Chapter 1

Basic Problem and Variants

1.1 Introduction

We are interested in a group of species, animal or plant, which, for various reasons, are threatened. They may thus disappear in the more or less near future. For example, the IUCN (International Union for Conservation of Nature) Red List provides information on whether or not a given species is threatened. This list classifies threatened species into three categories, according to their level of extinction risk: “Vulnerable”, “Endangered”, and “Critically Endangered”. This classification is made taking into account various factors such as the population size of the species in question, the rate of decline of this population, the loss and/or fragmentation of its habitat or its genetic erosion. It is also possible to look at a set of focal species, threatened or not, as their protection automatically leads to the protection of many other species. It should be noted that many species, although common, are also in decline and this should be reversed. We are also interested in a set of geographical zones, spread over a territory, that we can decide whether or not to protect, from a given moment on, in order to ensure a certain protection for the species in question and thus increase their chance of survival. The terms “sites”, “parcels”, “patches”, “tasks”, “areas”, and “islets” are also used to designate these parts of territory. In this book, we will essentially use the term “zone” which, because of its generality, is appropriate in many contexts. The focus here is on species protection, but all of the following could easily be adapted to other threatened aspects of biodiversity such as valuable habitats.

In sections 1.2–1.5 of this chapter, it is considered that there is only one level of protection for the zones. In other words, a zone is protected or not. Decisions on protective actions to be taken at the beginning of the time horizon (*e.g.*, 10 years, 50 years or 100 years) are made at the beginning of this horizon, at which time the candidate zones are in a certain state. Protecting a zone has a cost. It differs from one zone to another and may include monetary, ecological and social aspects.

We can look at the consequences of these decisions at the end of this horizon. For example, it can be assumed that a given species in a protected zone survives at the

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end of the considered time horizon and that this is not the case if it does not occur in a protected zone. The relevance of these hypotheses presupposes that a large amount of information is available, such as the life history and dynamics of the species studied and the size of their population. More simply, it is possible to assess the impact of the protection of a set of zones by the number of species concerned by this protection, without prejudging as precisely the future of these species. It is then only supposed that the chances of survival are greater in protected zones than in unprotected ones. Section 1.6 addresses a significantly different problem for the reason that different protection actions can be considered for each zone. The level of protection of the species present in a zone depends on these actions.

We denote by $S = \{s_1, s_2,\ldots, s_n\}$ the set of species of interest and $Z = \{z_1, z_2,\ldots, z_n\}$ the set of zones that are candidates for protection. To simplify the presentation, a set of protected zones, $R\subseteq Z$, is called a "reserve". For any reserve $R\subseteq Z$, we are interested in the number of species that are protected – at least in a certain way – because of the protection of the zones of R. It is therefore the criterion of species richness that is used here. Thus, this number, which may be difficult to estimate, may represent the number of species that will survive at the end of the chosen time horizon if it is decided to protect the zones of R or, less precisely, the number of species concerned by this protection. We are interested in the overall effect of the protection of the zones of R, *i.e.*, the species richness of these zones considered as a whole – complementarity principle. The cost associated with protecting zone z_i is denoted by c_i. As mentioned above, it can cover several aspects: monetary costs – or possibly gains – (*e.g.*, leasing or acquisition of the zone, potential restoration of the zone, removal of invasive species, zone management, compensation to third parties, income from nature tourism), ecological costs or gains (*e.g.*, habitat quality and ease of movement of the considered species through the zone, involuntary protection of invasive species) and also social costs or gains, which are often difficult to assess (*e.g.*, reduction in possible uses of the zone by the public, access road closures, welfare gains for certain social groups, cultural gains). This cost can also, more simply, represent the area of the zone. Generally, the protection cost of a set of zones, $R\subseteq Z$, is equal to the sum of the protection costs of each of the zones in that set; it is denoted by $C(R)$. The term $\underline{S}$ refers to the set of indices of the species considered and the term $\underline{Z}$ refers to the set of indices of the zones that can be selected for protection. We have thus $\underline{S} = \{1, 2,\ldots, m\}$ and $\underline{Z} = \{1, 2,\ldots, n\}$. It is considered here that any subset of Z can be a priori protected except when a limited budget must be taken into account, since in this case the total cost of protecting the selected zones must not exceed the available budget. It is assumed that the population size of each species in each zone is known. The population size of species s_k in zone z_i is denoted by n_{ik}.

