Sc}}$.

8.4 Reserve Minimizing the Maximal Relative Regret, for All Scenarios, About the Number of Protected Species, Taking into Account a Budgetary Constraint

Seeking to protect a set of zones in such a way that as many species as possible are protected in the worst-case scenario can have a significant drawback: if one of the scenarios is very "pessimistic" – from the viewpoint of species protection resulting from the protection of zones –, then the optimal solution of $\mathrm{P}_{8.2}$ will essentially take this only scenario into consideration. To overcome this drawback, it is possible to determine the zones to be protected – under a budgetary constraint – in such a way as to minimize the greatest regret, *i.e.,* the greatest relative gap, over all scenarios, between the number of protected species, given the zones selected, and the maximal number of species that could be protected in the scenario considered, by possibly retaining another set of zones. This problem can be written $\min_{R \subseteq Z, C(R) \leq B}$ $\{\max_{\omega \in \underline{\text{Sc}}}[(\text{Nb}_f^\omega(R_f^{*\omega}) - \text{Nb}_f^\omega(R))/\text{Nb}_f^\omega(R_f^{*\omega})]\}$ where the set of zones of maximal interest for scenario sc_ω is designated by $R_f^{*\omega}$ and when the interest of a reserve, R, is assessed by $\text{Nb}_f^\omega(R)$ – f is equal to 1 or 2. To solve this problem, it is first necessary to determine the maximal interest – here, the maximal number of protected species – that can be obtained by protecting a set of zones in the case of scenario sc_ω, for all scenarios.

8.4.1 Case Where the Interest of Protecting a Reserve, R, If Scenario sc_ω is Realized, is Assessed by $\text{Nb}_1^\omega(R)$

In this case, the maximal number of species that can be protected under scenario sc_ω can be calculated by solving the linear program in 0–1 variables $\text{P}_{8.3}(\omega)$.

$$\text{P}_{8.3}(\omega): \begin{cases} \max \quad N_{\max}^\omega = \sum_{k \in \underline{S}} y_k^\omega \\ \text{s.t.} \left| \begin{array}{llll} y_k^\omega \leq \sum_{i \in \underline{Z}_k^\omega} x_i \quad k \in \underline{S} & (8.3_\omega.1) & | \quad x_i \in \{0,1\} \quad i \in \underline{Z} & (8.3_\omega.3) \\ \sum_{i \in \underline{Z}} c_i x_i \leq B & (8.3_\omega.2) & | \quad y_k^\omega \in \{0,1\} \quad k \in \underline{S} & (8.3_\omega.4) \end{array} \right. \end{cases}$$

Because of the economic function to be maximized, $\sum_{k \in \underline{S}} y_k^\omega$, and constraints $8.3_\omega.1$, variable y_k^ω takes the value 1, at the optimum of $\text{P}_{8.3}(\omega)$, if and only if $x_i = 1$ for at least one index i of $\underline{Z}_k^\omega$, *i.e.*, if at least one of the zones that allow species s_k to be protected under scenario sc_ω is selected to be protected. Otherwise, variable y_k^ω can only take the value 0. The value of the economic function at the optimum of $\text{P}_{8.3}(\omega)$ is therefore well equal to the maximal number of species that can be protected if scenario sc_ω is realized, under the budgetary constraint expressed by constraints $8.3_\omega.2$.

Example 8.4. Let us take again the instance built from figure 8.1 and described by figure 8.2, and suppose that the budget available for the protection of the zones is equal to 7 units. If we think that scenario sc_1 will be realized, then the optimal reserve allows 10 species to be protected. It should be noted that if, contrary to the forecasts, scenario sc_2 is realized, then the reserve selected only allows for the protection of 8 species (figure 8.5). If, on the contrary, we think that scenario sc_2 will be realized, then the optimal reserve allows 9 species to be protected. Again, it should be noted that if, contrary to the forecasts, scenario sc_1 is realized, then the reserve selected only allows for the protection of 8 species (figure 8.6).

Once $N_{\max}^\omega$ is determined for all scenarios sc_ω, *i.e.,* for all $\omega \in \underline{\text{Sc}}$, the optimal solution to the problem considered – minimization of the maximal regret – can be calculated by solving the linear program in Boolean variables $\text{P}_{8.4}$.

