Chapter 4

Compactness

4.1 Introduction

As we have seen in the previous chapters, the spatial configuration of a nature reserve is a determining factor for the survival of the species that live there. Chapter 2 deals with fragmentation and chapter 3 with connectivity – or contiguity. In this chapter, we discuss another spatial aspect of a reserve, compactness. This aspect, which can be assessed in several ways and is not completely distinct from the notion of fragmentation, takes into account the distance separating the different zones. The smaller these distances, the easier it is for species to move within the reserve. It can therefore be said that the more compact a reserve is, the more effective the means devoted to its protection. On the other hand, a compact reserve is generally easier to manage than a non-compact reserve.

We will also look at the length of the reserve boundaries (which we also call the reserve perimeter), *i.e.*, the length of the transition zones between the reserve and the surrounding matrix, this criterion being, in a way, related to compactness (figure 4.1).

4.2 Compactness Measures of a Reserve

The compactness of a reserve, *i.e.*, a set of zones selected for some protection, can be assessed in many ways. For example, the diameter of the reserve, *i.e.*, the maximal distance between two points of the reserve, can be considered. The minimization of this criterion leads to the selection of a set of zones with an external contour that is close to a circle. The minimal distance between two zones can also provide some information on compactness. Another measure of the reserve compactness is its total perimeter. Minimizing this latter criterion allows to obtain groups of zones whose shape is close to a square or a circle, but the distance between the groups is not controlled. The compactness of a reserve can also be measured by the sum of the

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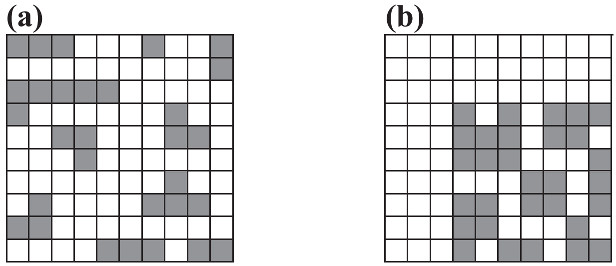

FIG. 4.1 – Two reserves defined on a set of 100 candidate zones represented by a grid of 10×10 square and identical zones whose length of the sides is equal to one unit. The area of these 2 reserves – made up of grey zones – is equal to 30 units. (a) A very fragmented and uncompact reserve with a perimeter of 80 units. (b) A fragmented but relatively compact reserve with a perimeter of 64 units.

distances between all the pairs of selected zones or by its total perimeter divided by its total area. In the latter case, the aim is to minimise the value of the corresponding ratio. Minimizing this ratio also has the effect of reducing the edge effect, which is generally considered as unfavourable to biodiversity protection.

Let $S = \{s_1, s_2,\ldots, s_m\}$ be the set of concerned species, $Z = \{z_1, z_2,\ldots, z_n\}$ be the set of zones that can be protected, $\underline{S}$ be the set of indices of the species of S and $\underline{Z}$ be the set of indices of the zones of Z. Let us specify some supplementary data – and corresponding notations – that we use in this chapter: l_i, the perimeter of zone z_i, a_i, the area of zone z_i, l_{ij}, the length of the border common to zones z_i and z_j, and d_{ij}, the distance "as the crow flies" between zones z_i and z_j. We denote by $\mathrm{Comp}(R)$ the compactness of a reserve, R, and we examine several measures of this compactness.

It is assumed here that we know, among the zones of Z, those whose protection leads to the protection of species s_k (*e.g.*, its survival), and this for all species, *i.e.*, for all $k \in \underline{S} = \{1, 2,\ldots, m\}$. This subset of Z is denoted by Z_k and the corresponding set of indices is denoted by $\underline{Z}_k$. Thus, the protection of species s_k is ensured if and only if at least one of the zones of Z_k is protected and the number of protected species is noted $\mathrm{Nb}_1(R)$ (chapter 1, section 1.1). For example, it is considered here that the protection of a zone allows all the species present in that zone to be protected, provided that their population size is greater than or equal to a certain threshold value. This value is denoted by v_{ik} for zone z_i and species s_k. In other words, $Z_k = \{z_i \in Z : n_{ik} \geq v_{ik}\}$ where n_{ik} refers to the population size of species s_k in zone z_i.

4.3 Some Problems of Selecting Compact Reserves and their Mathematical Programming Formulation

As with the other spatial criteria we examined, several zone selection problems may arise with the objective of obtaining a compact set of zones. We note R the set of selected zones, *i.e.*, the reserve obtained, and $\underline{R}$ the set of indices of the zones forming reserve R. Some of these problems are discussed below, by way of example. Let us recall that we denote by $\mathrm{Comp}(R)$ the value of the compactness criterion associated with reserve R. As we have seen, this criterion can correspond to different aspects of the compactness of a reserve – and can, therefore, be calculated in several different ways. Table 4.1 summarizes the compactness criteria considered in this chapter and that we seek to minimize.

To simplify the presentation, we assume that the set of candidate zones are represented by a grid with nr rows and nc columns. The zones are then designated by z_{ij} where i is the row index and j is the column index. It should be noted that everything presented in the rest of this chapter can be applied directly to any other set of candidate zones.

Take again the reserves in figure 4.1 to illustrate these 3 criteria. First of all, the distance between two candidate zones – represented by two squares whose length is equal to one unit – is defined by the distance, in a straight line, separating the centre of these two zones. Other definitions of the distance between two zones could be considered. Let us compare the compactness of the two reserves in figure 4.1 using the 3 criteria in table 4.1. Using criterion No. 1, the compactness of the reserve in figure 4.1a is equal to the distance between the centres of zones $z_{1,1}$ and $z_{10,10}$, *i.e.*, 12.73 ($\sqrt{162}$) while the compactness of the reserve in figure 4.1b is equal to the distance between zones $z_{4,4}$ and $z_{10,10}$, *i.e.*, 8.49 ($\sqrt{72}$). Using criterion No. 2, the compactness of the reserve in figure 4.1a is equal to 2.67 since its total perimeter is equal to 80 and its total area to 30, while the compactness of the reserve in figure 4.1b is equal to 2.13 since its total perimeter is equal to 64 and its total area to 30. Finally, using criterion No. 3, the compactness of the reserve in figure 4.1a is equal to 2,514.79 while the compactness of the reserve in figure 4.1b is equal to 1,716.43.

Different problems of determining an optimal compact reserve can be considered. For each of these problems, the 3 compactness criteria that we have just defined can be taken into account. Table 4.2 presents the 3 problems we will consider. Recall that $\mathrm{Nb}_1(R)$ represents the number of species protected by reserve R and that species s_k is considered to be protected by reserve R if at least one of the zones of Z_k belongs to R where, for any k of $\underline{S}$, Z_k is a known subset of Z (see section 4.2).

Tab. 4.1 – Three compactness criteria for a reserve, R.

Criterion number	Statement	Formulation
1	Diameter of the reserve, *i.e.*, maximal distance between two zones of the reserve.	$\max\{d_{ij} : (i,j) \in \underline{R}^2, i<j\}$
2	Total perimeter of the reserve, divided by the total area of the reserve.	$\left(\sum_{i\in\underline{R}} l_i - 2\sum_{(i,j)\in\underline{R}^2,\, i<j} l_{ij}\right) \Big/ \sum_{i\in\underline{R}} a_i$
3	Sum of the distances between all pairs of zones in the reserve.	$\sum_{(i,j)\in\underline{R}^2,\, i<j} d_{ij}$

TAB. 4.2 – Three reserve selection problems with a compactness objective.