Two different situations are considered, in which a given species, s_k, is protected by a reserve, R. In the first, the protection of an adequate zone is sufficient to protect s_k. In the second, s_k is protected by R if its total population size in R is greater than or equal to a certain threshold value. The number of species protected by a reserve, R, is thus calculated in two different ways. In the first, the result of which is denoted by $\mathrm{Nb}_1(R)$, it is assumed that all the zones whose protection ensures the protection of the species (*e.g.*, its survival) are known for all the species, *i.e.*, for all k of $\underline{S}$. This set is denoted by Z_k and the corresponding set of indices is denoted by $\underline{Z}_k$. In other words, for species s_k to be protected, it is necessary and sufficient that at least one of the zones of Z_k be protected. For example, it is considered here that the protection of a zone makes it possible to protect all the species present in that zone provided that their population sizes in that zone are greater than or equal to a certain threshold value. We note v_{ik} the threshold value associated with species s_k in zone z_i. In other words, $Z_k = \{z_i \in Z : n_{ik} \geq v_{ik}\}$ (see example 1.1 below). In the second way of calculating the number of species protected by a reserve, R, the result of which is denoted by $\mathrm{Nb}_2(R)$, a reserve is considered to protect species s_k, $k = 1, 2,\ldots, m$, if and only if the total population size of that species in the reserve is greater than or equal to a certain threshold value, denoted by θ_k (see example 1.1 below). It should be noted that data on the size of the different populations may be difficult to obtain. The models considered in this chapter are basic models. They can be considered as a starting point to help a decision-maker in thinking about a relevant set of zones to be protected. The fact that solutions are determined, as we will see, by solving a relatively simple mathematical program, facilitates the task. These models can then be extended to take into account different additional aspects. Here again, the mathematical programming approach makes it easy to take these additional aspects into account. We will see many examples of this approach in the rest of this book.

Example 1.1. Consider the instance described in figure 1.1. Suppose that zones z_1, z_2, and z_3 are protected – $R = \{z_1, z_2, z_3\}$ – and that v_{ik} is equal to 4 for any couple (i, k). We obtain $\mathrm{Nb}_1(R) = 4$. Indeed, if the protection of a zone makes it possible to protect the species that are present in that zone provided that their population size is greater than or equal to 4 units, species s_1, s_3, s_6, and s_{11} are protected by reserve $R = \{z_1, z_2, z_3\}$. If we look at the measure $\mathrm{Nb}_2(R)$, for the same reserve, we obtain, assuming that to be protected a species must be present on the reserve with a population whose total size is greater than or equal to 10 – $\theta_k = 10$ for all k –, $\mathrm{Nb}_2(R) = 2$. In this case, only species s_3 and s_6 are protected.

1.2 Protection by a Reserve of All the Considered Species

1.2.1 The Protection of Each Zone Ensures the Protection of a Given Set of Species; the Number of Species Protected by a Reserve, R, is then Denoted by $Nb_1(R)$

The first question that can be addressed is: what is the set of zones to be protected, at minimal cost, to protect all the species? This problem, which can be stated concisely as the minimization problem $\min_{R\subseteq Z, \mathrm{Nb}_1(R)=m} C(R)$, can be formulated as a linear program in Boolean variables by associating to each zone z_i, $i = 1,\ldots, n$, a Boolean variable x_i, *i.e.*, a variable that can only take the values 0 or 1 (see appendix at the end of this book). By convention, this decision variable takes the value 1 if and

