Chapter 2

Fragmentation

2.1 Introduction

The spatial configuration of a nature reserve plays an important role in the survival of the species that live there. In this chapter, we are interested in the fragmentation of a reserve, *i.e.*, the dispersion of the patches – or zones – that compose it, in relation to each other (see figure 2.1). This phenomenon, which is often associated with the decrease in the area of various patches, is considered to be one of the main causes of biodiversity loss. The fragmentation of a reserve is indeed one of the main factors preventing species from moving around the reserve as they should and could in a non-fragmented one. This habitat fragmentation, therefore, significantly increases the extinction risk of many species. It can be natural but more often results from a fragmentation of the space due to artificial phenomena such as the presence of urbanized zones, intensive agricultural zones or transport infrastructures. It should be noted that species are affected differently by habitat fragmentation. A reserve may appear to be very fragmented for some species, those that will have great difficulty moving from one patch to another, and not very fragmented for others, those that, despite some distance between patches, will still be able to travel most of these patches due, for example, to their ability to fly or cross obstacles such as roads or zones treated with pesticides. Fragmentation is also a handicap in terms of species' adaptation to climate change. It should be noted, however, that the ease of movement of species within a reserve is not always without its drawbacks: it can increase the risk of disease transmission between wildlife species in the reserve and also the transmission of these diseases to domestic species. It can also facilitate the proliferation of invasive species, a phenomenon currently considered to be one of the major causes of biodiversity loss. There has been much debate about the desirable size of protected zones: is it more interesting to have a single large protected zone or several small ones with the same total size – SLOSS: Single Large Or Several Small. This debate focuses mainly on ecological aspects, but it is worth noting that the

DOI: 10.1051/978-2-7598-2521-9.c002

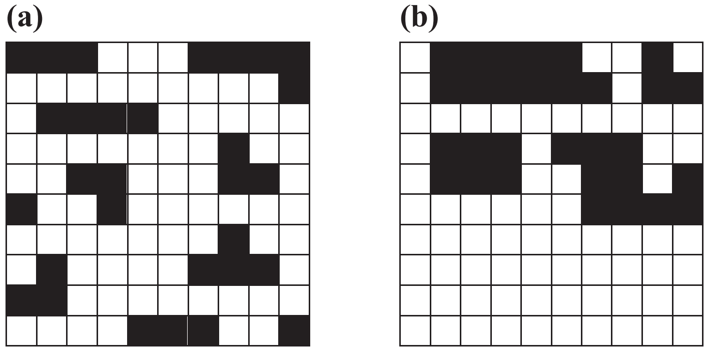

FIG. 2.1 – A hypothetical landscape represented by a grid of square and identical cells. Two reserves – in black – with a total area of 30 units. (a) A highly fragmented reserve. (b) A less fragmented reserve.

management of a fragmented set of zones is generally more difficult and costly than the management of a non-fragmented set.

Given a set of zones spread over a territory and such that any two zones have no common parts, many indicators of fragmentation can be associated with this set. We will examine, for example, the following indicators: the Mean Nearest Neighbour Distance (MNND), the Mean Shape Index (MSI), and the Mean Proximity Index (MPI).

The problem related to the notion of fragmentation, which naturally arises in the presence of a set of zones – without common parts – that can be protected, consists in selecting, among these zones and under certain constraints, a subset of zones to be protected that is optimal with regard to these indicators or that respects some of values of them.

2.2 The Indicators MNND (Mean Nearest Neighbour Distance), MSI (Mean Shape Index) and MPI (Mean Proximity Index)

First, let us look at the MNND indicator associated with a reserve, R, *i.e.*, a subset of zones of $Z = \{z_1, z_2,\ldots, z_n\}$. Let us denote by d_{ij} the distance between zones z_i and z_j. Here, it is the straight line distance between the two zones. More precisely, d_{ij} is defined as the shortest distance that can be found between a point in zone z_i and a point in zone z_j. The distance between two zones could very well be defined differently, taking into account, for example, the difficulty for the species under consideration to move from one zone to another. One could thus take into account the obstacles to be overcome or the inhospitable nature of the areas to be crossed, *i.e.*, the surrounding matrix and not only the distance to be covered. For each zone z_i of

R, we are interested in the distance between this zone and its nearest neighbour belonging to R. The index corresponding to this nearest neighbour is equal to $\min_{j \in \underline{R}, j \neq i} d_{ij}$ where $\underline{R}$ designates the set of indices of the zones of R. The MNND indicator associated with a reserve, R, can therefore be formulated as follows:

$$\text{MNND}(R) = \frac{1}{|R|} \sum_{i \in \underline{R}} \min_{j \in \underline{R}, j \neq i} d_{ij}.$$

The indicator MNND applied to reserve R concerns all the zones of R and is equal to the average of the distances between each zone of R and the zone closest to it. The dimension of MNND is a length. If the zones closest to each zone are further away, then MNND increases and the "inter-zone" movements of the different species concerned become more difficult. Low values of $\text{MNND}(R)$ correspond to a larger grouping of zones of R. We assume, for the definition of $\text{MNND}(R)$, that there are at least two zones in reserve R.