Problem no.	Problem statement	Formulation
I	Selection of a reserve, R, of minimal cost, allowing to protect, at a minimum, a given number of species, Ns, and such that the compactness indicator, Comp(R), is lower than or equal to a given value, ρ.	$\begin{cases} \min \ C(R) \\ \text{s.t.} \left\lvert \begin{array}{l} R \subseteq Z \\ \mathrm{Nb}_1(R) \geq \mathrm{Ns} \\ \mathrm{Comp}(R) \leq \rho \end{array} \right. \end{cases}$
II	Selection of a reserve, R, of cost less than or equal to a given value, B, allowing the greatest possible number of species to be protected, and whose value of the compactness indicator, Comp(R), is less than or equal to a given value, ρ.	$\begin{cases} \max \ \mathrm{Nb}_1(R) \\ \text{s.t.} \left\lvert \begin{array}{l} R \subseteq Z \\ C(R) \leq B \\ \mathrm{Comp}(R) \leq \rho \end{array} \right. \end{cases}$
III	Selection of a reserve, R, of cost less than or equal to a given value B, allowing to protect, at a minimum, a given number of species, Ns, and minimizing the value of the compactness indicator, Comp(R).	$\begin{cases} \min \ \mathrm{Comp}(R) \\ \text{s.t.} \left\lvert \begin{array}{l} R \subseteq Z \\ C(R) \leq B \\ \mathrm{Nb}_1(R) \geq \mathrm{Ns} \end{array} \right. \end{cases}$

Also remember that $C(R)$ refers to the cost of reserve R: $C(R) = \sum_{i \in \underline{R}} c_i$ where c_i is the cost associated with zone z_i.

4.3.1 Problem I: Protection, at the Lowest Cost, of at Least Ns Species of a Given Set, with a Compactness Constraint

This problem can be formulated as the mathematical program $\mathrm{P}_{4.1}$ in which ρ designates the value that the compactness indicator of the selected reserve must not exceed. As in all the programs we have studied, the Boolean variable x_i is equal to 1 if and only if zone z_i is selected to form the reserve.

$$\mathrm{P}_{4.1}: \begin{cases} \min \ \sum\limits_{i \in \underline{Z}} c_i x_i \\ \text{s.t.} \left\lvert \begin{array}{llll} y_k \leq \sum\limits_{i \in \underline{Z}_k} x_i & k \in \underline{S} & (4.1.1) & \mid \quad x_i \in \{0,1\} \quad i \in \underline{Z} \quad (4.1.4) \\ \sum\limits_{k \in \underline{S}} y_k \geq \mathrm{Ns} & & (4.1.2) & \mid \quad y_k \in \{0,1\} \quad k \in \underline{S} \quad (4.1.5) \\ \mathrm{Comp}(R) \leq \rho & & (4.1.3) & \mid \end{array} \right. \end{cases}$$

The economic function to be minimized represents the cost of the reserve. According to constraints 4.1.1, variable y_k can take the value 1 if and only if at least one zone in Z_k is protected. Constraint 4.1.2 expresses that the number of protected species must be greater than or equal to Ns. Constraint 4.1.3 imposes a compactness index less than or equal to ρ. Note that if we seek to protect all the species – Ns $= m$ – we can replace constraints 4.1.1 and 4.1.2 by the single family of constraints $\sum_{i\in \underline{Z}_k} x_i \geq 1,\;\; k \in \underline{S}$. If we want to obtain, among the optimal solutions of $\mathrm{P}_{4.1}$, a solution that maximizes the number of protected species, we only need to subtract from the economic function to be minimized the quantity $\varepsilon \sum_{k\in \underline{S}} y_k$ where ε is a sufficiently small constant. Similarly, if one wants to obtain, among the optimal solutions of $\mathrm{P}_{4.1}$, a solution that minimizes the value of the compactness criterion, it is sufficient to add to the economic function to be minimized the quantity $\varepsilon\mathrm{Comp}(R)$ where ε is a sufficiently small constant. Let us now study constraint 4.1.3 according to the criterion retained to measure compactness. Recall that $R = \{z_i : i = 1,..., n; x_i = 1\}$.

Criterion No. 1. The compactness of a reserve is measured by the diameter of the reserve, *i.e.*, by the maximal distance between two zones of the reserve (see appendix at the end of the book). In this case, $\mathrm{P}_{4.1}$ solves the problem by replacing the – generic – constraint 4.1.3 with one of the specific constraint sets $\mathrm{C}_{4.1}$ or $\mathrm{C}_{4.2}$:

$$\mathrm{C}_{4.1} : x_i + x_j \leq 1 \quad (i, j) \in \underline{Z}^2,\; i<j,\; d_{ij} > \rho;$$

$$\mathrm{C}_{4.2} : x_i + \sum_{j\in \underline{Z},\, j\,>\,i,\, d_{ij}\,>\,\rho} x_j \leq 1 + M(1 - x_i) \quad i \in \underline{Z}.$$

Constraints $\mathrm{C}_{4.1}$ express that if the distance between any two zones, z_i and z_j, is greater than ρ then these two zones cannot both be part of the reserve. In other words, in this case, variables x_i and x_j cannot simultaneously take the value 1. According to constraints $\mathrm{C}_{4.2}$, if zone z_i is selected – $x_i = 1$ – then none of the zones located at a distance greater than ρ from z_i can belong to the reserve. In case zone z_i is not retained – $x_i = 0$ – the corresponding constraint is inactive provided that the constant M is chosen large enough.

Criterion No. 2. Let us now consider the case where the compactness of a reserve, R, is measured by the total perimeter of the reserve divided by its total area: $\mathrm{Comp}(R) = \left(\sum_{i\in \underline{R}} l_i - 2\sum_{(i,j)\in \underline{R}^2, i<j} l_{ij}\right) \Big/ \sum_{i\in \underline{R}} a_i$. In this case, the problem considered can be solved by program $\mathrm{P}_{4.1}$ by replacing the – generic – constraint 4.1.3 by the specific constraint $\mathrm{C}_{4.3}$:

$$\mathrm{C}_{4.3} : \frac{\sum_{i\in \underline{Z}} l_i x_i - 2\sum_{(i,j)\in \underline{Z}^2, i<j} l_{ij} x_i x_j}{\sum_{i\in \underline{Z}} a_i x_i} \leq \rho.$$

The perimeter of the reserve is calculated by summing the perimeters of all the zones that constitute the reserve, $\sum_{i\in\underline{Z}} l_i x_i$, and by subtracting twice the sum of the lengths of the borders common to each pair of zones of the reserve, $2\sum_{(i,j)\in\underline{Z}^2,\, i<j} l_{ij} x_i x_j$. Note that, in the latter expression, many terms l_{ij} are equal to 0. The quantity $\sum_{i\in\underline{Z}} a_i x_i$ represents the sum of the areas of each zone constituting the reserve, *i.e.*, the total area of the reserve. Constraint $C_{4.3}$ is equivalent to constraint $C_{4.4}$ in which the first member is quadratic and the second member is linear:

$$C_{4.4} : \sum_{i\in\underline{Z}} l_i x_i - 2 \sum_{(i,j)\in\underline{Z}^2, i<j} l_{ij} x_i x_j \leq \rho \sum_{i\in\underline{Z}} a_i x_i.$$

It is possible to replace $C_{4.4}$ with equivalent linear constraints (see appendix at the end of the book). To do this, each product $x_i x_j$ is replaced in $C_{4.4}$ by variable u_{ij} and 2 families of linearization constraints are added to force variables u_{ij} to be equal to the products $x_i x_j$, at the optimum of the obtained program. Finally, the problem can be solved by program $P_{4.1}$ by replacing the – generic – constraint 4.1.3 by the set of specific constraints $C_{4.5}$:

$$C_{4.5} : \begin{cases} \sum_{i\in\underline{Z}} l_i x_i - 2 \sum_{(i,j)\in\underline{Z}^2, i<j} l_{ij} u_{ij} \leq \rho \sum_{i\in\underline{Z}} a_i x_i \\ u_{ij} \leq x_i \quad (i,j) \in \underline{Z}^2, i<j,\ l_{ij} > 0 \\ u_{ij} \leq x_j \quad (i,j) \in \underline{Z}^2, i<j,\ l_{ij} > 0 \end{cases}.$$

Note that if the compactness criterion is the perimeter of the reserve and not the perimeter-to-area ratio, it is sufficient to replace the first constraint of $C_{4.5}$ by the constraint $\sum_{i\in\underline{Z}} l_i x_i - 2\sum_{(i,j)\in\underline{Z}^2, i<j} l_{ij} u_{ij} \leq \rho$ where ρ now refers to the maximal allowed perimeter.