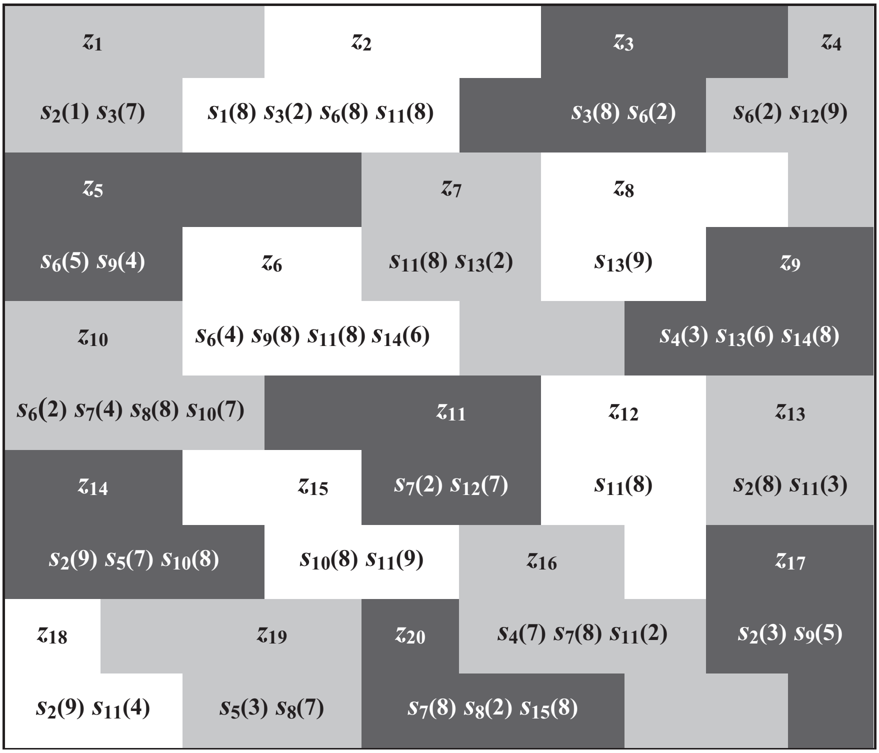

FIG. 1.1 – Twenty zones, z_1, z_2,…, z_{20}, are candidates for protection and fifteen species, s_1, s_2, …, s_{15}, living in these zones, are concerned. For each zone, the species present and their population size – in brackets – are indicated. The cost of protecting the white zones is equal to 1 unit, the cost of protecting the light grey zones is equal to 2 units and the cost of protecting the dark grey zones is equal to 4 units. For example, species s_6, s_9, s_{11}, and s_{14} are present in zone z_6, their population size is equal to 4, 8, 8, and 6 units, respectively, and the cost of protecting this zone is equal to 1 unit.

only if zone z_i is selected for protection. Program $\mathrm{P}_{1.1}$ corresponds to the determination of a reserve of minimal cost allowing all the species to be protected. Program $\mathrm{P}_{1.1}$ can admit several optimal solutions, *i.e.*, there may be several reserves allowing all the species to be protected at the lowest cost. In this case, the examination of all the optimal solutions and their evaluation using additional criteria may be necessary to determine the reserve that will finally be selected.

$$\mathrm{P}_{1.1}: \begin{cases} \min \displaystyle\sum_{i \in \underline{Z}} c_i x_i \\ \text{s.t.} \left| \begin{array}{lll} \displaystyle\sum_{i \in \underline{Z}_k} x_i \geq 1 & k \in \underline{S} & (1.1.1) \\ x_i \in \{0,1\} & i \in \underline{Z} & (1.1.2) \end{array} \right. \end{cases}$$

The objective function of $P_{1.1}$ expresses the total cost associated with protecting the selected zones. Indeed, the cost associated with zone z_i is equal to $c_i x_i$. If we decide to protect zone z_i, which corresponds to $x_i = 1$, then this cost is equal to c_i; if we decide not to protect zone z_i, which corresponds to $x_i = 0$, then this cost is equal to 0. Constraints 1.1.1 express that, for any species s_k, at least one zone of Z_k must be selected for protection. Indeed, at least one zone of Z_k is selected to be protected if and only if at least one of variables x_i – corresponding to zone z_i of Z_k – takes the value 1. Remember that the set Z_k is defined as follows: $Z_k = \{z_i \in Z : n_{ik} \geq v_{ik}\}$. Constraints 1.1.2 specify the Boolean nature of the variables x_i. The problem associated with $P_{1.1}$ is known, in operational research, as the set-covering problem (see appendix at the end of the book).