Let us now look at the indicator MSI. It reflects a relationship between the perimeter of a zone and its area. More precisely, for each zone of the set R considered, we use the ratio between the perimeter of this zone and the square root of its area, all this multiplied by the coefficient 0.25. The value of the indicator MSI associated with a reserve, R, is then equal to the average of these values over all the zones of R. By noting, respectively, l_i and a_i the perimeter and the area of zone z_i, the indicator MSI associated with R is written

$$\text{MSI}(R) = \frac{1}{|R|} \sum_{i \in \underline{R}} \frac{0.25\, l_i}{\sqrt{a_i}}.$$

For example, the value of this indicator is 0.89 for a circular zone, 1 for a square zone and 1.74 for a rectangular zone ten times longer than wide. MSI is dimensionless and minimal when all the zones have regular contours – circles. MSI increases with the irregularity of the contours of the zones.

Let us now look at the indicator MPI. Although the indicator MNND is useful for assessing the isolation of zones, considering only the zone closest to a given zone may not adequately represent the ecological neighbourhood of the zone under consideration. To remedy this weakness, we can consider the mean proximity index, MPI. This index takes into account both the proximity and the area of zones whose distance to a given zone is less than or equal to a certain value, d. The contribution of each zone to this index is calculated by summing, over all the zones within a given radius, the area of the zone divided by the square of the distance from the zone under consideration. The value of the index associated with a subset, R, of Z is then equal to the average of the values obtained for each zone of R. We obtain

$$\text{MPI}\,(R, d) = \frac{1}{|R|} \sum_{i \in \underline{R}} \sum_{j \in I_i(R,d)} \frac{a_j}{d_{ij}^2},$$

where $I_i(R, d) = \{j \in \underline{R} : j \neq i, d_{ij} \leq d\}$. The contribution of a zone of R that does not have neighbouring zones – belonging to R – located at a distance less than or equal to the threshold distance, d, is equal to 0. $\text{MPI}(R, d)$ is dimensionless and

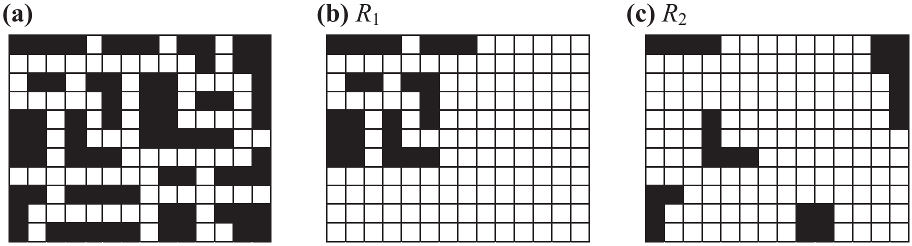

FIG. 2.2 – (a) The hypothetical landscape is represented by a grid of square and identical cells; 17 zones (in black) are candidates for protection. Two examples of reserves with the same area, R_1 and R_2, built from these zones: (b) MNND(R_1) = 1, MSI(R_1) = 1.18, MPI (R_1,2) = 11.5; (c) MNND(R_2) = 2.73, MSI(R_2) = 1.23, MPI(R_2,2) = 0.9.

increases with the size and proximity of the surrounding zones. This indicator measures the relative isolation of zones within a landscape.

Figure 2.2 illustrates the calculation of the 3 indicators MNND, MSI, and MPI on a small instance with 17 candidate zones.