Criterion No. 3. Let us now consider the case where the compactness of a reserve is measured by the sum of the distances between all the pairs of zones of the reserve: $\mathrm{Comp}(R) = \sum_{(i,j)\in\underline{R}^2,\, i<j} d_{ij}$. In this case, the problem considered can be solved by program $P_{4.1}$ by replacing the – generic – constraint 4.1.3 by the specific constraint $C_{4.6}$:

$$C_{4.6} : \sum_{(i,j)\in\underline{Z}^2, i<j} d_{ij} x_i x_j \leq \rho.$$

Constraint $C_{4.6}$ is quadratic since it involves the products $x_i x_j$. As in the case of $C_{4.3}$, it is possible to replace this constraint by a set of linear constraints (see appendix at the end of the book). To do this, each product $x_i x_j$ is replaced in $C_{4.6}$ by variable u_{ij} and 2 sets of linearization constraints are added to force variables u_{ij} to be equal to products $x_i x_j$ – at the optimum of the obtained program. Finally, the problem can be solved by program $P_{4.1}$ by replacing constraint 4.1.3 by the set of constraints $C_{4.7}$:

$$\mathrm{C}_{4.7}:\begin{cases}\sum_{(i,j)\in\underline{Z}^2,\ i<j} d_{ij}u_{ij}\leq\rho & \\ 1-x_i-x_j+u_{ij}\geq 0 & (i,j)\in\underline{Z}^2, i<j \\ u_{ij}\geq 0 & (i,j)\in\underline{Z}^2, i<j\end{cases}.$$

4.3.2 Problem II: Protection, Under a Budgetary Constraint, of the Largest Possible Number of Species of a Given Set, with a Compactness Constraint

This problem can be formulated as the mathematical program $\mathrm{P}_{4.2}$.

$$\mathrm{P}_{4.2}:\begin{cases}\max \sum_{k\in\underline{S}} y_k \\ \text{s.t.}\ \left|\begin{array}{llll|lll} \sum_{i\in\underline{Z}} c_i x_i\leq B & & (4.2.1) & & x_i\in\{0,1\} & i\in\underline{Z} & (4.2.4) \\ y_k\leq\sum_{i\in\underline{Z}_k} x_i & k\in\underline{S} & (4.2.2) & & y_k\in\{0,1\} & k\in\underline{S} & (4.2.5) \\ \mathrm{Comp}(R)\leq\rho & & (4.2.3) & & & & \end{array}\right.\end{cases}$$

The economic function, to be maximized, expresses the number of protected species. Constraint 4.2.1 reflects the budgetary constraint: the cost associated with the reserve retained must not exceed the budget, B. For the meaning of the other constraints (4.2.2–4.2.5), the reader may refer to program $\mathrm{P}_{4.1}$. To solve the problem with the 3 compactness criteria considered, it is sufficient to replace in $\mathrm{P}_{4.2}$ constraint 4.2.3 by the appropriate constraints: $\mathrm{C}_{4.1}$ or $\mathrm{C}_{4.2}$ for criterion No. 1, $\mathrm{C}_{4.5}$ for criterion No. 2, and $\mathrm{C}_{4.7}$ for criterion No. 3 (see section 4.3.1). As in program $\mathrm{P}_{4.1}$, zone z_i belongs to reserve R if and only if $x_i = 1$.

4.3.3 Problem III: Protection, Under a Budgetary Constraint, of at Least Ns Species of a Given Set, with Optimal Compactness

This problem can be formulated as the mathematical program $\mathrm{P}_{4.3}$.

$$\mathrm{P}_{4.3}:\begin{cases}\min\ \mathrm{Comp}(R) \\ \text{s.t.}\ \left|\begin{array}{llll|lll} y_k\leq\sum_{i\in\underline{Z}_k} x_i & k\in\underline{S} & (4.3.1) & & x_i\in\{0,1\} & i\in\underline{Z} & (4.3.4) \\ \sum_{i\in\underline{Z}} c_i x_i\leq B & & (4.3.2) & & y_k\in\{0,1\} & k\in\underline{S} & (4.3.5) \\ \sum_{k\in\underline{S}} y_k\geq \mathrm{Ns} & & (4.3.3) & & & & \end{array}\right.\end{cases}$$

The economic function, to be minimized, expresses the compactness of the reserve. As in programs $\mathrm{P}_{4.1}$ and $\mathrm{P}_{4.2}$, zone z_i belongs to reserve R if and only if

$x_i = 1$. The meaning of the constraints in $P_{4.3}$ is presented in the two previous sections. Let us now look at how to solve the problem with the 3 compactness criteria considered.

Criterion No. 1. The compactness of a reserve is measured by the diameter of the reserve, *i.e.*, the maximal distance between two zones of the reserve. To solve the problem, variable α is introduced, the – generic – economic function of $P_{4.3}$ is replaced by α and the set of constraints $C_{4.8}$ is added:

$$C_{4.8} : \alpha \geq d_{ij} x_i x_j \qquad (i,j) \in \underline{Z}^2,\ i<j.$$

These constraints express that if zones z_i and z_j are retained – $x_i x_j = 1$ – then the value of variable α must be greater than or equal to the distance between these two zones. This results in a program with a linear economic function but with some quadratic constraints. These constraints can be replaced by the equivalent set of linear constraints $C_{4.9}$:

$$C_{4.9} : \begin{cases} \alpha \geq d_{ij}(-1 + x_i + x_j) & (i,j) \in \underline{Z}^2,\ i<j \\ \alpha \geq 0 \end{cases}.$$

Criterion No. 2. The compactness of reserve R is measured by the ratio of the total perimeter of the reserve divided by its total area. In this case, the problem considered can be solved by program $P_{4.3}$ by replacing the – generic – economic function of this program with the expression $\left(\sum_{i \in \underline{Z}} l_i x_i - 2\sum_{(i,j) \in \underline{Z}^2, i<j} l_{ij} x_i x_j\right) \Big/ \sum_{i \in \underline{Z}} a_i x_i$. The numerator of this expression is quadratic and the denominator is linear (see appendix at the end of the book). This expression can be transformed into a ratio of two linear functions by replacing each product $x_i x_j$ with variable u_{ij} and adding the linear constraints $u_{ij} \leq x_i$ and $u_{ij} \leq x_j$. The problem can thus be reformulated as program $P_{4.4}$.

$$P_{4.4} : \begin{cases} \min \ \left(\sum_{i \in \underline{Z}} l_i x_i - 2\sum_{(i,j) \in \underline{Z}^2,\, i<j} l_{ij} u_{ij}\right) \Big/ \sum_{i \in \underline{Z}} a_i x_i \\ \text{s.t.} \left| \begin{array}{lll} \sum\limits_{i \in \underline{Z}} c_i x_i \leq B & & (4.4.1) \\ y_k \leq \sum\limits_{i \in \underline{Z}_k} x_i & k \in \underline{S} & (4.4.2) \\ \sum\limits_{k \in \underline{S}} y_k \geq \text{Ns} & & (4.4.3) \\ u_{ij} \leq x_i;\ u_{ij} \leq x_j & (i,j) \in \underline{Z}^2, i<j, l_{ij} > 0 & (4.4.4) \\ x_i \in \{0,1\} & i \in \underline{Z} & (4.4.5) \\ y_k \in \{0,1\} & k \in \underline{S} & (4.4.6) \end{array} \right. \end{cases}$$

