Chapter 6

Biological Corridors

6.1 Introduction

As we have pointed out in previous chapters, landscape fragmentation is an important cause of biodiversity loss. This fragmentation is mainly due to urbanization, agriculture and forestry. It prevents species from moving as they should because they would have to cross often inhospitable zones. These zones may, for example, lack food resources or may host many predators. The viability of the species concerned by this fragmentation then depends strongly on how the fragments can be connected. This connectivity between habitat zones within a landscape has become an essential element for biodiversity conservation. One of the options commonly used to establish – or restore – this connectivity is the establishment of corridors. Thus, the "trame verte et bleue" is a key measure of the Grenelle Environnement (set of political meetings organised in France in 2007 concerning actions to be undertaken in favour of the environment and in particular biodiversity) aimed at halting the decline of biodiversity through the preservation and restoration of ecological continuities or biological corridors. In the biological conservation literature, corridors have multiple definitions – and functions. They are natural spaces, generally linear, *i.e.*, longer than wide, allowing species to move through a fragmented set of zones that are natural habitats for them. These are therefore routes used by species to move, reproduce, flee, migrate, etc. They are highly dependent on the species of interest. They do not necessarily imply the notion of contiguous spaces. In other words, some routes can be easily used by some species – and thus be considered as corridors for these species – but not by others. For example, it will be difficult for some species to overcome obstacles such as transport infrastructure or zones treated with pesticides, which will not be the case for species capable of flying. However, it should be noted that the latter may face hunting when they move from one protected site to another. Another example is lighting, which can be a real obstacle, but only for certain nocturnal species (black corridors). The fact that species can move between the different zones without too many difficulties is an

DOI: 10.1051/978-2-7598-2521-9.c006

essential element for their survival. Indeed, these corridors allow, for example, the increase in population sizes, the resettlement of certain species in certain zones, the maintenance of genetic diversity, the access to different habitats and the increase in places for food. Corridors can also be used as a refuge for species when their usual habitat zones are threatened. In addition, some authors have also highlighted the value of corridors in the context of climate change, since it will force many species to migrate in order to conserve favourable habitats. The creation – or restoration – of these corridors is, therefore, one of the major strategies for protecting species threatened by habitat fragmentation. These corridors must themselves be zones favourable to the life of the species concerned, to enable them to feed, rest and protect themselves from predators during their movements. They can be of very different form and nature. Some studies clearly distinguish between habitat and travel functions in the characteristics of a corridor. Corridors or fractions of corridors can exist naturally. This is the case, for example, for agricultural hedges, riversides or old railway lines. They can be implemented through the protection of certain zones of the landscape. They may also include completely artificial elements such as wildlife crossings built above or below transport infrastructure. To fulfil their functions, these corridors must be made up of zones that benefit from some protection. It should be noted that the fact that biodiversity reservoirs are linked by a network of corridors may have certain disadvantages. Indeed, this network facilitates the movement between the reservoirs and is also an entry point to these reservoirs. It can therefore facilitate the spread of diseases, parasites, invasive species and predators from one reservoir to another, but also facilitate their introduction into the reservoirs. In addition, these corridors, which are often very long, are more difficult to control than reservoirs, which are generally more compact zones. This control concerns the threats we have just mentioned, but also, for example, hunting, poaching, and tourism. Also because of the ease of movement provided by the corridors, wildlife species that are present in reservoirs can become pests in other habitats such as agricultural or livestock zones. These species can also transmit diseases to non-wild species such as livestock and vice versa. It should also be noted that efforts to maintain the effectiveness of a corridor network consume significant human and financial resources. These could possibly be better used to protect other habitat zones, for example zones where the ratio expressing the area of the zone, divided by the length of its edge, is more important (see chapter 4). Finally, if the corridors are not well designed, they can present a high risk of mortality for the species that use them and thus contribute to their extinction. This mortality risk can come from predators encountered during the use of these corridors or from accidents occurring in crossing dangerous zones such as roads. It should be noted that the movements of certain species in certain corridors can take several years and even several generations. The reader can consult the many references cited at the end of this chapter for an in-depth discussion on corridor design and evaluation, a careful examination of the balance between ecological benefits and economic costs associated with maintaining or implementing corridors, and a presentation of the software available to assist in the design of these corridors. In this chapter, we present two optimization problems that we believe are representative of the design of a new corridor network or the restoration of an existing corridor network.

6.2 Least Cost Design of Corridor Networks

6.2.1 The Problem

We are interested in a landscape with a set of well-identified biodiversity reservoirs $BR_1, BR_2, ..., BR_N$. These reservoirs are protected zones that provide habitat for a given set of species. To simplify the presentation, it is assumed here that any route that can be considered as a corridor for one of the species concerned can also be considered as a corridor for all the species concerned. Each reservoir is in one piece, meaning that all the species considered can fully traverse it without leaving it. In other words, these reservoirs can be considered as connected reserves (see chapter 3). Outside these reservoirs, the landscape has two types of zones to consider: zones already protected and providing habitat favourable to the species under consideration, and unprotected zones that can become protected zones and provide habitat favourable to the species under consideration.

Both types of zones can, therefore, contribute to the constitution of corridors. The second type corresponds either to completely new zones – from the protection point of view – or to old zones to be restored. A cost is associated with this second type of zones (figure 6.1). This cost can cover many aspects: monetary costs (rental or acquisition, possible restoration, management of the zones), ecological costs (travel facilities for species through the zone, mortality risk, distance travelled) and also social costs (negative or positive social impact generated by the selection of the zone to constitute a corridor). The consideration of monetary costs is obviously a key issue since financial resources are of course limited. In the following, we consider that the cost associated with the first type of zones is zero, but it would be very easy to consider a non-zero cost corresponding, for example, to the management of the zone.

The aim is to determine type 2 zones to be protected in order to connect, possibly using type 1 zones, all the biodiversity reservoirs. Two reservoirs are said to be connected if the species can move from one to the other only through either type 1 zones, type 2 zones that have been decided to be protected, or through one of the reservoirs. The selected type 2 zones, possibly with the addition of type 1 zones, form a network of corridors that link all the reservoirs. The problem we are studying here is to build this network of corridors at the lowest cost (figure 6.2). To simplify the presentation of the general problem of developing a network of corridors linking a set of biodiversity reservoirs, it is considered that the landscape is represented by a grid of nr $\times$ nc square and identical zones. Each zone of this landscape is denoted by z_{ij} where i denotes its row index and j, its column index. This landscape includes N biodiversity reservoirs, $BR_1, BR_2, ..., BR_N$, each reservoir being formed by a connected subset of zones. These reservoirs are disjoint. It should be noted that the method we are going to propose would easily adapt to any other set of zones and reservoirs. As mentioned above, some of the zones that do not belong to the reservoirs are already protected and can provide habitat favourable to the species under consideration, while others can be protected and possibly restored to also provide habitat favourable to the species under consideration. The cost of protecting

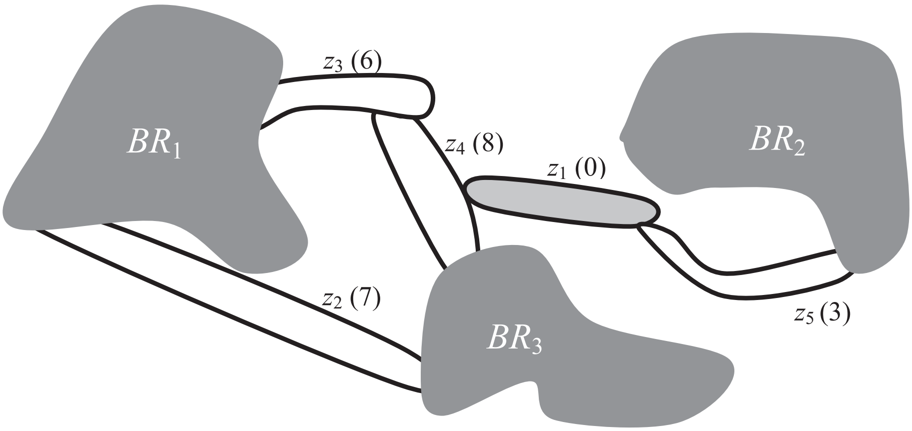

FIG. 6.1 – A hypothetical landscape with 3 biodiversity reservoirs, BR_1, BR_2, and BR_3, an already protected zone, z_1, which provides habitat favourable to the species under consideration, and 4 zones, z_2, z_3, z_4, and z_5, which can be protected and thus provide habitat favourable to the species under consideration, after possible restoration, with the associated costs indicated in brackets.

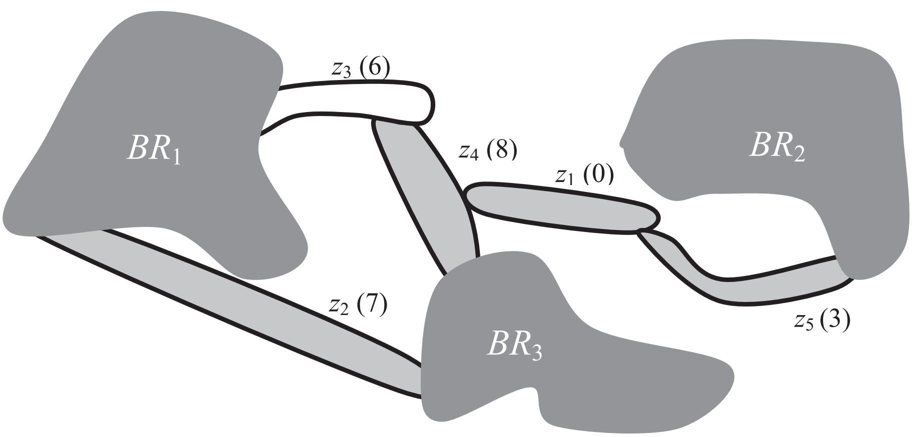

FIG. 6.2 – Zones z_1, z_2, z_4, and z_5 were selected. The corresponding cost is 18. The corridor connecting BR_1 to BR_2 consists of z_2, BR_3, z_4, z_1, and z_5, the corridor connecting BR_1 to BR_3 consists of the single zone z_2, and the corridor connecting BR_2 to BR_3 consists of z_5, z_1, and z_4.

zone z_{ij} is denoted by c_{ij}. In the case where z_{ij} is an already protected zone – not part of a reservoir – and provides habitat favourable to the species under consideration, this cost is zero. There are also zones in the considered landscape that, for different reasons, cannot be protected and, therefore, cannot contribute to the development of corridors (figure 6.3).