Example 1.2. Take again the instance described in figure 1.1, with $v_{ik} = 4$ for each couple (i, k). The cheapest strategy for protecting all the species is provided by the resolution of program $P_{1.1}$ – more precisely by the version corresponding to this example – and consists in protecting the 9 zones z_1, z_2, z_4, z_6, z_8, z_{10}, z_{14}, z_{16}, and z_{20}; it costs 19 units. Are there other reserves that cost 19 and protect all the species? This question can be answered simply by looking for a solution that satisfies constraints 1.1.1 and 1.1.2 as well as the 2 additional constraints $\sum_{i \in \underline{Z}} c_i x_i = 19$ and $x_1 + x_2 + x_4 + x_6 + x_8 + x_{10} + x_{14} + x_{16} + x_{20} \leq 8$. This new set of constraints allows for a feasible reserve – of cost 19 – consisting of zones z_1, z_2, z_4, z_6, z_8, z_{14}, z_{16}, z_{19}, and z_{20}.

A variant of this first problem is to consider that, in order to be protected, species s_k must be present – with a sufficient population size – not in at least one protected zone, but in at least β_k protected zones. Indeed, an effective way to guard against random events that could affect a zone (*e.g.*, storm, fire, pollution) and thus eliminate the species present in that zone is to protect several zones for each species. This increases the chances of survival of this species (replication principle). Figure 1.1 shows that, if $\beta_k = 2$ for any k, then the protected zones in the first solution of example 1.2 only protect species s_6, s_7, s_{10}, and s_{11}. It may be noted that, in this example, it is not possible to protect a set of zones in such a way that each species is present – with a sufficient population size – in at least 2 zones of the set. This problem can be formulated as a linear program in 0–1 variables by replacing in $P_{1.1}$ constraints 1.1.1, $\sum_{i \in \underline{Z}_k} x_i \geq 1,\ k \in \underline{S}$, by the constraints $\sum_{i \in \underline{Z}_k} x_i \geq \beta_k,\ k \in \underline{S}$. Indeed, these latter constraints require that, among the variables x_i – corresponding to zone z_i of Z_k – at least β_k of these variables take the value 1.

Other economic functions representing the cost of a reserve may be taken into account. For example, the candidate zones for protection can be considered as a set of q clusters, $\mathrm{Cl} = \{\mathrm{Cl}_1, \mathrm{Cl}_2, \ldots, \mathrm{Cl}_q\}$. More precisely, the q clusters form a partition of the set of zones, Z. Thus, each zone belongs to one and only one cluster and every cluster includes at least one zone. Let us denote by $\underline{\mathrm{Cl}}$ the set of cluster indices. In this case, the cost of protecting a zone consists of two costs: a cost associated specifically with the zone (*e.g.*, acquisition, restoration) and a cost associated with the cluster. The cost associated with cluster Cl_j, which we denote by d_j, is to be supported as soon as one of its zones is selected for protection. On the other hand, if several zones of the same cluster are selected for protection, the cost associated with the cluster is to be supported only once. This cost corresponds, for example, to the delivery of human and material resources to the cluster. The problem of protecting all the species at the lowest cost can then be formulated as the linear program in Boolean variables $P_{1.2}$. To do this, we associate, as before, a Boolean variable x_i to each zone z_i. In addition, with each cluster Cl_j is associated a Boolean variable, u_j, which, by convention, is equal to 1 if and only if at least one zone of cluster Cl_j is selected to be protected.