We can therefore use the algorithms of fractional programming to solve $P_{4.4}$, for example the Dinkelbach algorithm (see appendix at the end of the book). The core of this algorithm is to solve the auxiliary problem $P_{4.5}(\lambda)$ which is a linear program in 0–1 variables.

$$P_{4.5}(\lambda) : \begin{cases} \min \sum_{i \in \underline{Z}} l_i x_i - 2 \sum_{(i,j) \in \underline{Z}^2, i<j} l_{ij} u_{ij} - \lambda \sum_{i \in \underline{Z}} a_i x_i \\ \text{s.t.} | \ (4.4.1) - (4.4.6) \end{cases}$$

Criterion No. 3. Finally, consider the case where the compactness of a reserve is measured by the sum of the distances between all the pairs of zones of the reserve: $\mathrm{Comp}(R) = \sum_{(i,j) \in \underline{R}^2,\, i<j} d_{ij}$. In this case, the problem considered can be solved by program $P_{4.3}$ by replacing its – generic – economic function with the expression $\sum_{(i,j) \in \underline{Z}^2,\, i<j} d_{ij} x_i x_j$. We then obtain a mathematical program whose economic function is quadratic and whose constraints are linear (see appendix at the end of the book). One way to solve the resulting program is to linearize the economic function, and there are many techniques to do so. A simple technique that we have already presented consists of replacing products $x_i x_j$ by variables u_{ij} and adding the set of linear constraints $C_{4.10}$ that force variable u_{ij} to be equal to product $x_i x_j$:

$$C_{4.10} : \begin{cases} 1 - x_i - x_j + u_{ij} \geq 0 & (i,j) \in \underline{Z}^2, i<j \\ u_{ij} \geq 0 & (i,j) \in \underline{Z}^2, i<j \end{cases}.$$

Another technique consists in rewriting the economic function as $(1/2) \sum_{i \in \underline{Z}} x_i \sum_{j \in \underline{Z}} d_{ij} x_j$ then replacing, for any i of $\underline{Z}$, the expression $x_i \sum_{j \in \underline{Z}} d_{ij} x_j$ by the real, non-negative variable t_i. The new economic function – to be minimized – is therefore written $(1/2) \sum_{i \in \underline{Z}} t_i$. Then we must add the set of linear constraints $C_{4.11}$ which force variable t_i to be equal, at the optimum, to the expression $x_i \sum_{j \in \underline{Z}} d_{ij} x_j$:

$$C_{4.11} : \begin{cases} t_i \geq \sum_{j \in \underline{Z}} d_{ij} x_j - M(1 - x_i) & i \in \underline{Z} \\ t_i \geq 0 \quad i \in \underline{Z} \end{cases}.$$

If variable x_i is equal to 1 then, because of the first family of constraints of $C_{4.11}$ and the fact that we seek to minimize the expression $\sum_{i \in \underline{Z}} t_i$, variable t_i takes the value $\sum_{j \in \underline{Z}} d_{ij} x_j$. On the contrary, if variable x_i is equal to 0, then the set of constraints $C_{4.11}$ and the fact that we seek to minimize the expression $\sum_{i \in \underline{Z}} t_i$ force variable t_i to take the value 0. M denotes a sufficiently large constant.

4.4 Computational Experiments

A hypothetical set of candidate zones represented by a grid of 10×10 square and identical zones is considered. The cost associated with each zone of the grid is randomly drawn, in a uniform way, from the set $\{5, 6, \ldots, 10\}$ and is shown in

	1	2	3	4	5	6	7	8	9	10
1	10	7	7	9	7	6	10	5	9	6
2	9	6	8	10	6	9	7	7	10	9
3	7	7	8	8	8	5	5	9	8	7
4	5	10	8	9	8	4	6	9	7	4
5	7	5	8	7	5	8	5	10	9	9
6	7	10	4	10	9	6	6	4	7	5
7	7	5	5	10	5	9	5	10	7	7
8	9	6	7	9	6	8	8	6	6	5
9	8	9	8	10	6	6	5	9	5	4
10	7	8	5	5	7	9	4	7	8	7

Fig. 4.2 – Cost associated with each of the 100 candidate zones.

figure 4.2. The available budget is 150 units and 100 species are considered. The presence of a species in a given zone – with a sufficient abundance to be protected if the corresponding zone is protected – is randomly drawn, with a probability equal to 0.1. Figure 4.3 shows, for each candidate zone, the list of the species that are protected if the zone is itself protected. It should be noted that, in this example, 9 of the 100 species considered cannot be protected. These are species s_7, s_{13}, s_{16}, s_{23}, s_{33}, s_{61}, s_{73}, s_{83}, and s_{90}.

Table 4.3 presents the results obtained for Problem I of table 4.2 with the compactness criteria No. 1 and No. 2 of table 4.1 (see section 4.3.1). All instances were resolved in less than one second of computation. Some of the reserves obtained are shown in figure 4.4.

Table 4.4 presents the results obtained for Problem II of table 4.2 with the compactness criteria No. 1 and No. 2 of table 4.1 and when the available budget, B, is equal to 150 (see section 4.3.2). All the instances considered were resolved in less than one second of computation. Some of the reserves obtained are shown in figure 4.5.

Table 4.5 presents the results obtained for Problem III of table 4.2 with the compactness criteria No. 1 and No. 2 of table 4.1 and an available budget, B, equal to 150 (see section 4.3.3). All the instances considered were resolved in less than one second of computation. Some of the obtained reserves are presented in figure 4.6.

4.5 Compactness Measure Specific to a Connected Reserve: Protection of a Maximal Number of Species of a Given Set by a Connected and Compact Reserve, under a Budgetary Constraint

In this section, we focus on reserves that must, on the one hand, be connected and, on the other hand, meet a compactness criterion. In a connected reserve, the species can move through the whole reserve without leaving it (see chapter 3). With regard to compactness, we consider here a different measure from those studied in the

	1	2	3	4	5	6	7	8	9	10
1	15 75	4 11 18 69 98	3 5 85 89	-	14 31	47 80	34 85	1 22 38	35 37 77 88	27 87
2	4 79	28 92	31 42 65	35 37	6 25 68 84	19 32 63 99	11 25 40	29 53 85 86	34 51 99	95
3	42	17 63	-	20	41 99 100	21 44	50	-	2	-
4	8 40	-	-	67	63 95	6 26 79	9 44 54	-	8 40 54 58 69	43
5	98	35 66	36	46 72 77 87	25 76	18 32 53	88 96	53	69	26 44
6	52	67	28 82	-	44	27 37 41 88	19 66	84	5 59 64 88	57
7	71 85	43 48	76 78	10 37	-	46 59	14 62	10 12 27 59 91	15 25 59 60 70	87
8	69 85	41 45 80 86 91	46	48 49	30 39 94	46 76	8 14 97	3 70	-	19
9	63 99	12 24 39 64 68	2 5 14 25 88	98	26 94	99	76	18	8 50 72 81	75
10	3 47 71 74 97	81	17 58	48 93	55	19	12 60 85	53	18	56

FIG. 4.3 – For each of the 100 candidate zones, list of the indices of the species protected due to the protection of the zone. For example, the protection of zone z_{67} leads to the protection of species s_{19} and s_{66}.