2.3 Reserve Minimizing the Indicator MNND

With regard to the indicator MNND, the basic problem is to select, under certain constraints, a subset of zones that minimizes this indicator. Consider, for example, the problem of selecting, under a budgetary constraint, a subset of zones, $R \subseteq Z$, which allows to protect, at a minimum, a certain number, Ns, of species and which minimizes MNND. The set of species considered is $S = \{s_1, s_2, \ldots, s_m\}$. Let us situate ourselves in the case where the number of species protected by a reserve, R, is estimated by the quantity $\mathrm{Nb}_1(R)$ (see chapter 1, section 1.1). Recall that, in the calculation of $\mathrm{Nb}_1(R)$, it is assumed that the protection of a zone allows all the species present in that zone to be protected, provided that their population size is greater than or equal to a certain threshold value. We note Z_k the set of zones whose protection results in the protection of species s_k and $\underline{Z}_k$ the corresponding set of indices. We assume that we know the set Z_k for all $k \in \underline{S} = \{1, 2, \ldots, m\}$. Let us adopt the following notations: $\underline{Z} = \{1, \ldots, n\}$, $I_i = \{j \in \underline{Z} : j \neq i\}$ for all $i \in \underline{Z}$ and, for all vector x of $\{0,1\}^n$, $I(x) = \{i \in \underline{Z} : x_i = 1\}$, and $I_i(x) = \{j \in \underline{Z} : j \neq i,\ x_j = 1\}$ for all $i \in \underline{Z}$. Note that if x is the characteristic vector of reserve R ($x_i = 1 \Leftrightarrow z_i \in R$) then $I(x) = \underline{R}$ and $I_i(x) = \underline{R} - \{i\}$. The problem considered can be formulated as the fractional mathematical program in Boolean variables $\mathrm{P}_{2.1}$ (see appendix at the end of the book).

$$\mathrm{P}_{2.1}: \begin{cases} \min \sum_{i \in I(x)} \min_{j \in I_i(x)} d_{ij} \Big/ \sum_{i \in \underline{Z}} x_i \\ \text{s.t.} \left| \begin{array}{llcl} \sum_{i \in \underline{Z}} c_i x_i \leq B & (2.1.1) & | & x_i \in \{0,1\} \quad i \in \underline{Z} \quad (2.1.4) \\ \sum_{k \in \underline{S}} y_k \geq \mathrm{Ns} & (2.1.2) & | & y_k \in \{0,1\} \quad k \in \underline{S} \quad (2.1.5) \\ y_k \leq \sum_{i \in \underline{Z}_k} x_i \quad k \in \underline{S} & (2.1.3) & | & \end{array} \right. \end{cases}$$

This program consists in determining the values of variables x_i and y_k that respect constraints 2.1.1–2.1.5 and that minimize an economic function expressed as a fraction whose denominator is a linear function. We will see how to also express the numerator of this fraction by a linear function in order to finally obtain an economic function expressed as the ratio of two linear functions. Lemma 2.1 below shows how to express, for any vector x of $\{0,1\}^n$, the value of the expression $\sum_{i\in I(x)} \min_{j\in I_i(x)} d_{ij}$ as the optimal value of an integer linear program including the decision variables x_i, $i \in \{1,\ldots,n\}$, and the additional Boolean "working" variables t_{ij}, $(i,j) \in \underline{Z}^2, i \neq j$. By definition, variable t_{ij} is equal to 1 if and only if, on the one hand, zones z_i and z_j are selected and, on the other hand, zone z_j is, among the selected zones, the one closest to z_i.

Lemma 2.1. For all vector x of $\{0,1\}^n$,

$$\sum_{i\in I(x)} \min_{j\in I_i(x)} d_{ij} = \min\Bigg\{ \sum_{\substack{(i,j)\in \underline{Z}^2 \\ i\neq j}} d_{ij}t_{ij} : t \in \{0,1\}^{n\times n}, \sum_{j\in I_i} t_{ij} = x_i, t_{ij} \leq x_j \ ((i,j)\in \underline{Z}^2, i\neq j)\Bigg\}.$$

Proof.

$$\begin{aligned}
\sum_{i\in I(x)} \min_{j\in I_i(x)} d_{ij} &= \sum_{i\in \underline{Z}} x_i \min_{j\in I_i(x)} d_{ij} \\
&= \sum_{i\in \underline{Z}} x_i \min\Bigg\{\sum_{j\in I_i} d_{ij}t_{ij} : t\in\{0,1\}^{n\times n}, \sum_{j\in I_i} t_{ij} = 1, t_{ij}\leq x_j \ (j\in I_i)\Bigg\} \\
&= \sum_{i\in \underline{Z}} \min\Bigg\{\sum_{j\in I_i} d_{ij}t_{ij} : t\in\{0,1\}^{n\times n}, \sum_{j\in I_i} t_{ij} = x_i, t_{ij}\leq x_j \ (j\in I_i)\Bigg\} \\
&= \min\Bigg\{\sum_{(i,j)\in \underline{Z}^2, i\neq j} d_{ij}t_{ij} : t\in\{0,1\}^{n\times n}, \sum_{j\in I_i} t_{ij} = x_i \ (i\in \underline{Z}), t_{ij}\leq x_j \ ((i,j)\in\underline{Z}^2, i\neq j)\Bigg\}.
\end{aligned}$$