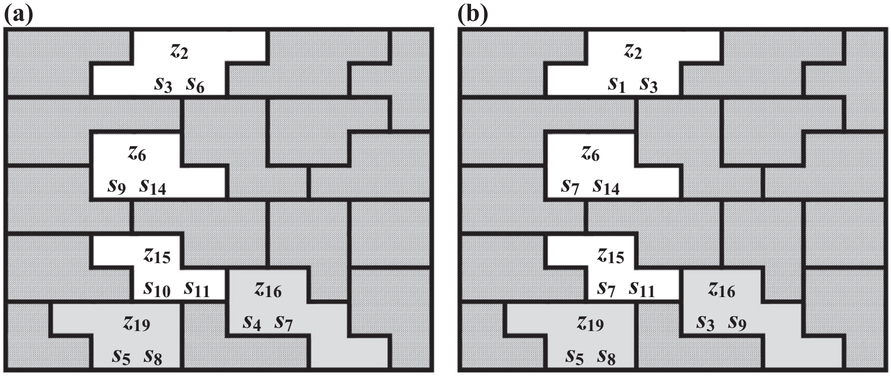

FIG. 8.5 – The budget available for the protection of the zones is equal to 7 units. (a) The optimal solution for scenario sc_1 of the instance described in figures 8.1 and 8.2 is to protect the 5 non-hatched zones z_2, z_6, z_{15}, z_{16}, and z_{19}, which costs 7 units and protects the 10 species s_3, s_4, s_5, s_6, s_7, s_8, s_9, s_{10}, s_{11}, and s_{14}. (b) If scenario sc_2 is realized, the protection of the 5 zones z_2, z_6, z_{15}, z_{16}, and z_{19} protects the 8 species s_1, s_3, s_5, s_7, s_8, s_9, s_{11}, and s_{14}.

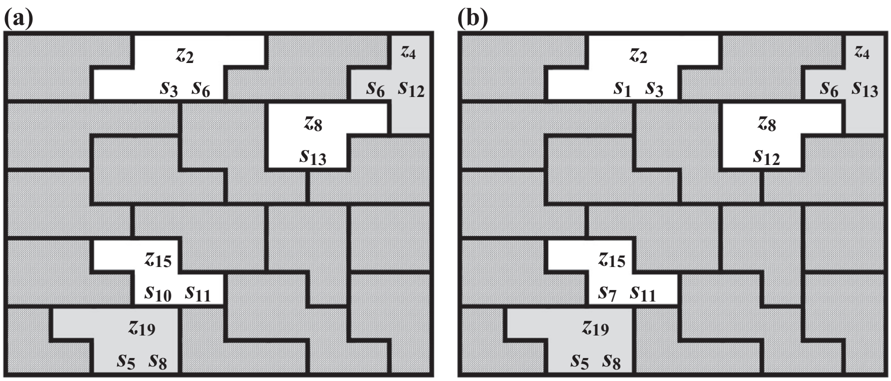

FIG. 8.6 – The budget available for the protection of the zones is equal to 7 units. (a) The optimal solution for scenario sc_2 of the instance described in figures 8.1 and 8.2 is to protect the 5 unhatched zones z_2, z_4, z_8, z_{15}, and z_{19}, which costs 7 units and protects the 9 species s_1, s_3, s_5, s_6, s_7, s_8, s_{11}, s_{12}, and s_{13}. (b) If scenario sc_1 occurs, the protection of the 5 zones z_2, z_4, z_8, z_{15}, and z_{19} protects the 8 species s_3, s_5, s_6, s_8, s_{10}, s_{11}, s_{12}, and s_{13}.

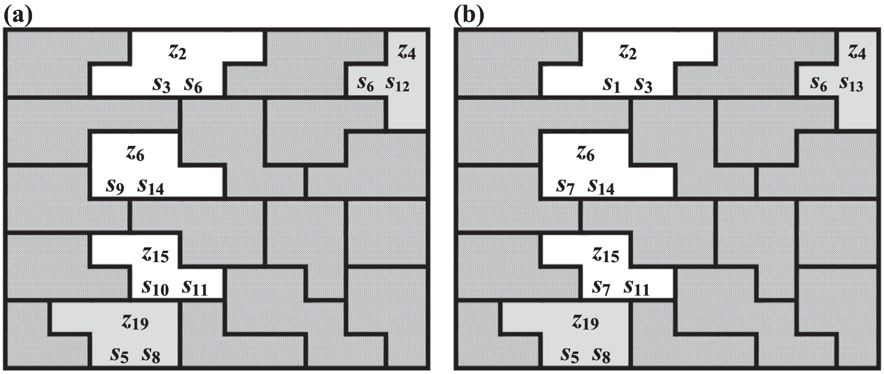

FIG. 8.7 – If the budget available for zone protection is 7 units, the optimal solution of $P_{8.4}$, for the example described in figures 8.1 and 8.2, is to protect zones z_2, z_4, z_6, z_{15}, and z_{19}. (a) The protection of these zones allows the protection of the 9 species s_3, s_5, s_6, s_8, s_9, s_{10}, s_{11}, s_{12}, and s_{14}, if scenario sc_1 is realized. (b) The protection of these zones allows the protection of the 9 species s_1, s_3, s_5, s_6, s_7, s_8, s_{11}, s_{13}, and s_{14} if scenario sc_2 is realized.