previous sections and which can only be applied to connected reserves, in contrast to the 3 measures presented in table 4.1. To measure the compactness of such reserves, we define the distance between two zones z_i and z_j as the shortest distance to travel from zone z_i to zone z_j without leaving the reserve. As announced, this definition of the distance between two zones of a reserve implies that this reserve is connected, in contrast to the definition of the distance between two zones used in the compactness measures No. 1 and No. 3 presented in table 4.1. The zones outside the connected and compact reserves in which we are interested should also form a set of connected zones. In other words, it must be possible to cross all the areas outside the reserve without crossing the reserve in order to protect it from external disturbances. The problem considered is to select a set of zones included in the set of candidate zones, $Z = \{z_1, z_2, \ldots, z_n\}$, to constitute a connected reserve that maximizes the number of species protected by this reserve while respecting a compactness criterion and budgetary constraint. The compactness criterion used here is described in detail

TAB. 4.3 – Results obtained for Problem I of table 4.2 with the compactness criteria No. 1 and No. 2 of table 4.1, and for the instance described in figures 4.2 and 4.3. We consider 2 different values of the minimal number of species to be protected, Ns, 30 and 60. For each pair (Ns, No. of the compactness criterion) considered, we study 2 values of the compactness criterion, ρ, that should not be exceeded.

Problem I							
Ns	No. of the compactness criterion considered	ρ	Number of selected zones	Cost of the selected reserve	Number of protected species	Actual value of the criterion	Associated figure
30	1	4	9	62	30	3.6	4.4a
		7	8	48	30	6.1	4.4b
	2	0.9	20	125	31	0.9	–
		1.5	10	68	30	1.5	–
60	1	4	–	–	–	–	–
		7	26	170	60	6.7	–
	2	0.9	32	219	60	0.9	4.4c
		1.5	24	159	60	1.5	4.4d

– No feasible solution.

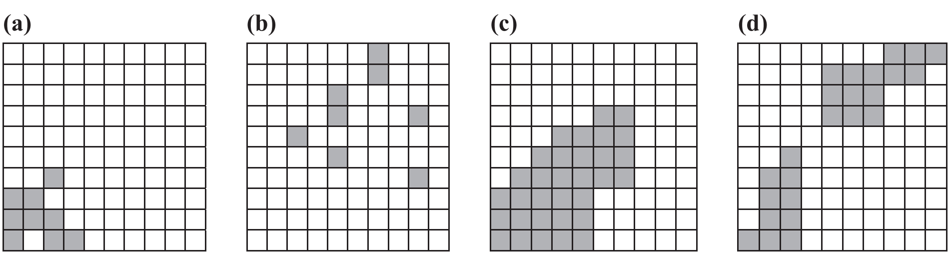

FIG. 4.4 – Obtained reserves for 4 instances of Problem I (see table 4.3).

below. It can be assumed that the non-selected zones, *i.e.*, those located outside the reserve, will be used for urban, industrial or agricultural development. We are interested in a set, $S = \{s_1, s_2, \ldots, s_m\}$, of rare or threatened species present in these zones. For each zone z_i we know the list of the species present in this zone and, for each species, its population size. The population size of species s_k in zone z_i is denoted by n_{ik} and reserve, R, is considered to protect species s_k if and only if the total population size of species s_k in this reserve is greater than or equal to a certain threshold value, θ_k. Thus, the interest in protecting reserve, R, is measured by the quantity $\mathrm{Nb}_2(R)$ which expresses the number of species whose total population size in the reserve is greater than or equal to the threshold value (chapter 1, section 1.1). It is assumed that the movements of the species under consideration are only

TAB. 4.4 – Results obtained for Problem II of table 4.2 with the compactness criteria No. 1 and No. 2 of table 4.1, for the instance described in figures 4.2 and 4.3, and when the value of the available budget, B, is equal to 150. Two values of ρ, the compactness criterion value that should not be exceeded are studied for each considered compactness criterion.

Problem II						
No. of the compactness criterion considered	ρ	Number of selected zones	Actual cost of the selected reserve	Number of protected species	Actual criterion value	Associated figure
1	4	14	101	35	3.6	4.5a
	7	23	149	57	6.7	4.5b
2	0.9	20	142	41	0.9	–
	1.5	24	150	57	1.5	–

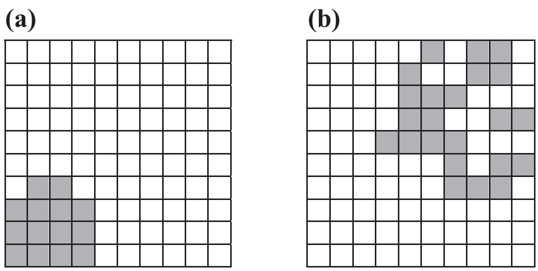

FIG. 4.5 – Obtained reserves for 2 instances of Problem II (see table 4.4).

TAB. 4.5 – Results obtained for Problem III of table 4.2 with the compactness criteria No. 1 and No. 2 of table 4.1, for the instance described in figures 4.2 and 4.3, and when the value of the available budget, B, is equal to 150. We consider 2 different values, 30 and 60, of the minimal number of species to be protected, Ns.

Problem III						
Ns	No. of the compactness criterion considered	Number of selected zones	Actual cost of the selected reserve	Number of protected species	Value of the criterion	Associated figure
30	1	11	82	30	3.2	4.6a
	2	23	148	31	0.9	4.6c
60	1	25	150	60	7.3	4.6b
	2	22	150	60	1.7	4.6d

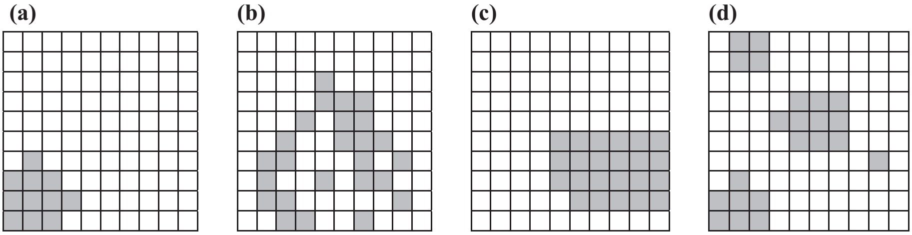

FIG. 4.6 – Obtained reserves for 4 instances of Problem III (see table 4.5).

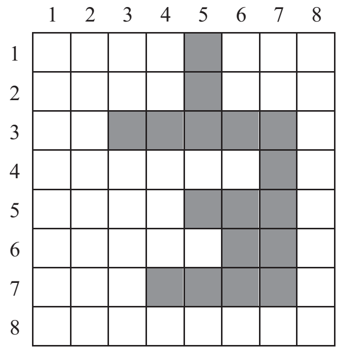

FIG. 4.7 – The candidate zones form a grid of dimension 8×8. Each zone is a square whose side length is equal to one unit. Among the 64 candidate zones, 17 are selected and form a connected reserve.

possible within the reserve – the outside of the reserve being a too hostile environment – and these movements are made from a zone of the reserve to an adjacent zone of this reserve. This notion of adjacency is considered to be the same for all species considered. To facilitate the presentation of the examples, all the candidate zones form a grid and two zones are considered adjacent if they share a common side. It is then assumed, for measuring the length of a route, that the movements are made gradually from the centre of a zone to the centre of an adjacent zone. We are looking for a connected and compact reserve. In such a reserve, thanks to connectivity, the species can circulate throughout the reserve without leaving it (figure 4.7) and, thanks to compactness, the distance they have to travel within the reserve to get from one zone to another is not too long. Let us now look at the precise definition of compactness that we have chosen here.