Lemma 2.1 allows program $P_{2.1}$ to be rewritten as program $P_{2.2}$.

$$
\mathrm{P}_{2.2}\ :\ \begin{cases} \min \displaystyle\sum_{(i,j)\in \underline{Z}^2,\ i\neq j} d_{ij}t_{ij} \Big/ \sum_{i\in \underline{Z}} x_i & \\ \text{s.t.} \left| \begin{array}{lll} \displaystyle\sum_{j:(i,j)\in \underline{Z}^2,\ i\neq j} t_{ij} = x_i & i\in \underline{Z} & (2.2.1) \\ t_{ij} \leq x_j & (i,j)\in Z^2, i\neq j & (2.2.2) \\ \displaystyle\sum_{i\in \underline{Z}} c_i x_i \leq B & & (2.2.3) \\ \displaystyle\sum_{k\in \underline{S}} y_k \geq \mathrm{Ns} & & (2.2.4) \\ y_k \leq \displaystyle\sum_{i\in \underline{Z}_k} x_i & k\in \underline{S} & (2.2.5) \\ x_i \in \{0,1\} & i\in \underline{Z} & (2.2.6) \\ y_k \in \{0,1\} & k\in \underline{S} & (2.2.7) \\ t_{ij} \in \{0,1\} & (i,j)\in Z^2, i\neq j & (2.2.8) \end{array} \right. \end{cases}
$$

Program $\mathrm{P}_{2.2}$ consists of minimizing the ratio of two linear functions whose variables are subject to linear constraints. This problem can be solved using the algorithms of fractional programming, for example the Dinkelbach algorithm (see appendix at the end of the book). In this case, the auxiliary problem associated with the – combinatorial – fractional program $\mathrm{P}_{2.2}$ consists in minimizing the linear function, parameterized by the scalar λ, $\sum_{(i,j)\in Z^2,\, i\neq j} d_{ij}t_{ij} - \lambda \sum_{i\in \underline{Z}} x_i$, under the same constraints as those of program $\mathrm{P}_{2.2}$.This auxiliary problem is a linear program in Boolean variables.

2.4 Examples of Reserves Minimizing the Indicator MNND

Consider a set of 20 rectangular zones spread over a 15 km square territory (figure 2.3). The total area of these 20 zones is 79 km^2 and the value of the indicator MNND for these 20 zones is 1.05 km.

We are interested in 10 species and, for each of the zones, we know all the species that live there in sufficient numbers to ensure that the protection of the zone will lead to the protection of this set of species. We also know the cost associated with protecting each zone. This information is summarized in table 2.1. We are looking for a subset of zones, R, which minimizes MNND(R), which protects, at a minimum, a fixed number of species, Ns, and whose cost is less than or equal to the available budget, B. The results obtained by solving program $\mathrm{P}_{2.2}$ are presented in table 2.2 for different values of B and Ns.

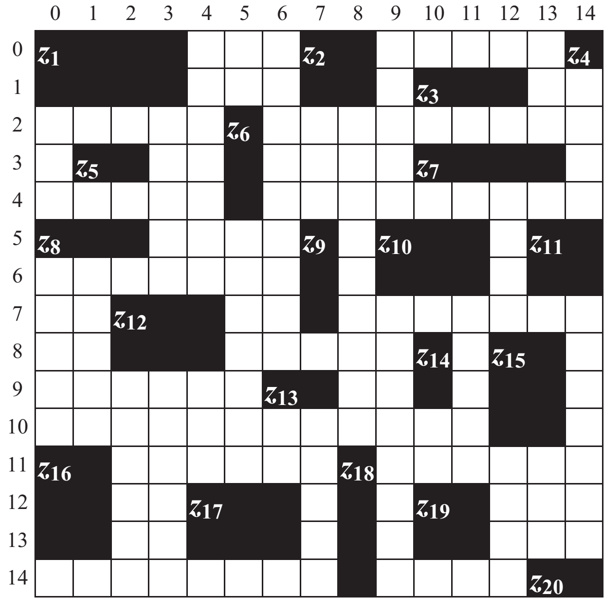

FIG. 2.3 – A set of 20 rectangular zones, z_1, z_2,..., z_{20}, distributed over a 15 km square territory represented by a grid of 15 × 15 identical square cells whose area is equal to 1 km^2. The total area of these 20 zones is 79 km^2 and the value of the indicator MNND for these 20 zones is 1.05 km.

TAB. 2.1 – Cost associated with protecting each zone of figure 2.3 and list of the species living in each of these zones in sufficient numbers.