$$P_{1.2}: \begin{cases} \min \sum_{i \in \underline{Z}} c_i x_i + \sum_{j \in \underline{\mathrm{Cl}}} d_j u_j \\ \text{s.t.} \left| \begin{array}{llll|lll} \sum_{i \in \underline{Z}_k} x_i \geq 1 & k \in \underline{S} & & (1.2.1) & x_i \in \{0,1\} & i \in \underline{Z} & (1.2.3) \\ u_j \geq x_i & (j,i) \in \underline{\mathrm{Cl}} \times \underline{Z} : z_i \in \mathrm{Cl}_j & & (1.2.2) & u_j \in \mathbb{R} & j \in \underline{\mathrm{Cl}} & (1.2.4) \end{array} \right. \end{cases}$$

The first part of the economic function represents the cost associated with protecting the zones selected for protection (see $P_{1.1}$) and the second part represents the cost associated with the clusters concerned by this protection, *i.e.*, clusters in which at least one of the zones is selected for protection. Constraints 1.2.1 express that all the species must be protected (see $P_{1.1}$). Because of constraints 1.2.2 and the fact that we are seeking to minimize the costs, the real variable u_j takes the value 0 at the optimum of $P_{1.2}$ if none of the zones of Cl_j is selected for protection and the value 1 in the opposite case. Constraints 1.2.3 specify the Boolean nature of the variables x_i. Note that it is not necessary to further constrain the real variables u_j, $j \in \underline{\mathrm{Cl}}$. Indeed, because of the fact that we are seeking to minimize the quantity $\sum_{j \in \underline{\mathrm{Cl}}} d_j u_j$ and taking into account constraints 1.2.2, the variable u_j takes, at the optimum of $P_{1.2}$, either the value 0 or the value 1.

1.2.2 A Species is Protected by a Reserve, R, if its Total Population Size in R Exceeds a Certain Value; the Number of Species Protected by R is then Denoted by Nb₂(R)

In this case, the basic problem, which consists in selecting a set of zones, of minimal cost, whose protection ensures the protection of all the species, corresponds to the minimization problem $\min_{R \subseteq Z, \mathrm{Nb}_2(R)=m} C(R)$ and can be formulated as the linear program in Boolean variables $P_{1.3}$.

$$P_{1.3}: \begin{cases} \min \sum_{i \in \underline{Z}} c_i x_i \\ \text{s.t.} \left| \begin{array}{lll} \sum_{i \in \underline{Z}} n_{ik} x_i \geq \theta_k & k \in \underline{S} & (1.3.1) \\ x_i \in \{0,1\} & i \in \underline{Z} & (1.3.2) \end{array} \right. \end{cases}$$

As in the previous models, the reserve retained is formed by zone z_i such that $x_i = 1$. The economic function expresses the cost of the reserve (see $\mathrm{P}_{1.1}$). Constraints 1.3.1 express that the total population size of species s_k in the reserve, $\sum_{i\in \underline{Z}} n_{ik}x_i$, must be greater than or equal to the minimal value required for the survival of this species, θ_k, and this for any k of $\underline{S}$.

Example 1.3. Let us take the instance described in figure 1.1 and set θ_k to 7 for any k of $\underline{S}$. The least costly strategy for protecting all the species, when the number of species protected by a reserve R is assessed by $\mathrm{Nb}_2(R)$, is provided by the solution of $\mathrm{P}_{1.3}$. This strategy consists of protecting the 10 zones z_1, z_2, z_4, z_6, z_8, z_9, z_{10}, z_{14}, z_{16}, and z_{20}, and costs 23 units.

1.3 Protection by a Reserve of a Maximal Number of Species of a Given Set Under a Budgetary Constraint

A second basic problem is to determine the zones to be protected, taking into account an available budget, in order to protect, at least in a certain way, the greatest possible number of species. This problem, which consists in maximizing the species richness of the selected reserve, can be expressed in the form of the maximization problem $\max_{R\subseteq Z, C(R)\le B} \mathrm{Nb}_1(R)$ or $\max_{R\subseteq Z, C(R)\le B} \mathrm{Nb}_2(R)$, depending on the method of calculating the number of species protected by reserve R. B is the available budget.