Compactness. The compactness indicators usually use the Euclidean distance between zones. This is the case for the criteria No. 1 and No. 3 of section 4.3, and also for a measure of the compactness of a reserve equal to the radius of the smallest circle containing the whole reserve (*e.g.*, all the centres of each zone). On the reserve in figure 4.8, we see that this radius is equal to $\sqrt{8}$ and the centre of the corresponding circle is located in the centre of the zone located at the intersection of row 5 and column 5. This structural measure may not be relevant from a functional point of view. Indeed, two zones may be relatively close as the crow flies but distant when trying to travel from one to the other without leaving the reserve. This is the case for the two light grey zones of the reserve shown in figure 4.8. The distance to be covered to join these two zones is equal to 2 units – assuming that the distance between two adjacent zones is equal to one unit – but the associated route leaves the reserve. On the other hand, the minimal distance to be covered to join these two zones without leaving the reserve is equal to 14 units. We will therefore use a more realistic measure of compactness than the radius of the smallest circle containing the whole reserve. Denote by R the reserve, *i.e.*, the set of zones that constitute it. Let $d_{ij}(R)$ be the minimal distance that the species must cover to get from zone z_i to zone z_j without leaving the reserve. To each zone z_i of the reserve R, its eccentricity is associated. It is denoted by $\mathrm{ecc}(z_i,R)$ and defined as follows: $\mathrm{ecc}(z_i, R) = \max\{d_{ij}(R) : z_j \in R\}$. The eccentricity of zone z_i in reserve R is therefore equal to the distance – defined above – between zone z_i and the zone which is furthest from z_i. Finally, we define the compactness of a reserve by the minimal value of this quantity, *i.e.*, $\mathrm{Comp}(R) = \min\{\mathrm{ecc}(z_i, R) : z_i \in R\}$. This measure of the compactness of R is also called the radius of R. With this definition, the compactness of the reserve shown in figure 4.8 is equal to 7 – the eccentricity of the zone located at the intersection of row 5 and column 6.

Connectivity of the zones outside the reserve. When a set of zones is selected to form a reserve, it may be desirable, in order to minimize disturbance of the reserve, to be able to move through all the zones not belonging to the reserve without crossing it. Indeed, some species and/or their habitats can be very sensitive to human presence. This is the case, for example, for plant species that are damaged by trampling (*e.g.*, plants to stabilize dunes), animals whose normal behaviour is easily

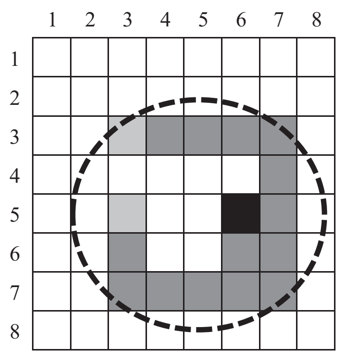

FIG. 4.8 – The candidate zones form a grid of dimension 8 × 8. Each zone is a square whose side length is equal to one unit. Among the 64 candidate zones, 17 are selected and form a connected reserve. The structural distance between the 2 zones z_{33} and z_{53} is equal to 2 units while the functional distance between these two zones is equal to 14 units.

disturbed, or species that are particularly affected by introduced diseases or invasive species. The zones that do not belong to the reserve are made up of unselected candidate zones to which one or more zones representing the territory located outside the candidate zones are added. It is therefore necessary that this set of zones that do not belong to the reserve be connected. It is assumed here that off-reserve movements, such as movements within the reserve, can only be made by gradually moving from one zone to an adjacent one. Consider figure 4.9 where the candidate zones are represented by a grid of 64 square and identical zones. The selected reserve contains 18 grey zones. Non-reserve zones are those zones of the grid that are not selected, to which a zone representing the outside of the grid is added. Zones of the grid that touch the outside of the grid are considered to be adjacent to the zone representing the outside of the grid. The reserve shown in this figure is not an admissible reserve because it is impossible, for example, to join the two zones marked with a "x" without crossing the reserve.

On the other hand, the reserve shown in figure 4.10 is admissible since it is possible to move to all the zones outside the reserve – including the zone outside the grid – without crossing the reserve.

Some definitions of graph theory (see appendix at the end of the book). Let $G = (V, E)$ be a connected graph where $V = \{v_1, v_2, \ldots, v_n\}$ is the set of vertices and E is the set of edges. Denote by d_{ij} the length of the minimal length chain connecting vertices v_i and v_j. The eccentricity of vertex v_i, $\text{ecc}(v_i)$, is equal to the quantity $\max_{j:v_j \in V} d_{ij}$. The centre of G is, by definition, a vertex of minimal eccentricity and its eccentricity is called the radius of the graph. In other words, the radius is the smallest possible value that satisfies the following property: the distance between a vertex – to be determined – and any other vertex is less than or equal to this value. A connected graph has one or more centres. The graph in figure 4.11 includes 3 centres, v_2, v_5, and v_8.

A tree is a connected graph without cycles. To work on a tree, it can be interesting to particularize one of its vertices to make it a root. For tree $\mathcal{A}$ of root r, the father of vertex v is the vertex adjacent to v and belonging to the chain connecting

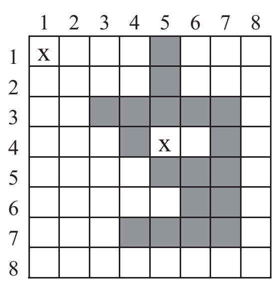

FIG. 4.9 – The 64 candidate zones form a grid of size 8 × 8. 18 zones are selected to form a connected reserve. This reserve is not admissible because the zones that are not affected to the reserve are not all connected.

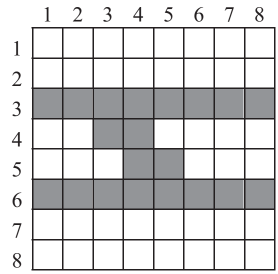

FIG. 4.10 – The 64 candidate zones form a grid of size 8 × 8. 20 zones are selected to form a connected reserve. This reserve is admissible because the zones that are not affected to the reserve are all connected – possibly through the zone outside the grid.

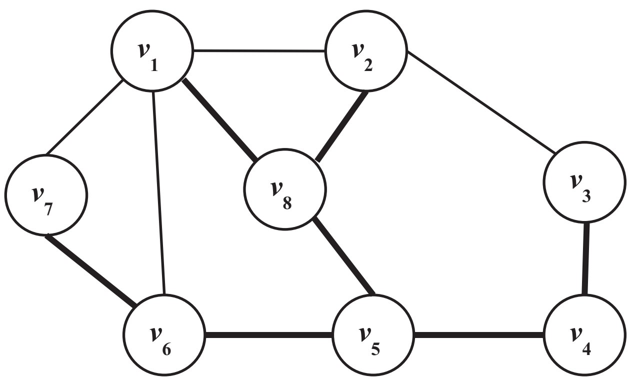

FIG. 4.11 – A connected graph, of radius 2, whose centres are v_2, v_5, and v_8. The edges drawn in bold define a spanning tree of height 2 if v_5 is selected as the root of this tree.

the root to v. Root r is the only vertex of $\mathcal{A}$ without a father. The sons of a vertex v are the vertices adjacent to v that are not the father of v. A leaf of $\mathcal{A}$ is a vertex without sons and its degree is therefore equal to 1. The height of $\mathcal{A}$, that we denote by $h(\mathcal{A})$, is the length of the chain of maximal length that connects the root to a leaf. If all the edges of $\mathcal{A}$ are transformed into arcs oriented from the chosen root, r, towards the leaves, we obtain an arborescence. A spanning tree of $G = (V, E)$, is a tree whose all edges belong to G, and which connects – covers or spans – all the vertices of G. A spanning tree of G is therefore a tree, $\mathcal{A} = (V, E_{\mathcal{A}})$, such that $E_{\mathcal{A}} \subseteq E$. An induced sub-graph of G is a sub-graph of G defined by a subset of vertices of V. More precisely, H is an induced sub-graph of G if, for any pair of vertices $\{v_i, v_j\}$ of H, v_i is connected to v_j by an edge of H if and only if v_i is connected to v_j by an edge of G.