Zone	Cost	Species living in the zone	Zone	Cost	Species living in the zone
z_1	2	s_5	z_{11}	3	s_4
z_2	2	s_5	z_{12}	3	s_9 s_{10}
z_3	1	s_6	z_{13}	4	s_4
z_4	5	s_7	z_{14}	3	s_4
z_5	1	s_1 s_8	z_{15}	4	s_1 s_4
z_6	2	s_2	z_{16}	3	s_{10}
z_7	2	s_9	z_{17}	3	s_1 s_2
z_8	4	s_1 s_3	z_{18}	2	s_4
z_9	5	s_1	z_{19}	4	s_5
z_{10}	5	s_1 s_4	z_{20}	1	s_8

TAB. 2.2 – Results corresponding to the minimization of the indicator MNND for the instance described in figure 2.3 and table 2.1, for different values of the minimal number of species to be protected, Ns, and the available budget, B.

Number of species to be protected (Ns)	B	Protected species	Budget used	Zones selected	Protected area in % of the initial area	MNND(R) (km)
5	5	s_1 s_6 s_8 s_9 s_{10}	5	z_3 z_5 z_{12}	13.92	4.36
5	8	s_1 s_3 s_8 s_9 s_{10}	8	z_5 z_8 z_{12}	13.92	1.00
7	9	s_1 s_2 s_5 s_6 s_8 s_9 s_{10}	9	z_2 z_3 z_5 z_6 z_{12}	22.78	1.40
7	12	s_1 s_2 s_3 s_5 s_6 s_8 s_9	12	z_2 z_3 z_5 z_6 z_7 z_8	24.05	1.00
8	10	–	–	–	–	–
8	12	s_1 s_2 s_4 s_5 s_6 s_8 s_9 s_{10}	12	z_2 z_3 z_5 z_6 z_{11} z_{12}	27.85	1.67
10	20	all	20	z_1 z_3 z_4 z_5 z_6 z_8 z_{12} z_{18}	37.97	1.33
10	21	all	21	z_1 z_3 z_4 z_5 z_8 z_{12} z_{17} z_{18}	41.77	1.00

– No solution.

2.5 Reserve Minimizing the Indicator MNND with a Constraint on the Indicator MSI

Several optimization problems can be considered with regard to the indicator MSI. For example, we consider the following problem: determine, under a budgetary constraint, a subset of zones that can protect at least a certain number of species, Ns, whose MSI value is less than or equal to a given value, $\mathrm{MSI}_{\max}$, and which minimizes the value of the indicator MNND. As in sections 2.3 and 2.4, the number of species protected by a reserve, R, is estimated by $\mathrm{Nb}_1(R)$. This optimization problem can be formulated as the fractional combinatorial program $\mathrm{P}_{2.2}$ to which is added the linear constraint $0.25 \sum_{i \in \underline{Z}} (l_i/\sqrt{a_i})\, x_i \leq \mathrm{MSI}_{\max} \times \sum_{i \in \underline{Z}} x_i$. The obtained program can be solved, like $\mathrm{P}_{2.2}$, by the Dinkelbach algorithm. The auxiliary program associated with the fractional program obtained consists in minimizing the parameterized linear function $\sum_{(i,j) \in Z^2,\, i \neq j} d_{ij} y_{ij} - \lambda \sum_{i \in \underline{Z}} x_i$ under the set of constraints of $\mathrm{P}_{2.2}$ plus the constraint on the maximal MSI value.

2.6 Examples of Reserves Minimizing the Indicator MNND with a Constraint on the Indicator MSI

Let us take the same instance as described in section 2.4 and look for a subset of zones, R, of minimal fragmentation, *i.e.,* minimizing $\mathrm{MNND}(R)$, which protects, at a minimum, a fixed number of species, Ns, and whose associated MSI indicator value is less than or equal to a given value, $\mathrm{MSI}_{\max}$. The results obtained are presented in table 2.3.

2.7 Reserve Maximizing the Indicator MPI

Many optimization problems can arise in connection with this indicator. Consider, for example, the following problem: determine, under a budgetary constraint, a set, R, of zones to be protected in order to protect at least Ns species, while maximizing $\mathrm{MPI}(R,\, d)$. As in the previous sections, the number of species protected by reserve R is estimated by $\mathrm{Nb}_1(R)$. To formulate this problem, simply replace the objective of $\mathrm{P}_{2.1}$ by the function $\left(\sum_{i \in I(x)} \sum_{j \in I_i(x,d)} \left(a_j/d_{ij}^2\right)\right) \Big/ \sum_{i \in \underline{Z}} x_i$ to be maximized where, for any i of $\underline{Z}$ and any x of $\{0,1\}^n$, $I_i(x,\, d) = \{j \in \underline{Z} : j \neq i, x_j = 1, d_{ij} \leq d\}$. We will see how to reformulate the program obtained as a fractional combinatorial program consisting in maximizing the ratio of two linear functions under linear constraints.