1.3.1 The Number of Species Protected by a Reserve, R, is Assessed by Nb₁(R)

The problem can be formulated as a linear program with Boolean variables. As in the previous programs, a Boolean decision variable, x_i, is associated with each zone z_i. With each species s_k is also associated a "working" Boolean variable, y_k, which, by convention, takes the value 1 if and only if at least one of the zones selected to be protected protects species s_k. Thus, when the number of species protected by a reserve, R, is evaluated by $\mathrm{Nb}_1(R)$, the problem considered can be formulated as the mathematical program $\mathrm{P}_{1.4}$.

$$\mathrm{P}_{1.4}: \begin{cases} \max \sum\limits_{k\in\underline{S}} y_k \\ \text{s.t.} \left| \begin{array}{lllll} y_k \le \sum\limits_{i\in\underline{Z}_k} x_i & k\in\underline{S} & (1.4.1) & \mid & x_i\in\{0,1\} \quad i\in\underline{Z} \quad (1.4.3) \\ \sum\limits_{i\in\underline{Z}} c_i x_i \le B & & (1.4.2) & \mid & y_k\in\{0,1\} \quad k\in\underline{S} \quad (1.4.4) \end{array} \right. \end{cases}$$

The objective of $\mathrm{P}_{1.4}$ is to maximize the expression $\sum_{k\in\underline{S}} y_k$, *i.e.*, the number of protected species. Indeed, according to constraints 1.4.1 and considering that we are seeking to maximize the quantity $\sum_{k\in\underline{S}} y_k$, variable y_k, which is a Boolean variable, necessarily takes the value 0 if $\sum_{i\in\underline{Z}_k} x_i = 0$, *i.e.*, if no zone of Z_k is selected, and the value 1, at the optimum of $\mathrm{P}_{1.4}$, if $\sum_{i\in\underline{Z}_k} x_i \ge 1$, *i.e.*, if at least one zone of Z_k is selected. Variable y_k therefore takes, as it should, at the optimum of $P_{1.4}$, the value 1 if and only if the zones selected for protection allow to protect species s_k. Note that constraints 1.4.4 could be replaced by constraints $y_k \leq 1,\ k \in \underline{S}$. The quantity $\sum_{i \in \underline{Z}} c_i x_i$ expresses the cost associated with the reserve and constraint 1.4.2, therefore, expresses the budgetary constraint. Note that if one wishes to obtain, among the optimal solutions of $P_{1.4}$, a lowest cost solution, one way is to solve program $P_{1.4}$ with the modified economic function, $\sum_{k \in \underline{S}} y_k - \varepsilon \sum_{i \in \underline{Z}} c_i x_i$, where ε is a sufficiently small constant. This technique can be applied in many cases when two criteria are considered, one in the economic function – here, the number of species – and the other in a constraint – here the cost.

Example 1.4. Let us take the instance described in figure 1.1 with $v_{ik} = 4$ for each couple (i, k) and assume that we have a budget of 8 units. The optimal use of this budget is provided by the resolution of $P_{1.4}$. It consists of protecting the zones z_1, z_2, z_6, z_8, z_{10}, and z_{18}, which protects 11 species, all the species except s_4, s_5, s_{12}, and s_{15}. The totality of the available budget is used.