The above definitions allow us to state Property 4.1 below that we use in the formulation of the problem.

Property 4.1. The radius of a connected graph G is less than or equal to ρ if and only if G admits a spanning tree with a height less than or equal to ρ.

Proof. If G admits a spanning tree of height less than or equal to ρ, then the eccentricity of its root, in G, is less than or equal to ρ, and the radius of G is, therefore, itself less than or equal to ρ. Conversely, if the radius of G is less than or equal to ρ, the graph composed of the shortest chains connecting a centre of G to all the other vertices of G is, by definition, a spanning tree with a height less than or equal to ρ.

Expression of the problem in terms of graphs. Let us associate to the set of candidate zones $Z=\{z_1, z_2,\ldots, z_n\}$ a non-oriented graph whose vertices are $\underline{Z} = \{1, 2, \ldots, n\}$ and such that there is an edge between vertex i and vertex j if two zones z_i and z_j are adjacent. Defining a reserve whose compactness is less than or equal to ρ consists in selecting a subset of zones, *i.e.*, a subset of vertices of the graph associated with the candidate zones, such that the sub-graph induced by this subset is connected and with a radius less than or equal to ρ. With each vertex of the graph – candidate zone – is associated a cost and the cost of a sub-graph is equal, by definition, to the sum of the costs of its vertices.

The problem can then be formulated as follows: determine a subset of vertices, with a cost less than or equal to the available budget, B, which induces a connected sub-graph of radius less than or equal to ρ and which allows the greatest possible number of species to be protected. Using property 4.1 above, the problem can be reformulated as follows: determine a subset of vertices, of cost less than or equal to the available budget, B, which admits a spanning tree with a height less than or equal to ρ, and which allows the greatest possible number of species to be protected. The consideration of the connectivity constraint for zones not belonging to the reserve is considered later in section 4.5.2.

4.5.1 Mathematical Programming Formulation

The Boolean variables t_{ih}, $i \in \underline{Z}, h = 1,\ldots, \rho+1$, are used, which take the value 1 if and only if zone z_i is selected and assigned to level h of the searched spanning tree. Level 1 corresponds to the root of the tree and the vertices of level $h+1$ are connected to the root by a chain of length h. Thus, $t_{ih} = 1$ implies that there is a chain, of length less than or equal to $h-1$, from z_i to the root and passing only through the selected vertices. We also use the Boolean variables y_k, $k \in \underline{S}$, which take the value 1 if and only if species s_k is protected by the selected reserve. In other words, and taking into account the conditions for a species to be protected, $y_k = 1$ if and only if the total population size of species s_k present in the zones selected to form the reserve is greater than or equal to the threshold value, θ_k. To simplify the presentation, we also use the working Boolean variables x_i which can be simply expressed as a function of variables t_{ih} and which take the value 1 if and only if zone

z_i is affected to the reserve. We can now formulate the problem as the linear program in Boolean variables $\mathrm{P}_{4.6}$.

$$
\mathrm{P}_{4.6}: \begin{cases} \max \sum_{k \in \underline{S}} w_k y_k & \\ \text{s.t.} \left| \begin{array}{lll} x_i = \sum_{h=1}^{\rho+1} t_{ih} & i \in \underline{Z} & (4.6.1) \\ \theta_k y_k \leq \sum_{i \in \underline{Z}} n_{ik} x_i & k \in \underline{S} & (4.6.2) \\ \sum_{i \in \underline{Z}} t_{i1} = 1 & & (4.6.3) \\ t_{ih} \leq \sum_{j \in \mathrm{Adj}_i} t_{j,h-1} & i \in \underline{Z}, h = 2,\ldots,\rho+1 & (4.6.4) \\ \sum_{i \in \underline{Z}} c_i x_i \leq B & & (4.6.5) \\ x_i \in \{0,1\} & i \in \underline{Z} & (4.6.6) \\ t_{ih} \in \{0,1\} & i \in \underline{Z},\ h = 1,\ldots,\rho+1 & (4.6.7) \\ y_k \in \{0,1\} & k \in \underline{S} & (4.6.8) \end{array} \right. \end{cases}
$$

The economic function measures the weighted number of protected species – w_k refers to the weight associated with species s_k. Indeed, due to constraints 4.6.2, the Boolean variable y_k is necessarily equal to 0 if the total population size of species s_k in the reserve is lower than the threshold value, θ_k. Otherwise, it takes the value 1 – at the optimum of $\mathrm{P}_{4.6}$ – since the aim is to maximize the value of the economic function. Constraints 4.6.1 express variables x_i as a function of variables t_{ih}: $x_i = 1$ if and only if zone z_i is assigned to one (and only one) of the $\rho + 1$ levels of the tree. Constraint 4.6.3 corresponds to the choice of the root vertex: the root must be chosen in one and only one vertex. According to constraints 4.6.4, if zone z_i is selected and assigned to level h of the tree, then at least one of its adjacent zones must be selected and assigned to level h–1. Adj_i refers to all the indices of the zones adjacent to zone z_i. Constraint 4.6.5 expresses the budgetary constraint. Finally, constraints 4.6.6–4.6.8 specify the Boolean nature of all variables in program $\mathrm{P}_{4.6}$.

4.5.2 Connectivity of Zones Outside the Reserve

To determine an optimal reserve that takes this constraint into account, we can proceed as follows: (1) solve $\mathrm{P}_{4.6}$ without taking it into account, (2) if the constraint is satisfied by the solution obtained, then this solution is the optimal solution to the problem, (3) if not, solve $\mathrm{P}_{4.6}$ again, but with an additional constraint to prohibit the obtained configuration. The process is repeated until an admissible reserve is obtained. Computational experiments have shown that, at least under our experimental conditions, an admissible reserve is obtained directly – *i.e.*, by solving $\mathrm{P}_{4.6}$

once – in more than one case out of three and that if this is not the case, a few iterations are sufficient to obtain an admissible solution. Let us look more precisely at a way of implementing point (3). Given a reserve, a set of "isolated" zones is defined as a set of candidate zones, not belonging to the reserve, in one piece – connected – that cannot be reached from outside the grid without crossing the reserve and that is maximal in the inclusion sense. A set of "isolating" zones is associated with any set of isolated zones as follows: any reserve containing all the zones of the set of isolating zones is admissible only if it contains all the zones of the set of isolated zones. In fact, we associate to any subset of isolated zones a set of isolating zones, minimal in the inclusion sense. If Z^a denotes a set of isolated zones and Z^b, the set of isolating zones associated with Z^a, then any admissible reserve must satisfy constraint $\mathrm{C}_{4.11}$. It should be noted that the reserve that had been obtained is no longer admissible.

$$\mathrm{C}_{4.11}: \sum_{i\,:\,z_i \in Z^a} x_i \geq |Z^a| \left(1 - \sum_{i:\,z_i \in Z^b} (1 - x_i)\right).$$

Indeed, if all the zones of the set of isolating zones are selected, constraint $\mathrm{C}_{4.11}$ becomes $\sum_{i\,:\,z_i \in Z^a} x_i \geq |Z^a|$ and imposes that all the zones of the set of isolated zones are selected in the reserve. On the contrary, if $q\ (\geq 1)$ zones of the set of isolating zones are not selected, this constraint becomes $\sum_{i\,:\,z_i \in Z^a} x_i \geq |Z^a|\ (1 - q)$ and is then inactive – always satisfied.