Lemma 2.2. Program $\mathrm{P}_{2.3}$ is equivalent to the fractional linear program $\mathrm{P}_{2.4}$.

$$\mathrm{P}_{2.3} : \begin{cases} \max \ \left(\sum_{i \in I(x)} \sum_{j \in I_i(x,d)} \left(a_j/d_{ij}^2\right)\right) \Big/ \sum_{i \in \underline{Z}} x_i \\ \text{s.t.}| \ x_i \in \{0,1\} \quad i \in \underline{Z} \quad (2.3.1) \end{cases}$$

TAB. 2.3 – Results associated with the instance described in figure 2.3 and table 2.1: Minimization of the indicator MNND for different values of the minimal number of species to be protected, Ns, and the available budget, B, with a maximal value of the indicator MSI, MSI_{max}.

Number of species to be protected (Ns)	MSI_{max}	B	Protected species	Selected zones	Budget used	Protected area in % of the initial area	MNND(R) (km)	MSI(R)
5	1.02	8	s_1 s_2 s_5 s_9 s_{10}	z_2 z_{12} z_{17}	8	20.25	3.80	1.01
	1.02	12	s_1 s_4 s_5 s_9 s_{10}	z_{12} z_{15} z_{19}	12	20.25	2.61	1.01
	1.50	5	s_1 s_6 s_8 s_9 s_{10}	z_3 z_5 z_{12}	5	13.92	4.36	1.08
	1.50	8	s_1 s_2 s_5 s_6 s_8 s_9	z_1 z_3 z_5 z_6 z_7	8	25.32	1.00	1.14
8	1.02	12	–	–	–	–	–	–
	1.02	17	s_1 s_2 s_4 s_5 s_7 s_8 s_9 s_{10}	z_2 z_4 z_5 z_{11} z_{12} z_{17}	17	29.11	3.52	1.02
	1.50	12	s_1 s_2 s_4 s_5 s_6 s_8 s_9 s_{10}	z_2 z_3 z_5 z_6 z_{11} z_{12}	12	27.85	1.67	1.07
	1.50	15	s_1 s_2 s_3 s_5 s_6 s_8 s_9 s_{10}	z_2 z_3 z_5 z_6 z_8 z_{12}	15	26.58	1.00	1.09

– No solution.

$$\mathrm{P}_{2.4}: \begin{cases} \max \quad \sum_{i \in \underline{Z}} v_i \Big/ \sum_{i \in \underline{Z}} x_i \\ \text{s.t.} \left| \begin{array}{llll} v_i \leq \sum\limits_{j \in I_i(d)} \dfrac{a_j}{d_{ij}^2} x_j & i \in \underline{Z} & (2.4.1) & \mid \quad v_i \geq 0 \qquad i \in \underline{Z} \quad (2.4.3) \\ v_i \leq M_i x_i & i \in \underline{Z} & (2.4.2) & \mid \quad x_i \in \{0,1\} \quad i \in \underline{Z} \quad (2.4.4) \end{array} \right. \end{cases}$$

where M_i is a constant greater than or equal to the value of the expression $\sum_{j \in I_i(d)} (a_j/d_{ij}^2) x_j$ in an optimal solution of $\mathrm{P}_{2.3}$. We can take, for example, $M_i = \sum_{j \in I_i(d)} (a_j/d_{ij}^2)$. By examining successively the two possible values of x_i, it can easily be verified that constraints 2.4.1 and 2.4.2 imply $v_i = x_i \sum_{j \in I_i(d)} \frac{a_j}{d_{ij}^2} x_j$, at the optimum of $\mathrm{P}_{2.4}$. The objective of $\mathrm{P}_{2.4}$ is therefore equivalent to maximizing the expression $\left(\sum_{i \in \underline{Z}} x_i \sum_{j \in I_i(d)} \frac{a_j}{d_{ij}^2} x_j \right) \Big/ \sum_{i \in \underline{Z}} x_i$. This last expression, to be maximized, is a rewriting of the economic function of $\mathrm{P}_{2.3}$, since it is easy to verify that $\sum_{i \in \underline{Z}} x_i \sum_{j \in I_i(d)} \frac{a_j}{d_{ij}^2} x_j = \sum_{i \in I(x)} \sum_{j \in I_i(x,d)} \frac{a_j}{d_{ij}^2}$. $\mathrm{P}_{2.4}$ is therefore equivalent to $\mathrm{P}_{2.3}$.