Here again, it can be considered that the chances of survival of each species s_k are only really increased if β_k zones that contribute to this increase are protected. This problem can be modelled by a linear program in Boolean variables by replacing in $P_{1.4}$ constraints 1.4.1, $y_k \leq \sum_{i \in \underline{Z}_k} x_i,\ k \in \underline{S}$, by constraints $\beta_k y_k \leq \sum_{i \in \underline{Z}_k} x_i,\ k \in \underline{S}$. Thus, if the number of selected zones in the set Z_k, $\sum_{i \in \underline{Z}_k} x_i$, is less than β_k, the Boolean variable y_k can only take the value 0. Otherwise, and because of the fact that we are seeking to maximize $\sum_{k \in \underline{S}} y_k$, variable y_k takes the value 1 at the optimum. Note that, in this case, constraints 1.4.4 cannot be replaced by constraints $y_k \leq 1,\ k \in \underline{S}$. It can also be considered, as in section 1.2.1, that the zones are divided into q clusters. The problem of protecting a maximal number of species under a budgetary constraint can then be formulated as program $P_{1.5}$.

$$
P_{1.5}: \begin{cases} \max \sum_{k \in \underline{S}} y_k \\ \text{s.t.} \left| \begin{array}{lll} y_k \leq \sum_{i \in \underline{Z}_k} x_i & k \in \underline{S} & (1.5.1) \\ u_j \geq x_i & (j, i) \in \underline{\mathrm{Cl}} \times \underline{Z} : z_i \in \mathrm{Cl}_j & (1.5.2) \\ \sum_{i \in \underline{Z}} c_i x_i + \sum_{j \in \underline{\mathrm{Cl}}} d_j u_j \leq B & & (1.5.3) \\ x_i \in \{0, 1\} & i \in \underline{Z} & (1.5.4) \\ y_k \leq 1 & k \in \underline{S} & (1.5.5) \\ u_j \in \mathbb{R} & j \in \underline{\mathrm{Cl}} & (1.5.6) \end{array} \right. \end{cases}
$$

Due to constraints 1.5.2, the real variable u_j must take a value greater than or equal to 0 if none of the zones of Cl_j is selected for protection and a value greater than or equal to 1 in the opposite case. Considering all constraints of $P_{1.5}$, this implies that variable u_j can take the value 0 if none of the zones of Cl_j are selected for protection and the value 1 in the opposite case. Constraint 1.5.3 expresses that the cost associated with the reserve (see $P_{1.2}$) must not exceed the available budget, B. If one wishes to obtain, among the optimal solutions of $P_{1.5}$, a minimal cost solution, one way to do so is to solve program $P_{1.5}$ with the modified economic function, $\sum_{k\in \underline{S}} y_k - \varepsilon(\sum_{i\in \underline{Z}} c_i x_i + \sum_{j\in \underline{Cl}} d_j u_j)$, where ε is a sufficiently small constant.

Example 1.5. Let us consider the 20 zones in figure 1.1, with $v_{ik} = 4$ for each pair (i, k), and assume that these 20 zones are divided into 5 clusters, Cl_1, Cl_2, Cl_3, Cl_4, and Cl_5, as follows: $Cl_1 = \{z_1, z_2, z_5, z_6, z_7, z_{10}\}$, $Cl_2 = \{z_3, z_4, z_8, z_9\}$, $Cl_3 = \{z_{11}, z_{16}, z_{20}\}$, $Cl_4 = \{z_{14}, z_{15}, z_{18}, z_{19}\}$, and $Cl_5 = \{z_{12}, z_{13}, z_{17}\}$. Suppose, moreover, that the cost associated with each cluster is equal to 2 units. The optimal strategy to protect a maximal number of species with an available budget of 11 units is provided by the resolution of $P_{1.5}$. This strategy consists of protecting the 5 zones z_1, z_2, z_6, z_{10}, and z_{18}. These zones, distributed over the 2 clusters Cl_1 and Cl_4, make it possible to protect the 10 species s_1, s_2, s_3, s_6, s_7, s_8, s_9, s_{10}, s_{11}, and s_{14}. We present below, for illustration purposes, a way to solve this example using the AMPL modelling language and the CPLEX solver. Three files, named respectively "Example-1.5.mod", "Example-1.5.dat" and "Example-1.5.run", are used. The first corresponds to the translation of program $P_{1.5}$ into the AMPL language, the second describes the data in this example that are not already defined in "Example-1.5.mod", *i.e.*, c_i, n_{ik}, and a_{ij}, and the third is to start the resolution by CPLEX and display the solution obtained. The Boolean parameter a_{ij} describes the composition of each cluster: $a_{ij} = 1$ if and only if zone z_i belongs to cluster Cl_j.