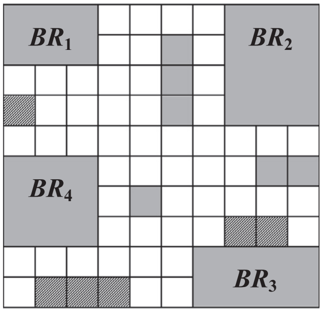

Fig. 6.3 – A hypothetical landscape represented by a grid of 10×10 square and identical zones. It includes 4 biodiversity reservoirs, BR_1, BR_2, BR_3, and BR_4, and 6 already protected zones that can contribute to the development of a corridor for the species under consideration, z_{26}, z_{36}, z_{46}, z_{69}, $z_{6,10}$, and z_{75}. The cost of protecting each zone – not yet protected – is equal to one unit; the cost associated with already protected zones is equal to 0; and finally, zones z_{41}, z_{88}, z_{89}, $z_{10,2}$, $z_{10,3}$, and $z_{10,4}$ cannot contribute to the development of a corridor.

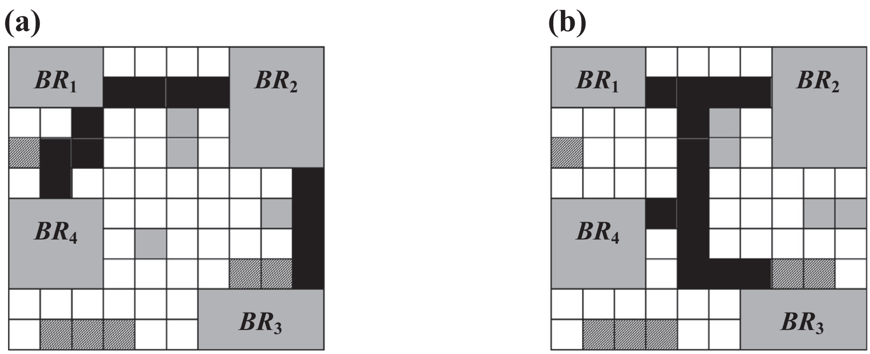

Fig. 6.4 – (a) The 12 black zones form a network of corridors linking the 4 biodiversity reservoirs BR_1, BR_2, BR_3, and BR_4. The associated cost is equal to 10 since, among these 12 zones, 2 were already protected. The length of the corridor connecting BR_3 and BR_4 is equal to 18. (b) The 13 black zones form a network of corridors linking the 4 reservoirs. The associated cost is 11 since, among these 13 zones, 2 were already protected. The length of the corridor connecting BR_3 and BR_4 is equal to 6.

The problem is to determine the zones to be protected, and possibly restored, in order to connect all the reservoirs at the lowest cost. Two reservoirs BR_i and BR_j are considered to be connected if it is possible, for the species under consideration, to move from BR_i to BR_j only through protected zones or zones belonging to a reservoir, and gradually moving from one zone to an adjacent one. Two zones are considered as adjacent if they share a common side. Figure 6.4 shows two different corridor networks to connect the 4 biodiversity reservoirs in figure 6.3.

The problem of connecting biodiversity reservoirs through a network of corridors has similarities to the problem of designing a connected reserve discussed in chapter 3. In both cases, the ultimate goal is to obtain a connected set of zones, *i.e.*, a set of zones in which species can move without leaving it. In chapter 3, the set of zones to be protected is determined in such a way as to ensure the best possible survival of certain species, taking into account protection costs. In this chapter, the set of zones to be protected is chosen in such a way as to link a set of already protected zones, at the lowest cost, and possibly taking into account certain constraints. Expressed in terms of graphs, both problems consist in determining, in a given graph, a subset of vertices inducing a connected sub-graph that checks certain constraints and takes into account certain costs.

6.2.2 Graph Optimization Formulation

Let us now look at how to state the problem as a graph optimization problem (see appendix at the end of the book). Let us associate to the grid of the nr × nc zones a graph, $G = (\underline{Z}, U)$, where the set of vertices, $\underline{Z}$, corresponds to the pairs of indices associated with a zone and where $((i, j), (k, l))$ is an arc of U if and only if zones z_{ij} and z_{kl} are adjacent – share a common side. For each biodiversity reservoir BR_k, let us choose one of its zones to represent it and denote by $z_{i(k),j(k)}$ this zone. The problem can be formulated as follows: determine a partial sub-graph of $G = (\underline{Z}, U),\ G' = (\hat{\underline{Z}}, A)$, checking the following properties: all the vertices associated with a zone representing a reservoir belong to $\hat{\underline{Z}}$ and, for all $r \in \{1,..., N-1\}$, there is in this graph a path from the vertex associated with the zone representing reservoir BR_r to the vertex associated with the zone representing reservoir BR_N. This problem is similar to the Steiner tree problem which, in a general way, can be expressed as follows: given a graph whose edges are assigned with a weight, and a subset S of vertices of this graph, find a subset of edges of minimal weight that induces a connected sub-graph containing all the vertices of S (see appendix at the end of the book).

6.2.3 Mathematical Programming Formulation

We give below a flow type formulation of this problem (see appendix at the end of the book). Let ϕ_{ijkl} be the Boolean variable which is equal to 1 if and only if at least one of the N−1 paths, from the vertex representing reservoir BR_r, $r = 1,\ldots, N-1$, to the vertex representing reservoir BR_N, follows the arc $((i, j), (k, l))$ and let μ^r_{ijkl} be the Boolean variable which is equal to 1 if and only if, among these paths, the one from BR_r to BR_N follows the arc $((i, j), (k, l))$. Denote by Adj_{ij} the set of index pairs associated with the zones adjacent to zone z_{ij} and define constant d^r_{ij}, $(i, j) \in \underline{Z}$, $r = 1,..., N-1$, as follows: $d^r_{ij} = 1$ if zone z_{ij} represents reservoir BR_r, $d^r_{ij} = -1$ if zone z_{ij} represents reservoir BR_N, and $d^r_{ij} = 0$ in all the other cases. The problem can then be formulated as the 0–1 linear program $\mathrm{P}_{6.1}$.

$$\mathrm{P}_{6.1}:\left\{\begin{array}{lll}\min \sum\limits_{(i,j)\,\in\,\underline{Z}} c_{ij} \sum\limits_{(k,l)\,\in\,\mathrm{Adj}_{ij}} \phi_{ijkl} & & \\ \mathrm{s.t.}\left|\begin{array}{l}\phi_{ijkl} \geq \mu^r_{ijkl} \\ \sum\limits_{(k,l)\in\,\mathrm{Adj}_{ij}} \mu^r_{ijkl} - \sum\limits_{(k,l)\in\,\mathrm{Adj}_{ij}} \mu^r_{klij} = d^r_{ij} \\ \phi_{ijkl} \in \{0,1\} \\ \mu^r_{ijkl} \in \{0,1\}\end{array}\right. & \begin{array}{l}((i,j),(k,l)) \in U, r = 1,\ldots,N-1 \\ (i,j) \in \underline{Z}, r = 1,\ldots,N-1 \\ ((i,j),(k,l)) \in U \\ ((i,j),(k,l)) \in U, r = 1,\ldots,N-1\end{array} & \begin{array}{l}(6.1.1) \\ (6.1.2) \\ (6.1.3) \\ (6.1.4)\end{array}\end{array}\right.$$

In this formulation $c_{ij} = 0$ if zone z_{ij} is already protected or belongs to a reservoir. We give below some indications to justify the formulation $\mathrm{P}_{6.1}$. First of all, it can be shown that the optimal solution to the problem has a tree structure. More precisely, the selected arcs, *i.e.*, the arcs $((i, j), (k, l))$ such that $\phi_{ijkl} = 1$ and the corresponding vertices satisfy the following property: any vertex (i, j) selected and different from the vertex $(i(N), j(N))$ is the initial end of one and only one selected arc. Thus $\sum_{(k,l)\,\in\,\mathrm{Adj}_{ij}} \phi_{ijkl} = 1$ for any vertex (i, j) selected and different from $(i(N), j(N))$, and $\sum_{(k,l)\,\in\,\mathrm{Adj}_{ij}} \phi_{ijkl} = 0$ for any vertex (i, j) not selected or when $(i, j) = (i(N), j(N))$. We deduce from this that the vertex (i, j), different from $(i(N), j(N))$, is selected if and only if $\sum_{(k,l)\,\in\,\mathrm{Adj}_{ij}} \phi_{ijkl} = 1$. It should also be noted that, for all $r \in 1,\ldots, N-1$, any vertex (i, j) is the initial end of at most one arc of the path from the vertex representing reservoir BR_r to the vertex representing reservoir BR_N, and also the terminal end of at most one arc of the same path. Thus, for any vertex (i, j), $\sum_{(k,l)\in\,\mathrm{Adj}_{ij}} \mu^r_{ijkl} \leq 1$ and $\sum_{(k,l)\in Adj_{ij}} \mu^r_{klij} \leq 1$. Constraints 6.1.1 force variable ϕ_{ijkl} to take the value 1 if at least one of variables μ^r_{ijkl}, $r = 1,\ldots, N-1$, takes the value 1. Consider constraints 6.1.2 for the 3 types of zones. If zone z_{ij} represents reservoir BR_N – $d^r_{ij} = -1$ for all $r \in 1,\ldots, N-1$ – these constraints express, taking into account the above remarks, that for all $r \in 1,\ldots, N-1$, $\sum_{(k,l)\in Adj_{ij}} \mu^r_{klij} = 1$ and $\sum_{(k,l)\in\,\mathrm{Adj}_{ij}} \mu^r_{ijkl} = 0$. If zone z_{ij} represents reservoir BR_r, – $d^r_{ij} = 1$ – these constraints express that for all $(i,j) \in \underline{Z}$ and for all $r \in 1,\ldots, N-1$ such that z_{ij} represents reservoir BR_r, $\sum_{(k,l)\in A_{ij}} \mu^r_{klij} = 0$ and $\sum_{(k,l)\in A_{ij}} \mu^r_{ijkl} = 1$. Finally, if zone z_{ij} does not represent any of reservoirs – $d^r_{ij} = 0$ for all $r \in 1,\ldots, N-1$ – these constraints express that, for all $(i,j) \in \underline{Z}$ and for all $r \in 1,\ldots, N-1$, if (i, j) is the terminal end of an arc of the path from the vertex representing reservoir BR_r to the vertex representing reservoir BR_N, then (i, j) is also the initial end of an arc of the same path.