Finally, the problem considered – determining, taking into account an available budget, B, a set of zones, R, to be protected in order to protect at least Ns species, while maximizing MPI(R, d) – can be formulated as the fractional mathematical program $\mathrm{P}_{2.5}$.

$$\mathrm{P}_{2.5}: \begin{cases} \max \quad \sum_{i \in \underline{Z}} v_i \Big/ \sum_{i \in \underline{Z}} x_i \\ \text{s.t.} \left| \begin{array}{llll} v_i \leq \sum\limits_{j \in I_i(d)} \dfrac{a_j}{d_{ij}^2} x_j & i \in \underline{Z} & (2.5.1) & \mid \quad y_k \leq \sum\limits_{i \in \underline{Z}_k} x_i \qquad k \in \underline{S} \quad (2.5.5) \\ v_i \leq M_i x_i & i \in \underline{Z} & (2.5.2) & \mid \quad v_i \geq 0 \qquad i \in \underline{Z} \quad (2.5.6) \\ \sum\limits_{i \in \underline{Z}} c_i x_i \leq B & & (2.5.3) & \mid \quad x_i \in \{0,1\} \quad i \in \underline{Z} \quad (2.5.7) \\ \sum\limits_{k \in \underline{S}} y_k \geq \mathrm{Ns} & & (2.5.4) & \mid \quad y_k \in \{0,1\} \quad k \in \underline{S} \quad (2.5.8) \end{array} \right. \end{cases}$$

The auxiliary problem associated with $\mathrm{P}_{2.5}$ is to maximize the parameterized linear function $\sum_{i \in \underline{Z}} v_i - \lambda \sum_{i \in \underline{Z}} x_i$ under the same constraints as those of $\mathrm{P}_{2.5}$.

Example 2.1. Consider the instance described in figure 2.3 and table 2.1 and the problem of maximizing the indicator MPI for different values of the threshold distance, d, minimal number of species to be protected, Ns, and available budget, B. The results obtained, by solving program $\mathrm{P}_{2.5}$, are presented in table 2.4.

Tab. 2.4 – Results concerning the maximization of the indicator MPI for the instance described in figure 2.3 and table 2.1, for different values of Ns, d, and B.

Number of species to be protected (Ns)	d (km)	B	Protected species	Selected zones	Used budget	Protected area in % of the initial area	MPI(R, d) (km)
5	4	6	s_1 s_2 s_5 s_6 s_8	z_1 z_3 z_5 z_6	6	20.25	5.66
	4	10	s_1 s_2 s_5 s_6 s_8 s_9	z_1 z_2 z_3 z_5 z_6 z_7	10	30.38	8.23
	6	6	s_1 s_2 s_5 s_6 s_8	z_1 z_3 z_5 z_6	6	20.25	5.73
	6	10	s_1 s_2 s_5 s_6 s_8	z_1 z_2 z_3 z_5 z_6 z_7	10	30.38	8.34
	6	12	s_1 s_3 s_8 s_9 s_{10}	z_1 z_5 z_6 z_8 z_{12}	12	27.85	8.53
8	4	10	–	–	–	–	–
	4	12	s_1 s_2 s_4 s_5 s_6 s_8 s_9 s_{10}	z_1 z_3 z_5 z_6 z_{11} z_{12}	12	32.91	4.42
	6	10	–	–	–	–	–
	6	12	s_1 s_2 s_4 s_5 s_6 s_8 s_9 s_{10}	z_1 z_3 z_5 z_6 z_{11} z_{12}	12	32.91	4.57
	6	15	s_1 s_2 s_3 s_5 s_6 s_8 s_9 s_{10}	z_1 z_2 z_3 z_5 z_6 z_8 z_{12}	15	36.71	8.52

– No solution.

References and Further Reading

Bajalinov E.B. (2003) *Linear-fractional programming theory, methods, applications and software.* Springer-Science+Business Media, B.V.

Battisti C. (2003) Habitat fragmentation, fauna and ecological network planning: toward a theoretical conceptual framework, *Ital. J. Zool.* **70**, 241.

Bennett A.F., Saunders D.A. (2010) Habitat fragmentation and landscape change, *Conservation biology for all* (N.S. Sodhi, P.R. Ehrlich, Eds). Oxford University Press, Ch. 5, pp. 88–106.

Billionnet A. (2010) Optimal selection of forest patches using integer and fractional programming, *Oper. Res.: Int. J.* **10**, 1.

Collinge S.K. (2000) Effects of grassland fragmentation on insect species loss, colonization, and movement patterns, *Ecology* **81**, 2211.