$$
P_{8.4}: \begin{cases} \min\ \alpha \\ \text{s.t.} \left| \begin{array}{lll} \alpha \geq \left(N_{\max}^{\omega} - \sum_{k\in \underline{S}} y_k^{\omega}\right) \Big/ N_{\max}^{\omega} & \omega \in \underline{\mathrm{Sc}} & (8.4.1) \\ y_k^{\omega} \leq \sum_{i\in \underline{Z}_k^{\omega}} x_i & k \in \underline{S}, \omega \in \underline{\mathrm{Sc}} & (8.4.2) \\ \sum_{i\in \underline{Z}} c_i x_i \leq B & & (8.4.3) \\ x_i \in \{0,1\} & i \in \underline{Z} & (8.4.4) \\ y_k^{\omega} \in \{0,1\} & k \in \underline{S}, \omega \in \underline{\mathrm{Sc}} & (8.4.5) \end{array} \right. \end{cases}
$$

Since we are seeking to minimize variable α and because of constraints 8.4.1, the Boolean variable y_k^{ω} takes, at the optimum of $P_{8.4}$, the highest possible value. Because of constraints 8.4.2, it therefore takes the value 1 if and only if the zones selected to be protected ensure the protection of species s_k, in the case of scenario sc_{ω}. In other words, y_k^{ω} takes the value 1 if and only if at least one of the zones of Z_k^{ω} is selected. Because of the economic function, α, to be minimized and constraints 8.4.1, variable α takes, at the optimum of $P_{8.4}$, the largest of the values $\left(N_{\max}^{\omega} - \sum_{k\in \underline{S}} y_k^{\omega}\right) \Big/ N_{\max}^{\omega}$, on all scenarios sc_{ω}. The resolution of $P_{8.4}$, therefore, enables the selection of zones whose protection minimizes the maximal relative gap, over all scenarios sc_{ω}, between 1) the number of species that are protected in scenario sc_{ω} given the selected zones – zone z_i is selected if $x_i = 1$ – and 2) the maximal number of species that could have been protected – possibly by protecting another set of zones – in scenario sc_{ω} (figure 8.7).

Example 8.5. Let us take again the instance built from figure 8.1 and described by figure 8.2, and suppose that the budget available for the protection of the zones is equal to 7 units. The reserve associated with the optimal solution of $P_{8.4}$ costs 7 units and allows 9 species to be protected, if scenario sc_1 is realized, and also 9 species, if scenario sc_2 is realized. For scenario sc_1 the value of the expression $\left(N^{\omega}_{\max} - \sum_{k \in \underline{S}} y^{\omega}_k\right) \Big/ N^{\omega}_{\max}$ is equal to $(10-9)/10 = 0.1$ and for scenario sc_2 it is equal to $(9-9)/9 = 0$. The corresponding value of the economic function of $P_{8.4}$, α, is therefore equal, for this example, to $\max\{0.1,\, 0\} = 0.1$. In other words, regardless of which scenario sc_ω occurs, the relative gap between the number of species that are protected by protecting the zones corresponding to the optimal solution of $P_{8.4}$ rather than the zones corresponding to the best strategy for scenario sc_ω is less than or equal to 10%.

8.4.2 Case Where the Interest of Protecting a Reserve, R, If Scenario sc_ω Occurs, is Assessed By $Nb^{\omega}_2(R)$

In this case, the problem of determining the maximal interest, $N^{\omega}_{\max}$, that can be obtained by protecting a set of zones, in the case of scenario sc_ω, can be formulated as the program obtained by replacing in $P_{8.3}(\omega)$ the constraints $y^{\omega}_k \leq \sum_{i \in \underline{Z}^{\omega}_k} x_i,\ k \in \underline{S}$, by the constraints $\theta^{\omega}_k\, y^{\omega}_k \leq \sum_{i \in \underline{Z}} n_{ik} x_i,\ k \in \underline{S}$. Once $N^{\omega}_{\max}$ is determined for all scenarios sc_ω, *i.e.,* for all $\omega \in \underline{\mathrm{Sc}}$, one can calculate the optimal solution to the problem under consideration by solving the program obtained by replacing in $P_{8.4}$ the constraints $y^{\omega}_k \leq \sum_{i \in \underline{Z}^{\omega}_k} x_i,\ k \in \underline{S}, \omega \in \underline{\mathrm{Sc}}$, by the constraints $\theta^{\omega}_k y^{\omega}_k \leq \sum_{i \in \underline{Z}} n_{ik} x_i,\ k \in \underline{S}, \omega \in \underline{\mathrm{Sc}}$.

References and Further Reading

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