This type of formulation has been used to define a network of corridors suitable for grizzly bear movement in the northern Rocky Mountains of the United States. A disadvantage of this formulation is that the lengths of the corridors connecting two reservoirs cannot be controlled in the searched solution except for the pairs of reservoirs (BR_r, BR_N), $r = 1,\ldots, N-1$. In this case, it is sufficient to add the constraint $\sum_{(i,j,k,l)\in I_{r,N}} \mu^r_{ijkl} \leq L^{rN}_{\max}$, where $I_{r,N} = \{((i,j),(k,l)) \in U, z_{ij} \notin BR_r \cup BR_N\}$ and $L^{rN}_{\max}$ indicates the maximal authorised length for the corridor connecting BR_r to BR_N. In other words, $I_{r,N}$ refers to the set of arcs for which the zone associated with their initial end does not belong to either reservoir BR_r or reservoir BR_N.

We propose below a slightly different formulation of the corridor design problem that does not have this disadvantage. We keep variables μ^r_{ijkl} with their same meaning and replace variables ϕ_{ijkl} by variables x_{ij} that are equal to 1 if and only if at least one of the N−1 paths from, BR_r, $r = 1,\ldots, N-1$, to BR_N passes through the vertex (i, j). The result is program $P_{6.2}$, which has fewer variables and fewer constraints than $P_{6.1}$, and allows a limit to be imposed on the length of the corridor connecting any two reservoirs.

$$
P_{6.2}: \begin{cases} \min \sum_{(i,j)\in \underline{Z}} c_{ij}x_{ij} \\ \text{s.t.} \left| \begin{array}{lll} x_{ij} \geq \sum_{(k,l)\in \mathrm{Adj}_{ij}} \mu^r_{ijkl} & (i,j)\in \underline{Z}, r=1,\ldots,N-1 & (6.2.1) \\ \sum_{(k,l)\in \mathrm{Adj}_{ij}} \mu^r_{ijkl} - \sum_{(k,l)\in \mathrm{Adj}_{ij}} \mu^r_{klij} = d^r_{ij} & (i,j)\in \underline{Z}, r=1,\ldots,N-1 & (6.2.2) \\ x_{ij}\in\{0,1\} & (i,j)\in \underline{Z} & (6.2.3) \\ \mu^r_{ijkl}\in\{0,1\} & ((i,j),(k,l))\in U, r=1,\ldots,N-1 & (6.2.4) \end{array} \right. \end{cases}
$$

Constraints 6.2.1 express that, if at least one of the N−1 paths from the vertex representing reservoir BR_r to the vertex representing reservoir BR_N passes through an arc of initial end (i, j), then zone z_{ij} is retained. Constraints 6.2.2 are identical to constraints 6.1.2.

As in the previous formulation, a constraint can be introduced limiting the length of the corridor connecting BR_r and BR_N. To limit to $L^{st}_{\max}$ the length of the corridor connecting any two reservoirs, BR_s and BR_t, a new Boolean variable ψ^{st}_{ijkl} is defined, which is equal to 1 if and only the path from the vertex representing BR_s to the vertex representing BR_t follows the arc $((i, j), (k, l))$ and the set of constraints $C_{6.1}$ is added where $\delta^{st}_{ij} = 1$ if z_{ij} represents BR_s, $\delta^{st}_{ij} = -1$ if z_{ij} represents BR_t, and $\delta^{st}_{ij} = 0$ in the other cases.

$$
C_{6.1}: \begin{cases} \sum_{((i,j),(k,l))\in U, z_{ij}\notin BR_s \cup BR_t} \psi^{st}_{ijkl} \leq L^{st}_{\max} & \\ x_{ij} \geq \sum_{(k,l)\in \mathrm{Adj}_{ij}} \psi^{st}_{ijkl} & (i,j)\in \underline{Z} \\ \sum_{(k,l)\in \mathrm{Adj}_{ij}} \psi^{st}_{ijkl} - \sum_{(k,l)\in \mathrm{Adj}_{ij}} \psi^{st}_{klij} = \delta^{st}_{ij} & (i,j)\in \underline{Z} \end{cases}
$$

6.2.4 Example

The hypothetical landscape studied is represented by a grid of 20 × 20 square and identical zones and includes 7 biodiversity reservoirs, BR_1, BR_2,..., BR_7. Among the zones that do not belong to the reservoirs, some are already protected and provide habitat favourable to the species under consideration, others can be protected and possibly restored to also provide habitat favourable to the species under

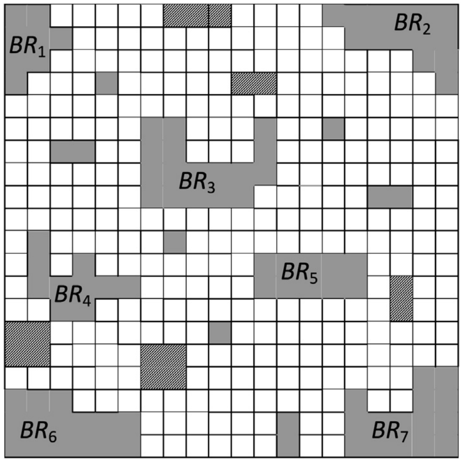

FIG. 6.5 – A hypothetical landscape represented by a grid of 20×20 square and identical zones. It includes 7 biodiversity reservoirs, BR_1, BR_2, BR_3 BR_4, BR_5, BR_6, and BR_7, and 11 zones already protected and providing habitat favourable to the species concerned, z_{45}, $z_{6,15}$, z_{73}, z_{74}, $z_{9,17}$, $z_{9,18}$, $z_{11,8}$, $z_{15,10}$, $z_{19,13}$, and $z_{20,13}$. The cost associated with the not already protected zones is equal to one unit and the cost associated with the already protected zones is equal to 0. Finally, zones z_{18}, z_{19}, $z_{1,10}$, $z_{4,11}$, $z_{4,12}$, $z_{13,18}$, $z_{14,18}$, $z_{15,1}$, $z_{15,2}$, $z_{16,1}$, $z_{16,2}$, $z_{16,7}$, $z_{16,8}$, $z_{17,7}$, and $z_{17,8}$ cannot be protected.

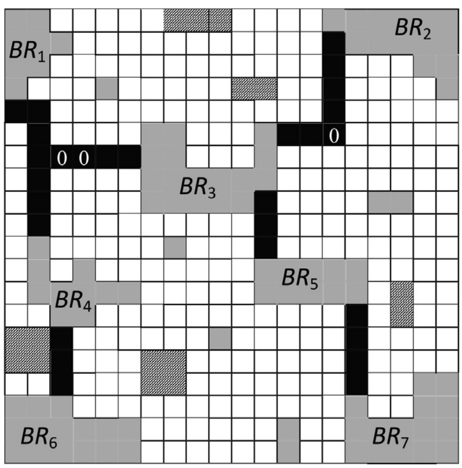

FIG. 6.6 – Optimal solution associated with the instance of figure 6.5. The corridors are shown in black. The protection and possible restoration of a total of 28 zones allows all the reservoirs to be connected. The cost associated with each black zone is equal to 1 except for the zones that were already protected. For these zones, the cost is 0 and is shown in the figure. The total cost of the corridor network is equal to 25 units. The length of the corridor connecting BR_4 and BR_5 is equal to 18.

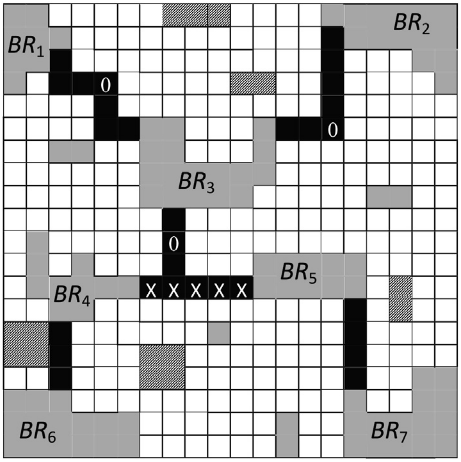

FIG. 6.7 – Optimal solution associated with the instance described in figure 6.5 when the length of the corridor connecting reservoirs BR_4 and BR_5 is required to be less than or equal to 9. The cost of this solution is equal to 26 units. The zones constituting the corridor linking BR_4 and BR_5 are marked with a cross. The length of this corridor is equal to 5.

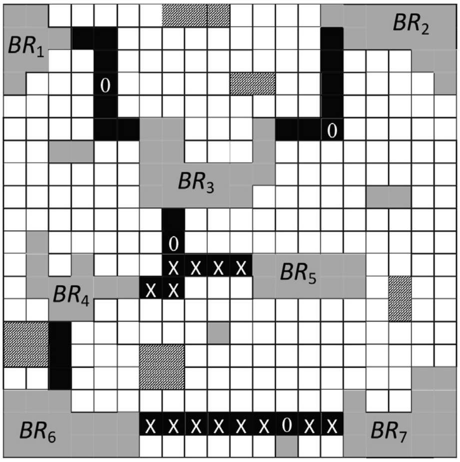

FIG. 6.8 – Optimal solution associated with the instance described in figure 6.5 when the length of the corridor connecting reservoirs BR_4 and BR_5 is required to be less than or equal to 9, and the length of the corridor connecting reservoirs BR_6 and BR_7 is required to be less than or equal to 11. The cost of this solution is equal to 30 units. The zones constituting the corridors connecting BR_4 to BR_5 and BR_6 to BR_7 are marked with a cross. The corresponding lengths are 6 and 9, respectively. On the other hand, the length of the corridor connecting reservoirs BR_5 and BR_7 increases significantly, compared to the previous solution, from 4 to 28.

consideration. In addition, some zones cannot be protected and will, therefore, not be able to contribute to the constitution of corridors (figure 6.5).