Dinkelbach W. (1967) On nonlinear fractional programming, *Manage. Sci.* **13**, 492.

Fahrig L. (2003) Effects of habitat fragmentation on biodiversity, *Annu. Rev. Ecol. Evol. Syst.* **34**, 487.

Fischer D.T., Church R.L. (2003) Clustering and compactness in reserve site selection: an extension of the biodiversity management area selection model, *Forest Sci.* **49**, 555.

Fischer J., Lindenmayer D.B. (2007) Landscape modification and habitat fragmentation: a synthesis, *Global Ecol. Biogeo.* **16**, 265.

Fletcher R.J. Jr. (2005) Multiple edge effects and their implications in fragmented landscapes, *J. Animal Ecol.* **74**, 342.

Hanski I., Ovaskainen O. (2000) The metapopulation capacity of a fragmented landscape, *Nature* **404**, 755.

Hargis C.D., Bissonette J.A., David J.L. (1998) The behaviour of landscape metrics commonly used in the study of habitat fragmentation, *Landscape Ecol.* **13**, 167.

Hargis C.D., Bissonette J.A., Turner D.L. (1999) The influence of forests fragmentation and landscape pattern on American martens, *J. Appl. Ecol.* **36**, 167.

Harrison S., Bruna E. (1999) Habitat fragmentation and large-scale conservation: what do we know for sure? *Ecography* **22**, 225.

Henle K., Davies K.F., Kleyer M., Margules C., Settele J. (2004) Predictors of species sensitivity to fragmentation, *Biodiver. Conserv.* **13**, 207.

Hilty J.A., Lidicker W.Z. Jr., Merenlender A.M. (2006) *Corridor ecology, the science and practice of linking landscapes for biodiversity conservation*. Island Press.

Lindenmayer D.B., Cunningham R.B., Pope M.L., Donnelly C.F. (1999) A large-scale "experiment" to examine the effects of landscape context and habitat fragmentation on mammals *Biol. Conserv.* **88**, 387.

Lindenmayer D.B., Fischer J. (2006) *Habitat fragmentation and landscape change: an ecological and conservation synthesis*. Island Press.

Marks B.J., McGarigal K. (1994) Fragstats: spatial pattern analysis program for quantifying landscape structure. Technical report, Forest Science Department, Oregon State University.

Moilanen A., Wilson K.A., Possingham H.P., Eds (2009) *Spatial conservation prioritization.* Oxford University Press.

Noss R.F. (1987) Corridors in real landscapes: a reply to Simberloff and Cox, *Conserv. Biol.* **1**, 159.

Ohman K., Lamas T. (2005) Reducing forest fragmentation in long-term forest planning by using the shape index, *Forest Ecol. Manage.* **212**, 346.

Önal H., Briers R.A. (2002) Incorporating spatial criteria in optimum reserve network selection, *Proc. Royal Soc. London B* **269**, 2437.

Radzik T. (1998) Fractional combinatorial optimization, *Handbook of combinatorial optimization* (D.-Z. Du, P.M. Pardalos, Eds), Springer, vol. 1, pp. 429–478.

Rebelo A.G., Siegfried W.R. (1992) Where should nature reserves be located in the Cape Floristic region, South Africa? Models for the spatial configuration of a reserve network aimed at maximizing the protection of floral diversity, *Conserv. Biol.* **6**, 243.

Ricketts T.H. (2001) The matrix matters: effective isolation in fragmented landscapes, *Am. Nat.* **158**, 87.

Saunders D.A., Hobbs R.J., Margules C.R. (1991) Biological consequences of ecosystem fragmentation: a review, *Conserv. Biol.* **5**, 18.

Schaible S. (1995) Fractional programming, *Handbook of global optimization* (R. Horst, P. Pardalos, Eds). Kluwer Academic Publishers, pp. 495–608.

Shafer C.L. (2001) Inter-reserve distance, *Biol. Conserv.* **100**, 215.

Tischendorf L., Fahrig L. (2000) On the usage and measurement of landscape connectivity, *Oikos* **90**, 7.

Vemema H.D. (2005) Forest structure optimization using evolutionary programming and landscape ecology metrics, *Euro. J. Oper. Res.* **164**, 423.

Walters J.R. (1998) The ecological basis of avian sensitivity to habitat fragmentation, *Avian conservation: research and management* (J. Marzluff, R. Sallabanks, Eds), Island Press.

Williams J.C., ReVelle C.S., Levin S.A. (2005) Spatial attributes and reserve design models: a review, *Environ. Model. Assess.* **10**, 163.