In summary, if the solution obtained by $\mathrm{P}_{4.6}$ corresponds to a reserve with at least one set of isolated zones, it is necessary to add constraint $\mathrm{C}_{4.11}$ to the program and solve it again. The process must then be iterated – keeping the constraints already added – until a reserve is obtained without a set of isolated zones. Let us again take the example of figure 4.9 and assume that the resolution of $\mathrm{P}_{4.6}$ results in the reserve of this figure. It is then necessary to add to $\mathrm{P}_{4.6}$ the constraint $x_{45} + x_{46} \geq 2\,(1 + x_{35} + x_{36} + x_{44} + x_{47} + x_{55} + x_{56} - 6)$.

4.5.3 Computational Experiments

In order to test the effectiveness of the approach, we considered different instances of the problem and solved them with the mathematical program $\mathrm{P}_{4.6}$. We considered hypothetical instances constructed from a set of zones forming a grid of 20×20 identical square zones whose length of sides is equal to one unit, and 100 hypothetical species, which are divided into 3 groups whose weight in the economic function is equal to 10, 5, and 1, respectively.

- Group I (species numbered 1 to 20): These species are rare; they are present in only 10% of the candidate zones and their presence is randomly selected.
- Group II (species numbered 21 to 50): These species are relatively rare; they are present in only 20% of the candidate zones and their presence is randomly selected.

- Group III (species numbered 51 to 100): These species are relatively common; they are present in 30% of the candidate zones and their presence is randomly selected.

For each species present in a zone, its population size in that zone is chosen at random according to the uniform law, between 5 and 10 units. In order to simplify the presentation, the minimal size of the total population necessary for the survival of each of the 100 species considered is set to 25. The distance between two adjacent zones is equal to the distance between their centres, *i.e.*, one unit. The costs associated with each zone are randomly selected, according to the uniform law, between 5 and 20 units. With regard to compactness, 5 values of ρ are considered, 4, 5, 6, 7, and 8, and for each of these values, 3 values of the available budget, B, are considered, 150, 300, and 450. We also randomly select 3 zones that must necessarily belong to the reserve and, on the contrary, 20 zones that cannot be included in it. With these values, the reserves obtained allow for the protection of 0–100 species and the value of the economic function – the weighted number of protected species – varies from 0 to 400. The computation results are presented in table 4.6. All the instances considered could be solved. When $\rho = 3$, there are no admissible reserves, regardless of the value of B considered, and the resolution of $P_{4.6}$ for these 3 values of B requires less than one second of computation. As expected, the resolution of $P_{4.6}$ is very fast for small radius values, ρ, and slower for large values. Indeed, the number of Boolean variables t_{ijh} – associated with zone z_{ij} and level h – increases rapidly with the value of ρ; in our experiments, this number is equal to $400(\rho + 1)$. Several hours of computation are required to solve the problem when $\rho = 8$ and $B = 300$. It can also be seen that, for any fixed value of the radius, the CPU time increases with the budget up to a certain value and then decreases. The resolution of program $P_{4.6}$ provides, for 9 instances out of 15, a reserve with at least one enclave – zones outside the reserve and isolated. The results presented in table 4.6 show that, for these 9 instances, only a few iterations are sufficient to determine a reserve without an enclave. They also show that taking into account the connectivity constraint for the zones outside the reserve deteriorates only slightly the value of the economic function. Figure 4.12a shows the reserve obtained by solving $P_{4.6}$ with $\rho = 6$ and $B = 300$ without taking into account the connectivity constraint for the zones outside the reserve. We see that zone z_{66} is isolated. Figure 4.12b shows the optimal reserve without an enclave. Respecting the "no enclave" constraint does not significantly penalize the value of the solution: it decreases by only 5 units out of 310 – less than 2%. On the other hand, the structure of the reserve has been profoundly modified. A table such as table 4.6 can help a decision-maker to choose the level of compactness of the envisaged reserve. In this example, if he/she can only use a budget of 150 units, he/she can afford to look for a very compact reserve. Indeed, in this case, the compactness constraint does not influence the optimal value of the weighted number of protected species. In contrast, if he/she has a larger budget, such as 300 units, he/she must deal with a compromise between the compactness and the weighted number of protected species.

TAB. 4.6 – Results obtained by solving program $P_{4.6}$ for hypothetical instances, constructed from a grid 20 × 20 with 100 hypothetical species, when $\theta_k = 25$ for all $k \in \underline{S}$.

ρ	B	Number of protected species in each group (I, II, III)	Economic function value	CPU time (s)	Presence of enclaves in the obtained reserve	Number of iterations to obtain a reserve without enclaves	Final value of the economic function	Additional CPU time (s)
4	150	1, 11, 37	102	1	No	–	–	–
	300	7, 25, 50	245	1	Yes	4	235	3
	450	8, 30, 50	280	1	Yes	2	270	1
5	150	1, 11, 37	102	3	No	–	–	–
	300	10, 28, 50	290	11	Yes	1	290	4
	450	16, 30, 50	360	1	Yes	3	350	3
6	150	1, 11, 37	102	8	No	–	–	–
	300	11, 30, 50	310	174	Yes	2	305	292
	450	20, 30, 50	400	3	Yes	5	390	28
7	150	1, 11, 37	102	24	No	–	–	–
	300	13, 28, 50	320	1,046	Yes	1	320	881
	450	20, 30, 50	400	3	Yes	2	400	7
8	150	1, 11, 37	102	207	No	–	–	–
	300	13, 30, 50	330	10,267	Yes	3	325	27,292
	450	20, 30, 50	400	5	No	–	–	–

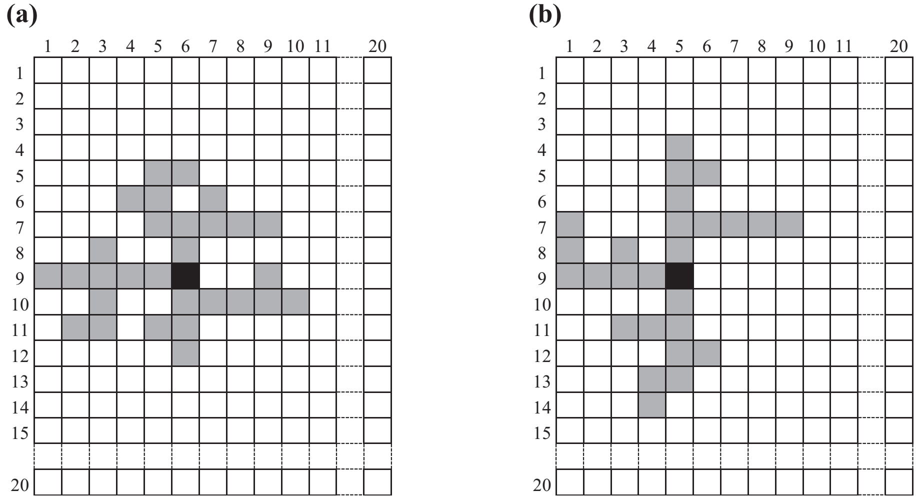

FIG. 4.12 – Two optimal connected reserves, with a radius of 6 or less and a cost of 300 or less. (a) The centre of the reserve is zone z_{96}. 11 species in Group I and all the species in Groups II and III are protected. The value of the solution is 310 but zone z_{66}, which does not belong to the reserve, is isolated by zones z_{56}, z_{67}, z_{76}, and z_{65}. (b) The centre of the reserve is zone z_{95}. 11 species in Group I and all the species in Groups II and III, except one species in Group II, are protected. The value of the solution is 305 and there is no enclave in this reserve.

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