Figure 6.6 shows the low-cost corridor network linking the 7 reservoirs. Figure 6.7 shows the least-cost network when the length of the corridor connecting BR_4 and BR_5 is limited to 9 and figure 6.8 presents the least-cost network when, in addition, the length of the corridor connecting BR_6 and BR_7 is limited to 11.

6.3 Optimizing the Permeability of an Existing Corridor Network Under a Budgetary Constraint

6.3.1 The Problem

This problem consists in improving and/or restoring an existing network of corridors with the best cost-effectiveness ratio. In other words, there is a certain budget available to carry out developments to improve the permeability of the network and the aim is to carry out these developments in such a way as to increase this permeability as much as possible, while respecting the financial constraint. Some authors have considered this type of problem but have proposed – approximate – resolutions based on simulation methods. We present here an – exact – resolution based on mixed-integer linear programming. We consider a network of corridors and a set of species all having the same behaviour in this network. The network is represented by a graph, $G = (\underline{BR}, C)$, where $\underline{BR}$ is the set of indices associated with the set of the N biodiversity reservoirs, BR_1, BR_2,..., BR_N, corresponding to habitats favourable to the species under consideration, and where C is the set of arcs. For any couple, $(i, j) \in \{1,..., N\}^2$, $i \neq j$, (i, j) is an arc of the graph if there is a corridor between BR_i and BR_j. Note that $G = (\underline{BR}, C)$ is a symmetric graph. For various reasons – road and rail infrastructure, urbanization, agriculture, etc. – the condition of these corridors is more or less deteriorated. The problem is to restore this network of corridors as efficiently as possible, *i.e.*, to optimize its permeability, under a budgetary constraint. This permeability is measured by the mathematical expectation of the distance travelled in the network by the species under consideration. It is assumed that when an animal is in reservoir BR_i, it randomly and equiprobably chooses one of the corridors leading to this reservoir – and thus also leaving this reservoir. It thus chooses the corridor $[BR_i, BR_j]$ with the probability $1/\mathrm{d}_i$ where d_i indicates the degree of the vertex associated with reservoir BR_i, and then tries, eventually, to use this corridor. A certain probability is associated with this possibility. If it decides to use the corridor, it is assumed that it succeeds in reaching the other end, *i.e.*, reservoir BR_j, also with a certain probability and that it does not succeed in reaching it, being killed beforehand, with the complementary probability. Restoring a corridor $[BR_i, BR_j]$ increases the last two probabilities – trying to follow the corridor and succeeding in its course. The more resources are devoted to restoration, the higher the values of these probabilities. For a given corridor, several levels of investment are possible. The set of these levels, for the corridor connecting reservoirs BR_i and BR_j, is designated by $H_{ij} = \{0, 1,..., h_{ij}\}$ with $h_{ij} = h_{ji}$. It is assumed that for each corridor, the possible values of the different probabilities mentioned above and the associated costs are known. The level 0 investment consists in doing nothing – the corridor remains in its current state – and costs 0. Denote by $r^1_{ijh}, (i,j) \in C, h \in H_{ij}$, the probability for an animal, located in BR_i, to try to use the corridor $[BR_i, BR_j]$ if level h investment is made in this corridor and $r^2_{ijh}, (i,j) \in C, h \in H_{ij}$, the probability, having chosen to use the corridor, to reach BR_j. These probabilities are not necessarily symmetric. Thus, probability r^1_{ijh} may be different from probability r^1_{jih} and probability r^2_{ijh} may be different from probability r^2_{jih}.

6.3.2 Associated Markov Chain

With the corridor network is associated a Markov chain (see appendix at the end of the book) whose set of states is made up of N transient states corresponding to the N reservoirs and a $(N + 1)$th, absorbing, state corresponding to the death of the animal. These states are denoted by 1, 2,…, N, $N + 1$. We denote by pr_{ij}, $i = 1,..., N+1, j = 1,..., N+1$, the transition probability from state i to state j. The probability $\text{pr}_{N+1,N+1}$ is equal to 1 and, for all $j \in \{1,..., N\}$, the probability $\text{pr}_{N+1,j}$ is equal to 0. The probability pr_{ij}, $i = 1,..., N, j = 1,..., N$, corresponds to the probability that an animal present in reservoir BR_i at time t is present in reservoir BR_j at time $t + 1$. Note that the probability pr_{ii}, $i = 1,…, N$, is to be considered. It corresponds to the fact that an animal, present in reservoir BR_i at time t, can give up using one of the corridors leaving BR_i and thus be again in BR_i at time $t + 1$. The probability $\text{pr}_{i,N+1}$, $i = 1,..., N$, corresponds to the probability that an animal in reservoir BR_i at time t is dead at time $t + 1$. It is assumed that at the initial moment there is an animal in each of the N transient states, *i.e.*, in each of the reservoirs. The duration of a transition will depend on the context of the study and in particular on the type of corridor networks and the type of species considered.

Let us consider the transition probability matrix, $\Pi = \begin{pmatrix} Z & D \\ 0 & 1 \end{pmatrix}$, Z corresponding to the transition probabilities between transient states and D, to the transition probabilities from transient states to the absorbing state. Let us denote by $\mathcal{N}$ the $N \times N$ – matrix whose general term, n_{ij}, represents the expected number of passages through transient state j for an animal starting from transient state i, before being absorbed, *i.e.*, before being in the state $N + 1$. According to Markov's chain theory, $\mathcal{N} = (I - Z)^{-1}$ where I denotes the $N \times N$ identity matrix. Let $w_i = \sum_{j=1}^N n_{ji}$, $i = 1,..., N$. The quantity w_i thus represents the expected total number of passages through state i, before being absorbed. We deduce that the expected total number of routes in the corridor $[BR_i, BR_j]$, from BR_i to BR_j, is equal to $w_i\text{pr}_{ij}$. We can show that the only solution of the system of equations $w_i - \sum_{j=1}^N w_j\text{pr}_{ji} = 1$, $i = 1,..., N$, in which the quantities w_i, $i = 1,…, N$, are the unknowns, checks $w_i = \sum_{j=1}^N n_{ji}$.

6.3.3 Mathematical Programming Formulation

The problem of choosing the investments to be made in each corridor in order to maximize the expected value of the total distance travelled by N animals, one animal being initially located in each of the N reservoirs, can therefore be formulated as the mathematical program $\mathrm{P}_{6.3}$.

$$
\mathrm{P}_{6.3}: \begin{cases} \max \displaystyle\sum_{(i,j)\in C,\, i<j} l_{ij}(w_i \mathrm{pr}_{ij} + w_j \mathrm{pr}_{ji}) \\ \text{s.t.} \left| \begin{array}{lll} w_i - \displaystyle\sum_{j=1}^{N} w_j \mathrm{pr}_{ji} = 1 & i = 1,\dots, N & (6.3.1) \\ \Pi \in \tilde{\Pi} & & (6.3.2) \end{array} \right. \end{cases}
$$

where $C = \{(i,j) : [BR_i, BR_j] \text{ is a corridor}\}$, l_{ij} is the length of the corridor $[BR_i, BR_j]$, w_i is a real variable that represents the expression $\sum_{j=1}^{N} n_{ji}$ and $\tilde{\Pi}$ is a set of stochastic matrices, of dimension $(N+1)\times(N+1)$, of general term pr_{ij} and admissible for the problem. It should be recalled that the set of possible investment levels in the corridor $[BR_i, BR_j]$ is $H_{ij} = \{1, 2,\dots, h_{ij}\}$ with $h_{ij} = h_{ji}$. Let x_{ijh}, $(i,j)\in C$, $h \in H_{ij}$, be the Boolean variable which is equal to 1 if and only if the level h investment is made in the corridor $[BR_i, BR_j]$ and c_{ijh}, be the cost of this investment. This cost is defined for $i < j$. Remember that for all $(i,j)\in C, i<j$, $c_{ij0} = 0$. We put $x_{ijh} = x_{jih}$. Let en_{ij}, $(i,j)\in C$, be the positive or zero variable that represents the expected total number of routes in the corridor $[BR_i, BR_j]$, from BR_i to BR_j, *i.e.*, the quantity $w_i \mathrm{pr}_{ij}$. The problem considered can then be formulated as program $\mathrm{P}_{6.4}$.

$$
\mathrm{P}_{6.4}: \begin{cases} \max \displaystyle\sum_{(i,j)\in C, i<j} l_{ij}(\mathrm{en}_{ij} + \mathrm{en}_{ji}) \\ \text{s.t.} \left| \begin{array}{lll} \displaystyle\sum_{(i,j)\in C, i<j, h\in H_{ij}} c_{ijh} x_{ijh} \le B & & (6.4.1) \\ \displaystyle\sum_{h\in H_{ij}} x_{ijh} = 1 & (i,j)\in C, i<j & (6.4.2) \\ \mathrm{en}_{ij} = \frac{w_i}{d_i} \displaystyle\sum_{h\in H_{ij}} r^1_{ijh}\, r^2_{ijh}\, x_{ijh} & (i,j)\in C & (6.4.3) \\ \frac{w_i}{d_i} \displaystyle\sum_{j:(i,j)\in C, h\in H_{ij}} r^1_{ijh}\, x_{ijh} & & \\ \quad = 1 + \displaystyle\sum_{j:\,(i,j)\in C} \mathrm{en}_{ji} & i = 1,\dots, N & (6.4.4) \\ x_{ijh} = x_{jih} & (i,j)\in C, i<j, h\in H_{ij} & (6.4.5) \\ x_{ijh} \in \{0,1\} & (i,j)\in C, h\in H_{ij} & (6.4.6) \\ w_i \ge 0 & i = 1,\dots, N & (6.4.7) \\ \mathrm{en}_{ij} \ge 0 & (i,j)\in C & (6.4.8) \end{array} \right. \end{cases}
$$

The economic function of $\mathrm{P}_{6.4}$ expresses the expected total distance travelled in the corridors. Constraint 6.4.1 expresses the financial constraint. Constraints 6.4.2

and 6.4.5 express that, for each corridor, only one level of investment must be selected. Constraints 6.4.3 express the expected number of routes in the corridor $[BR_i,\ BR_j]$, from BR_i to BR_j. Constraints 6.4.4 reflect constraints 6.3.1. Indeed, these last constraints can be written as $w_i = 1 + \sum_{j=1,\dots,N, j\neq i} w_j \mathrm{pr}_{ji} + w_i \mathrm{pr}_{ii}$ or alternatively $w_i(1-\mathrm{pr}_{ii}) = 1 + \sum_{j=1,\dots,N,j\neq i} \mathrm{en}_{ji}$. Let us express probability pr_{ii}, *i.e.*, the probability, for an animal present at time t in reservoir BR_i, of being present again in this reservoir at time $t+1$. This occurs when the animal chooses any corridor leaving from BR_i and renounces trying to travel that corridor. Remember that an animal present in BR_i chooses the corridor $[BR_i,\ BR_j]$ with probability $1/d_i$. Moreover, when it has chosen the corridor $[BR_i,\ BR_j]$, it tries to use it with probability r^1_{ijh} if level h investment has been made in this corridor. So we have $\mathrm{pr}_{ii} = \sum_{j:(i,j)\in C} (1 - \sum_{h\in H_{ij}} r^1_{ijh} x_{ijh})/d_i$ and constraints 6.3.1 can therefore be written $w_i(1 - \sum_{j:\,(i,j)\in C} (1 - \sum_{h\in H_{ij}} r^1_{ijh} x_{ijh})/d_i) = 1 + \sum_{j=1,\dots,N,j\neq i} \mathrm{en}_{ji}$ or alternatively $\frac{w_i}{d_i} \sum\limits_{j:(i,j)\in C, h\in H_{ij}} r^1_{ijh}\, x_{ijh} = 1 + \sum_{j=1,\dots,N,j\neq i} \mathrm{en}_{ji}$. $\mathrm{P}_{6.4}$ can be transformed into a mixed-integer linear program by linearizing the quadratic expressions $w_i x_{ijh}$, which are products of the real, non-negative variable w_i by the Boolean variable x_{ijh} (see appendix at the end of this book). To do this, we replace each product $w_i x_{ijh}$ with variable v_{ijh} and add the set of linear constraints $\mathrm{C}_{6.2}$ below to force v_{ijh} to be equal to $w_i x_{ijh}$, $(i,j) \in C, h \in H_{ij}$.

$$\mathrm{C}_{6.2} : \begin{cases} v_{ijh} \leq \mathrm{UB}_i\, x_{ijh} & (i,j)\in C, h\in H_{ij} \\ \sum\limits_{h\in H_{ij}} v_{ijh} = w_i & (i,j)\in C \\ v_{ijh} \geq 0 & (i,j)\in C, h\in H_{ij} \end{cases}$$

UB_i is a constant greater than or equal to the optimal value of w_i in program $\mathrm{P}_{6.4}$. By examining successively the two possible values of x_{ijh}, while taking into account constraints 6.4.2 and 6.4.5, we see that $v_{ijh} = w_i x_{ijh}$ if and only if the constraints of $\mathrm{C}_{6.2}$ are satisfied. Finally, the problem can be solved by program $\mathrm{P}_{6.5}$.

$$\mathrm{P}_{6.5} : \begin{cases} \max \sum\limits_{(i,j)\in C,\, i<j} l_{ij}\big(\mathrm{en}_{ij} + \mathrm{en}_{ji}\big) \\ \text{s.t.} \left| \begin{array}{lll} (6.4.1), (6.4.2), (6.4.5), (6.4.6), (6.4.7), (6.4.8) & & \\ \mathrm{en}_{ij} = \frac{1}{d_i} \sum\limits_{h\in H_{ij}} r^1_{ijh}\, r^2_{ijh}\, v_{ijh} & (i,j)\in C & (6.5.1) \\ \frac{1}{d_i} \sum\limits_{\substack{j:(i,j)\in C \\ h\in H_{ij}}} r^1_{ijh}\, v_{ijh} = 1 + \sum\limits_{j:(i,j)\in C} \mathrm{en}_{ji} & i = 1,\dots,N & (6.5.2) \\ v_{ijh} \leq \mathrm{UB}_i\, x_{ijh} & (i,j)\in C, h\in H_{ij} & (6.5.3) \\ \sum\limits_{h\in H_{ij}} v_{ijh} = w_i & (i,j)\in C & (6.5.4) \\ v_{ijh} \geq 0 & (i,j)\in C, h\in H_{ij} & (6.5.5) \end{array} \right. \end{cases}$$

6.3.4 Example 1

Consider the example described in figure 6.9 and table 6.1.

In this example, we assume that there are 4 possible types of restoration for each corridor to reduce the barrier effect – reflected by the probabilities r^1_{ijh} and r^1_{jih} – and mortality risk – reflected by the probabilities r^2_{ijh} and r^2_{jih}. Table 6.1 gives, for each corridor $[BR_i, BR_j]$, its length, l_{ij}, the probabilities r^1_{ijh}, r^1_{jih}, r^2_{ijh}, and r^2_{jih}, for $h = 0$, 1, …, 4, and the associated costs, c_{ijh}, for $h = 0$, 1,…, 4. By definition, c_{ij0} is equal to 0. In this example, the effects of the possible restorations for each corridor are not symmetric, neither with regard to the barrier effect since r^1_{ijh} may be different from r^1_{jih}, nor with regard to the mortality risk since r^2_{ijh} may be different from r^2_{jih}. Note that, in this example, r^1_{ijh}, r^1_{jih}, r^2_{ijh}, r^2_{jih}, and c_{ijh} are increasing as a function of h.

The computational experiments were conducted with different values of the available budget, B. The results are presented in table 6.2. Remember that, in this example, the effects of corridor restoration are not symmetric, neither in terms of barrier effect nor in terms of mortality risk. In order to obtain, among the equivalent solutions of $P_{6.5}$, a minimal cost solution, we subtract from the objective function the quantity $\varepsilon \sum_{(i,j)\in C, i<j, h\in H_{ij}} c_{ijh}x_{ijh}$, where ε is a sufficiently small constant.

If no restoration is carried out in the corridors, the expected total distance travelled is 60 km, and if the best possible restoration is carried out in view of the pursued objective – which requires a budget of 133 units – this expected distance becomes equal to 213 km. The results in table 6.2 show that, in some cases, it is not

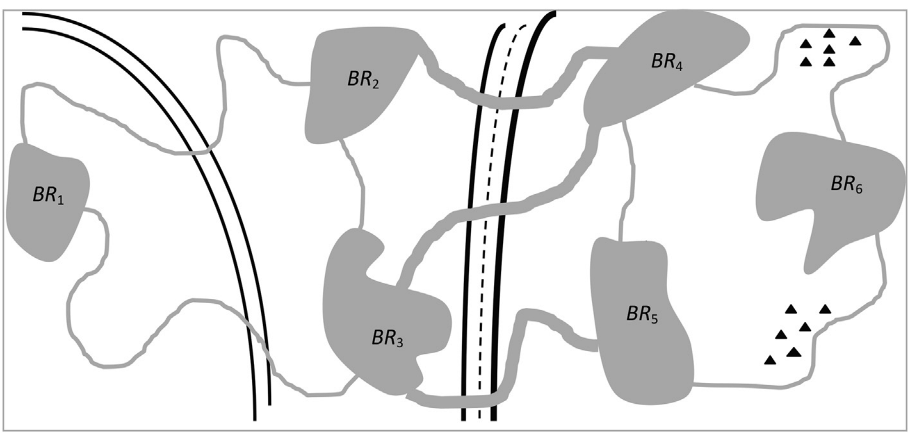

FIG. 6.9 – A hypothetical network of corridors associated with 6 biodiversity reservoirs. The corridors $[BR_1, BR_2]$, $[BR_1, BR_3]$, $[BR_4, BR_6]$, and $[BR_5, BR_6]$ are long and narrow. Dwellings are located near the corridors $[BR_4, BR_6]$ and $[BR_5, BR_6]$. The corridors $[BR_2, BR_3]$ and $[BR_4, BR_5]$ are short and narrow. The corridors $[BR_2, BR_4]$, $[BR_3, BR_4]$, and $[BR_3, BR_5]$ are relatively short and wide. These last 3 corridors are crossed by a main road and the corridors $[BR_1, BR_2]$ and $[BR_1, BR_3]$ are crossed by a small road.

TAB. 6.1 – Each cell in columns 2 to 6 shows, for a corridor $[BR_i, BR_j]$ of the network in figure 6.9 and for a given value of the investment level $h \in \{0, 1, 2, 3, 4\}$, the probabilities r^1_{ijh}, r^1_{jih}, r^2_{ijh}, r^2_{jih}, and the associated costs, c_{ijh}, in this order. For example, in the cell located at the intersection of row [3, 5] – associated with the corridor $[BR_3, BR_5]$ – and column $h = 3$, $r^1_{353} = 0.7$, $r^1_{533} = 0.7$, $r^2_{353} = 0.9$, $r^2_{533} = 0.8$, and $c_{353} = 12$. The last column of the table shows the length of each corridor, l_{ij}, in kilometres. The total cost associated with the maximal investments that can be made in each corridor is equal to 150.

$[i, j]$	$h = 0$	$h = 1$	$h = 2$	$h = 3$	$h = 4$	l_{ij}
[1, 2]	0.2/0.2/0.7/0.7/0	0.5/0.6/0.8/0.8/3	0.7/0.8/0.9/0.9/8	0.8/0.8/0.9/0.9/11	0.9/0.9/0.8/0.9/15	10
[1, 3]	0.2/0.2/0.6/0.6/0	0.3/0.4/0.7/0.7/4	0.6/0.5/0.7/0.7/10	0.7/0.7/0.7/0.8/13	0.9/0.8/0.8/0.9/17	10
[2, 3]	0.2/0.2/0.5/0.5/0	0.5/0.6/0.6/0.6/2	0.5/0.7/0.7/0.6/7	0.7/0.8/0.7/0.7/11	0.8/0.9/0.8/0.9/17	2
[2, 4]	0.2/0.2/0.6/0.7/0	0.4/0.6/0.7/0.7/4	0.7/0.7/0.7/0.7/9	0.8/0.8/0.9/0.8/13	0.9/0.9/0.8/0.9/16	5
[3, 4]	0.2/0.2/0.6/0.6/0	0.4/0.4/0.6/0.6/3	0.7/0.7/0.8/0.6/8	0.8/0.8/0.8/0.8/15	0.9/0.9/0.8/0.9/19	6
[3, 5]	0.2/0.3/0.5/0.7/0	0.4/0.5/0.6/0.7/5	0.6/0.7/0.8/0.7/9	0.7/0.7/0.9/0.8/12	0.9/0.9/0.8/0.9/18	5
[4, 5]	0.2/0.2/0.7/0.7/0	0.4/0.4/0.7/0.7/2	0.5/0.5/0.8/0.7/6	0.8/0.8/0.8/0.8/11	0.9/0.9/0.9/0.9/16	2
[4, 6]	0.2/0.3/0.7/0.5/0	0.5/0.5/0.7/0.7/4	0.7/0.7/0.7/0.7/9	0.8/0.8/0.7/0.7/12	0.9/0.9/0.7/0.9/15	7
[5, 6]	0.2/0.2/0.6/0.6/0	0.5/0.5/0.7/0.7/5	0.7/0.8/0.8/0.8/10	0.8/0.8/0.9/0.8/13	0.9/0.9/0.8/0.9/17	7

TAB. 6.2 – Results obtained, by solving program $P_{6.5}$, for the example described in figure 6.9 and table 6.1.

B	Expected total distance travelled (km)	Actual cost	$(\mathrm{en}_{ij} + \mathrm{en}_{ji})$ Min	Av	Max	CPU time (s)
0	60	0	0.86	1.06	1.37	0.0
30	147	29	0.74	2.03	5.54	0.1
60	181	60	0.70	2.65	5.18	0.2
90	203	87	0.64	3.11	4.81	0.2
120	211	117	0.79	3.63	4.29	0.1
150	213	133	3.21	4.94	4.43	0.1

TAB. 6.3 – Detailed results corresponding to the optimal solution of the example described in figure 6.9 and table 6.1 for a budget of 90 units.

	1	2	3	4	5	6
1		11	17			
2			0	0		
3				19	12	
4					0	15
5						13
6						

(a) c_{ijh}, amount of investment in each corridor - a total of 87 units.

	1	2	3	4	5	6
1		0.8/0.9	0.9/0.8			
2	0.8/0.9		0.2/0.5	0.2/0.6		
3	0.8/0.9	0.2/0.5		0.9/0.8	0.7/0.9	
4		0.2/0.7	0.9/0.9		0.2/0.7	0.9/0.7
5			0.7/0.8	0.2/0.7		0.8/0.9
6				0.9/0.9	0.8/0.8	

(b) r^1_{ijh} / r^2_{ijh}, probabilities associated with barrier effect, r^1_{ijh}, and mortality risk, r^2_{ijh}.

	1	2	3	4	5	6	7
1	0.15	0.36	0.36	0	0	0	0.13
2	0.24	0.60	0.03	0.04	0	0	0.09
3	0.18	0.02	0.35	0.18	0.16	0	0.11
4	0	0.03	0.20	0.45	0.03	0.16	0.12
5	0	0	0.19	0.05	0.43	0.24	0.09
6	0	0	0	0.40	0.32	0.15	0.12
7	0	0	0	0	0	0	1

(c) pr_{ij}, transition probabilities.

	1	2	3	4	5	6
1		4.81	4.52			
2			0.64	0.81		
3				4.49	3.58	
4					0.83	4.21
5						4.08
6						

(d) $(\mathrm{en}_{ij} + \mathrm{en}_{ji})$, expected number of routes along each corridor; average value=3.11.

	1	2	3	4	5	6
1	2.24	2.25	2.01	1.26	0.91	0.49
2	1.49	4.05	1.64	1.23	0.79	0.45
3	1.01	1.26	3.10	1.75	1.37	0.71
4	0.66	1.00	1.81	3.30	1.25	0.96
5	0.62	0.83	1.83	1.80	3.06	1.20
6	0.55	0.79	1.55	2.25	1.75	2.09

(e) n_{ij}, general term of $(I\text{-}Z)^{-1}$, expected number of passages through BR_j for an individual starting from BR_i - before its disappearance.

TAB. 6.4 – Detailed results corresponding to the optimal solution of the example described in figure 6.9 and table 6.1, in the case of a budget of 90 units and when the number of routes along each corridor must be greater than or equal to 1.5.

	1	2	3	4	5	6
1		8	17			
2			2	13		
3				15	12	
4					6	4
5						13
6						

(a) c_{ijh}, amount of investment in each corridor - a total of 90 units.

	1	2	3	4	5	6
1		0.7/0.9	0.9/0.8			
2	0.8/0.9		0.5/0.6	0.8/0.9		
3	0.8/0.9	0.6/0.6		0.8/0.8	0.7/0.9	
4		0.8/0.8	0.8/0.8		0.5/0.8	0.5/0.7
5			0.7/0.8	0.5/0.7		0.8/0.9
6				0.5/0.7	0.8/0.8	

(b) r^1_{ijh} / r^2_{ijh}, probabilities associated with barrier effect, r^1_{ijh}, and mortality risk, r^2_{ijh}.

	1	2	3	4	5	6	7
1	0.20	0.32	0.36	0	0	0	0.12
2	0.24	0.30	0.10	0.24	0	0	0.12
3	0.18	0.09	0.28	0.16	0.16	0	0.14
4	0	0.16	0.16	0.35	0.10	0.09	0.14
5	0	0	0.19	0.12	0.33	0.24	0.12
6	0	0	0	0.18	0.32	0.35	0.16
7	0	0	0	0	0	0	1

(c) pr_{ij}, transition probabilities.

	1	2	3	4	5	6
1		3.46	3.63			
2			1.55	3.24		
3				2.97	2.90	
4					1.85	1.82
5						3.69
6						

(d) $(\mathrm{en}_{ij} + \mathrm{en}_{ji})$, expected number of routes along each corridor; average value=2.79.

	1	2	3	4	5	6
1	2.05	1.42	1.69	1.21	0.80	0.46
2	1.04	2.42	1.39	1.51	0.79	0.50
3	0.88	1.03	2.54	1.36	1.08	0.58
4	0.60	1.04	1.30	2.63	1.06	0.75
5	0.50	0.70	1.29	1.33	2.54	1.12
6	0.41	0.62	0.99	1.36	1.54	2.29

(e) n_{ij}, general term of $(I\text{-}Z)^{-1}$, expected number of passages through BR_j for an individual starting from BR_i - before its disappearance.

worthwhile to use all the financial resources to optimize the permeability of the network. For example, when $B = 90$, the best solution – 203 km – is obtained by investing only 87 units. If the entire budget is required to be used by transforming in program $\mathrm{P}_{6.5}$ the inequality constraint $\sum_{(i,j)\in C, i<j, h\in H_{ij}} c_{ijh}x_{ijh} \leq B$ into the equality constraint $\sum_{(i,j)\in C, i<j, h\in H_{ij}} c_{ijh}x_{ijh} = B$, the best solution obtained corresponds to an expected distance of only 201 km. Table 6.3 gives detailed results when $B = 90$.

We see in table 6.3d that the corridors $[BR_2, BR_3]$, $[BR_2, BR_4]$, and $[BR_4, BR_5]$ are little used, compared to others. The mathematical programming approach allows additional constraints to be easily taken into account. For example, the expected number of routes along each corridor can be required to be greater than or equal to 1.5. To do this, simply add the constraints $\mathrm{en}_{ij} + \mathrm{en}_{ji} \geq 1.5$, $(i, j) \in C$, $i<j$, to program $\mathrm{P}_{6.5}$. In this case, the expected total distance travelled along the corridors becomes 165 km instead of 203 km. The detailed characteristics of this solution are given in table 6.4.

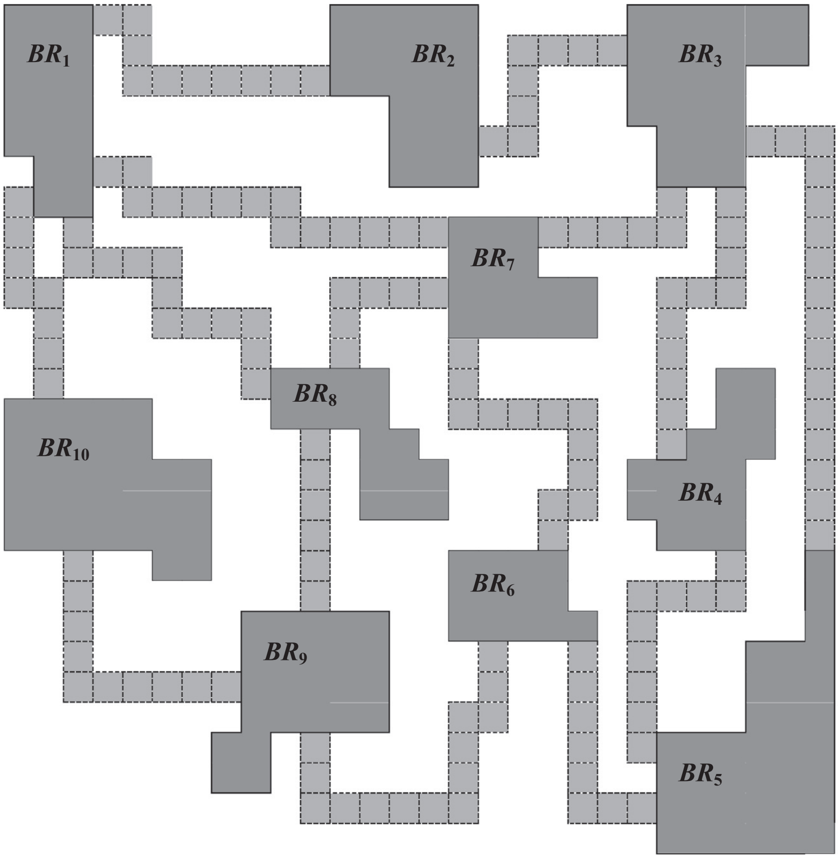

FIG. 6.10 – A hypothetical network with 10 reservoirs and 15 corridors.

6.3.5 Example 2

From a theoretical point of view, there is no limit in the size of the instances – number of reservoirs and number of corridors – that can be handled by program $P_{6.5}$. However, for large-sized instances, the computation time required to resolve them can become very important. We tested an instance with 10 reservoirs and 15 corridors and considered that there could be 7 levels of restoration for each of these corridors (figure 6.10). In this example, the landscape is represented by a grid of 28×28 square and identical zones, each side of which measures 500 m, and includes 10 biodiversity reservoirs, BR_1, BR_2,..., BR_{10}. As in the example in section 6.3.4, it is assumed that the effects of corridor restoration are not symmetric – r^1_{ijh} may be different from r^1_{jih} and r^2_{ijh} may be different from r^2_{jih}.

In figure 6.10, the length of the corridor connecting two reservoirs, BR_i and BR_j, is proportional to the number of grid cells that must be traversed along this corridor

TAB. 6.5 – This table shows, for each corridor $[BR_i, BR_j]$ of the network in figure 6.10 and for each possible value of the investment level $h \in \{0, 1, 2, 3, 4, 5, 6, 7\}$, the probabilities r^1_{ijh}, r^1_{jih}, r^2_{ijh}, r^2_{jih} and the associated costs, c_{ijh}, in this order. For example, the cell located at the intersection of row [3, 5] – associated with the corridor $[BR_3, BR_5]$ – and column $h = 6$, $r^1_{356} = 0.81$, $r^1_{536} = 0.81$, $r^2_{356} = 0.86$, $r^2_{536} = 0.86$, and $c_{356} = 31$. The last column of the table shows the length of each corridor, l_{ij}, in kilometres – 16 for the corridor $[BR_3, BR_5]$.

$[i, j]$	$h = 0$	$h = 1$	$h = 2$	$h = 3$	$h = 4$
[1, 2]	0.21/0.23/0.56/0.58/0	0.31/0.33/0.61/0.63/1	0.41/0.43/0.66/0.68/4	0.51/0.53/0.71/0.73/9	0.61/0.63/0.76/0.78/15
[1, 7]	0.22/0.22/0.57/0.57/0	0.32/0.32/0.62/0.62/1	0.42/0.42/0.67/0.67/4	0.52/0.52/0.72/0.72/9	0.62/0.62/0.77/0.77/15
[1, 8]	0.21/0.22/0.56/0.57/0	0.31/0.32/0.61/0.62/1	0.41/0.42/0.66/0.67/5	0.51/0.52/0.71/0.72/11	0.61/0.62/0.76/0.77/18
[1, 10]	0.22/0.23/0.57/0.58/0	0.32/0.33/0.62/0.63/1	0.42/0.43/0.67/0.68/5	0.52/0.53/0.72/0.73/11	0.62/0.63/0.77/0.78/18
[2, 3]	0.24/0.21/0.59/0.56/0	0.34/0.31/0.64/0.61/1	0.44/0.41/0.69/0.66/5	0.54/0.51/0.74/0.71/10	0.64/0.61/0.79/0.76/17
[3, 4]	0.22/0.23/0.57/0.58/0	0.32/0.33/0.62/0.63/1	0.42/0.43/0.67/0.68/5	0.52/0.53/0.72/0.73/10	0.62/0.63/0.77/0.78/17
[3, 5]	0.21/0.21/0.56/0.56/0	0.31/0.31/0.61/0.61/1	0.41/0.41/0.66/0.66/4	0.51/0.51/0.71/0.71/9	0.61/0.61/0.76/0.76/15
[3, 7]	0.21/0.21/0.56/0.56/0	0.31/0.31/0.61/0.61/1	0.41/0.41/0.66/0.66/4	0.51/0.51/0.71/0.71/8	0.61/0.61/0.76/0.76/13
[4, 5]	0.21/0.21/0.56/0.56/0	0.31/0.31/0.61/0.61/1	0.41/0.41/0.66/0.66/5	0.51/0.51/0.71/0.71/10	0.61/0.61/0.76/0.76/17
[5, 6]	0.22/0.23/0.57/0.58/0	0.32/0.33/0.62/0.63/1	0.42/0.43/0.67/0.68/4	0.52/0.53/0.72/0.73/9	0.62/0.63/0.77/0.78/15
[6, 7]	0.22/0.23/0.57/0.58/0	0.32/0.33/0.62/0.63/1	0.42/0.43/0.67/0.68/5	0.52/0.53/0.72/0.73/11	0.62/0.63/0.77/0.78/18
[6, 9]	0.24/0.22/0.59/0.57/0	0.34/0.32/0.64/0.62/1	0.44/0.42/0.69/0.67/4	0.54/0.52/0.74/0.72/9	0.64/0.62/0.79/0.77/15
[7, 8]	0.23/0.22/0.58/0.57/0	0.33/0.32/0.63/0.62/1	0.43/0.42/0.68/0.67/5	0.53/0.52/0.73/0.72/10	0.63/0.62/0.78/0.77/17
[8, 9]	0.21/0.23/0.56/0.58/0	0.31/0.33/0.61/0.63/1	0.41/0.43/0.66/0.68/4	0.51/0.53/0.71/0.73/7	0.61/0.63/0.76/0.78/12
[9, 10]	0.22/0.23/0.57/0.58/0	0.32/0.33/0.62/0.63/1	0.42/0.43/0.67/0.68/4	0.52/0.53/0.72/0.73/8	0.62/0.63/0.77/0.78/13

$[i, j]$	$h = 5$	$h = 6$	$h = 7$	l_{ij}
[1, 2]	0.71/0.73/0.81/0.83/23	0.81/0.83/0.86/0.88/32	0.91/0.93/0.91/0.93/42	10
[1, 7]	0.72/0.72/0.82/0.82/22	0.82/0.82/0.87/0.87/30	0.92/0.92/0.92/0.92/40	14
[1, 8]	0.71/0.72/0.81/0.82/27	0.81/0.82/0.86/0.87/38	0.91/0.92/0.91/0.92/50	12
[1, 10]	0.72/0.73/0.82/0.83/27	0.82/0.83/0.87/0.88/38	0.92/0.93/0.92/0.93/50	8
[2, 3]	0.74/0.71/0.84/0.81/25	0.84/0.81/0.89/0.86/35	0.94/0.91/0.94/0.91/47	8
[3, 4]	0.72/0.73/0.82/0.83/26	0.82/0.83/0.87/0.88/35	0.92/0.93/0.92/0.93/47	11
[3, 5]	0.71/0.71/0.81/0.81/22	0.81/0.81/0.86/0.86/31	0.91/0.91/0.91/0.91/41	16
[3, 7]	0.71/0.71/0.81/0.81/20	0.81/0.81/0.86/0.86/28	0.91/0.91/0.91/0.91/37	6
[4, 5]	0.71/0.71/0.81/0.81/26	0.81/0.81/0.86/0.86/36	0.91/0.91/0.91/0.91/47	10
[5, 6]	0.72/0.73/0.82/0.83/23	0.82/0.83/0.87/0.88/32	0.92/0.93/0.92/0.93/42	8
[6, 7]	0.72/0.73/0.82/0.83/27	0.82/0.83/0.87/0.88/37	0.92/0.93/0.92/0.93/49	12
[6, 9]	0.74/0.72/0.84/0.82/22	0.84/0.82/0.89/0.87/31	0.94/0.92/0.94/0.92/41	14
[7, 8]	0.73/0.72/0.83/0.82/26	0.83/0.82/0.88/0.87/36	0.93/0.92/0.93/0.92/47	6
[8, 9]	0.71/0.73/0.81/0.83/19	0.81/0.83/0.86/0.88/26	0.91/0.93/0.91/0.93/34	6
[9, 10]	0.72/0.73/0.82/0.83/19	0.82/0.83/0.87/0.88/27	0.92/0.93/0.92/0.93/36	10

TAB. 6.6 – Computational results for the example described in figure 6.10 and table 6.5.

B	Expected total distance travelled (km)	Actual cost	$(\text{en}_{ij} + \text{en}_{ji})$			CPU time (s)
			Min	Av	Max	
0	133	0	0.77	0.87	1.00	0
150	443	147	0.84	2.39	6.62	9
300	688	296	0.86	3.95	7.17	20
450	917	445	0.96	5.49	7.58	2
600	1,065	576	1.00	6.87	7.53	0.1
750	1,111	620	7.16	7.34	7.62	0.03

to get from BR_i to BR_j. For example, the length of the corridor connecting BR_1 to BR_2 is equal to 5 km and the length of the corridor connecting BR_6 to BR_7 is equal to 6 km. All data for this example are summarized in table 6.5.

We see in table 6.6 that the resolution of this instance is very fast for the six B values considered. The case that requires the most computation time is when the financial resources are limited to about 0.5 times the maximum potential investment, $\sum_{(i,j)\in C,\, i<j} c_{ij}^7 = 650$. We also see in this table that the difference between the extreme values of $(\text{en}_{ij} + \text{en}_{ji})$ is often significant. We solved the problem with $B = 450$ and the additional constraints, $\text{en}_{ij} + \text{en}_{ji} \geq 2$, $(i,j) \in C$, $i<j$. In this case, the minimal number of routes along a corridor is equal to 2.33 and the maximal number of routes along a corridor is equal to 6.15, but the expected total distance travelled along the corridors is only equal to 765 km. It should be noted that taking this constraint into account significantly increases the computation time, since it increases from 2 to 49 s. We also solved the problem with the constraints $\text{pr}_{ij} \geq 0.1$ – without constraints on the number of routes in each corridor. In this case, the minimal number of routes along a corridor is equal to 2.32, the maximal number of routes along a corridor is equal to 6.09 and the expected total distance travelled along the corridors is equal to 744 km (39 s of computation time). With the constraints $\text{pr}_{ij} \geq 0.15$, $(i,j) \in C$, the minimal number of routes along a corridor is equal to 2.86, the maximal number of routes along a corridor is equal to 5.47 and the expected total distance travelled along the corridors is equal to 676 km (1.12 s of computation time). Finally, there is no feasible solution when $\text{pr}_{ij} \geq 0.2$, $(i,j) \in C$. In this case, with the budgetary constraint corresponding to $B = 450$, it is impossible to make investments in the corridors in such a way that $\text{pr}_{ij} \geq 0.2$ for all the reservoir pairs connected by a corridor. Remember that pr_{ij} is the probability, for an animal leaving reservoir BR_i, of reaching the adjacent reservoir BR_j in one transition. The same applies to any available budget value less than or equal to 569. On the other hand, for any available budget value greater than or equal to 570, there is a feasible solution. For example, for the maximal potential investment – $B = 650$ – the minimal number of routes along a corridor is equal to 7.16, the maximal number of routes along a corridor is equal to 7.62, and the expected total distance travelled along the corridors is equal to 1,111 km (0.02 s of computation time).

References and Further Reading

Álvarez-Miranda E., Ljubić I., Mutzel P. (2013) The rooted maximum node-weight connected subgraph problem, *Integration of AI and OR techniques in constraint programming for combinatorial optimization problems* (C. Gomes, M. Sellmann, Eds). CPAIOR 2013. Lecture notes in computer science, Springer, vol. 7874.

Anderson A.B., Jenkins C.N. (2006) *Applying nature's design: corridors as a strategy for biodiversity conservation*. Columbia University Press.

Beale E.M.L. (1988) *Introduction to optimization*. Wiley.

Beier P., Noss R. (1998) Do habitat corridors provide connectivity? *Conserv. Biol.* **12**, 1241.

Bell S., Apostol D. (2008) *Designing sustainable forest landscapes*. Taylor & Francis, London.

Bergès L., Roche P., Avon C. (2010) Corridors écologiques et conservation de la biodiversité, intérêts et limites pour la mise en place de la Trame verte et bleue, *Sci. Eaux Territoires* **3**, 34.

Billionnet A. (2010) Integer programming for optimizing habitat network permeability, *Manage. Environ. Qual.* **21**, 570.

Billionnet A. (2013) Mathematical optimization ideas for biodiversity conservation, *Eur. J. Oper. Res.* **231**, 514.

Bond M. (2003) *Principles of wildlife corridor design*. Report Center for Biological Diversity, Idyllwild, California.

Carrière S.M., Hervé D., Andriamahefazafy F., Méral P. (2011) Les corridors, passage obligé ? L'exemple malgache (C. Aubertin, E. Rodary, Eds). *Aires protégées, espaces durables ?* IRD Editions.

Challéat S. (2017) *Empreinte lumineuse, trame nocturne et corridors noirs: quelques precisions*. RENOIR research group, Interdisciplinary Night-time Environment Observatory.

Clergeau P., Désiré G. (1999) Biodiversité, paysage et aménagement: du corridor à la zone de connexion biologique, *Mappemonde* **55**, 19.

Closset-Kopp D., Wasof S., Decocq G. (2016) Using process-based indicator species to evaluate ecological corridors in fragmented landscapes, *Biol. Conserv.* **201**, 152.

Conrad J., Gomes C., van Hoeve W.J., Sabharwal A., Suter J. (2012) Wildlife corridors as a connected subgraph problem, *J. Environ. Econ. Manage.* **63**, 1.

Conservation corridor, connectivity conservation specialist group, available at: https://conservationcorridor.org/.

Conservation corridor, Connecting Science to Conservation, Programs and tools. Available at: http://conservationcorridor.org/corridor-toolbox/programs-and-tools.

Cushman S.A., McRae B., Adriaensen F., Beier P., Shirley M., Zeller K. (2013) Biological corridors and connectivity, *Key topics in conservation biology 2* (D.W. Macdonald, K.J. Willis, Eds). Wiley-Blackwell, Hoboken, NJ, pp. 384-404.

Dilkina B., Gomes C.P. (2010) Solving connected subgraph problems in wildlife conservation. CPAIOR-10: International Conference on Integration of AI and OR Techniques in Constraint Programming for Combinatorial Optimization Problems, Bologna, Italy.

Downes S.J., Handasyde K.A., Elgar M.A. (1997) The use of corridors by mammals in fragmented Australian eucalypt forests, *Conserv. Biol.* **11**, 718.

Finke J., Sonnenschein M. (2008) Simulation and optimization of habitat network permeability, *Inf. Technol. Environ. Eng.* **1**, 12.

Finke J., Strein M., Sonnenschein M. (2008) A simulation framework for modeling anthropogenic disturbances in habitat networks, *Ecol. Inf.* **3**, 26.

Grinstead C.M., Snell J.L. (1997) *Introduction to probability*. American Mathematical Society (2nd edition 2006), Providence, USA.

Gustafsson L., Hansson L. (1997) Corridors as a conservation tool, *Ecol. Bull.* **46**, 182.

Harris L.D., Scheck J. (1991) From implications to applications: the dispersal corridor principle applied to the conservation of biological diversity, *The role of corridors* (D.A. Saunders, R.J. Hobbs, Eds). Surrey Beatty, Chipping Norton, pp. 189-200.

Harrison R. (1992) Toward a theory of inter-refuge corridor design, *Conserv. Biol.* **6**, 293.

Hilty J.A., Lidicker W.Z. Jr., Merenlender A.M. (2006) *Corridor ecology, the science and practice of linking landscapes for biodiversity conservation*. Island Press.

Hobbs R.J. (1992) The role of corridors in conservation: solution or bandwagon? *Trends Ecol. Evol.* **7**, 389.

Holzgang O., Pfister H.P., Heynen D., Blant M., Righetti A., Berthoud G., Marchesi P., Maddalena T., Muri H., Wendelspiess M., Dandliker G., Mollet P., Bornhauser-Sieber U. (2001) Les corridors faunistiques en Suisse. Cahier de l'environnement n° 326, Office fédéral de l'environnement, des forêts et du paysage (OFEFP), Société suisse de Biologie de la Faune (SSBF) & Station ornithologique suisse de Sempach, Bern, p. 120.

Jordan F. (2000) A reliability-theory approach to corridor design, *Ecol. Model.* **28**, 211.

Kemeny J.G., Snell J.L. (1983) *Finite Markov chains*. Springer.

Margules C.R., Nicholls A.O., Pressey R.L. (1988) Selecting networks of reserves to maximize biological diversity, *Biol. Conserv.* **43**, 63.

McEuen A. (1993) The wildlife corridor controversy: a review, *Endanger. Species Update* **10**, 1.

Menard P., Clergeau P. (2001) La notion de zone de connexion biologique, son application en aménagement du territoire, *Mappemonde* **64**, 24.

Ministère de la Transition écologique et solidaire, available at: https://www.ecologique-solidaire.gouv.fr/trame-verte-et-bleue.

Naidoo R., Kilian J.W., Du Preez P., Beytell P., Aschenborn O., Taylor R.D., Stuart-Hill G. (2018) Evaluating the effectiveness of local- and regional-scale wildlife corridors using quantitative metrics of functional connectivity, *Biol. Conserv.* **217**, 96.

Naro-Maciel E., Sterling E.J., Rao M. (2008) Protected areas and biodiversity conservation I: reserve planning and design, *Lessons Conserv.* **2**, 19.

Natural England. (2015) Green Bridges, a literature review. Natural England Commissioned Report (NECR181).

Noss R.F. (1987) Corridors in real landscapes: a reply to Simberloff and Cox, *Conserv. Biol.* **1**, 159.

Önal H., Briers R.A. (2006) Optimal selection of a connected reserve network, *Oper. Res.* **54**, 379.

Rosenberg D.K., Noon B.R., Meslow E.C. (1995) Towards a definition of biological corridor, *Integrating people and wildlife for a sustainable future* (J.A. Bissonette, P.R. Krausman, Eds). International Wildlife Management Congress, Bethesda, Maryland, pp. 436-439.

Salkin H.M., Mathur K. (1989) *Foundations of integer programming*. North-Holland.

Saunders D.A., Hobbs R.J., Eds (1991) *Nature conservation 2: the role of corridors*. Surrey Beatty & Sons.

Sessions J. (1992) Solving for network connections as a Steiner network problem, *For. Sci.* **38**, 203.

Simberloff D., Cox J. (1987) Consequences and costs of conservation corridors, *Conserv. Biol.* **1**, 63.

Simberloff D., Farr J.A., Cox J., Mehlman D.W. (1992) Movement corridors: conservation bargains or poor investments? *Conserv. Biol.* **6**, 493.

St John R. (2016) Explicitly controlling geometric characteristics of corridors in spatial optimization. PhD, University of Washington.

Suter J.F., Conrad J., Gomes C.P., van Hoeve W.J., Sabharwal A. (2008) Optimal corridor design for grizzly bear in the U.S. Northern Rockies. Paper presented at American Agricultural Economics Association Annual Meeting, July 27–29, Orlando, FL.

West S., Cairns R., Schultz L. (2016) What constitutes a successful biodiversity corridor? A Q-study in the Cape Floristic Region, South Africa. *Biol. Conserv.* **198**, 183.

Williams J.C. (1998) Delineating protected wildlife corridors with multi-objective programming, *Environ. Model. Assess.* **3**, 77.

Williams J.C., Snyder S.A. (2005) Restoring habitat corridors in fragmented landscape using optimization and percolation models, *Environ. Model. Assess.* **10**, 239.

Wolsey L.A. (1998) *Integer programming*. Wiley, New York, NY.