1.3.2 The Number of Species Protected by a Reserve, R, is Assessed by $Nb_2(R)$

In this case, the basic problem of selecting a set of zones with a cost less than or equal to B and whose protection ensures the protection of a maximal number of species can be formulated as the linear program in Boolean variables $P_{1.6}$.

$$P_{1.6}: \begin{cases} \max \sum_{k\in \underline{S}} y_k \\ \text{s.t.} \left| \begin{array}{llll} \theta_k y_k \le \sum_{i\in \underline{Z}} n_{ik} x_i & k \in \underline{S} & (1.6.1) & \mid \quad x_i \in \{0,1\} \quad i \in \underline{Z} \quad (1.6.3) \\ \sum_{i\in \underline{Z}} c_i x_i \le B & & (1.6.2) & \mid \quad y_k \in \{0,1\} \quad k \in \underline{S} \quad (1.6.4) \end{array} \right. \end{cases}$$

Let us examine constraints 1.6.1. There are two possibilities. Either $\sum_{i\in \underline{Z}} n_{ik} x_i < \theta_k$ and then the Boolean variable y_k can only take the value 0, or $\sum_{i\in \underline{Z}} n_{ik} x_i \ge \theta_k$ and then variable y_k takes the value 1 at the optimum of $P_{1.6}$ since we seek to maximize the expression $\sum_{k\in \underline{S}} y_k$. These constraints, therefore, reflect the fact that species s_k is protected if and only if the total population size of this species in the reserve is greater than or equal to θ_k. If one wishes to obtain, among the optimal solutions of $P_{1.6}$, a least-cost solution, one way to do this is to solve

Example-1.5.mod

```
#------Data----------------------------------------------------------------------------------
param c{i in 1..20};
param a{i in 1..20, j in 1..5} default 0;
param n{i in 1..20, k in 1..15} default 0;
param d{j in 1..5}:=2;
param nu{i in 1..20, k in 1..15}:=4;
param B:=11;
#------Variables----------------------------------------------------------------------------
var x{i in 1..20} binary;
var y{k in 1..15} <=1;
var u{j in 1..5} <= 1;
#------Model--------------------------------------------------------------------------------
maximize f: sum{k in 1..15} y[k];
subject to
C1{k in 1..15}: y[k]<=sum{i in 1..20 : n[i,k]>=nu[i,k]} x[i];          # (1.5.1)
C2{j in 1..5, i in 1..20 : a[i,j]=1}:    u[j]>=x[i];                  # (1.5.2)
C3: sum{i in 1..20} c[i]*x[i] + sum{j in 1..5} d[j]*u[j]<=B;          # (1.5.3)
#------Fin-----------------------------------------------------------------------------------
```

Example-1.5.dat

```
data;
#-------------------------------------------------------------------------------------------
param c:=
1 2, 2 1, 3 4, 4 2, 5 4, 6 1, 7 2, 8 1, 9 4, 10 2, 11 4,
12 1, 13 2, 14 4, 15 1, 16 2, 17 4, 18 1, 19 2, 20 4;
#-------------------------------------------------------------------------------------------
param a:=
1 1 1, 2 1 1, 3 2 1, 4 2 1, 5 1 1, 6 1 1, 7 1 1, 8 2 1, 9 2 1, 10 1 1, 11 3 1,
12 5 1, 13 5 1, 14 4 1, 15 4 1, 16 3 1, 17 5 1, 18 4 1, 19 4 1, 20 3 1;
#-------------------------------------------------------------------------------------------
param n:=
1 2 1, 1 3 7, 2 1 8, 2 3 2, 2 6 8, 2 11 8, 3 3 8, 3 6 2, 4 6 2, 4 12 9, 5 6 5, 5 9 4, 6 6 4,
6 9 8, 6 11 8, 6 14 6, 7 11 8, 7 13 2, 8 13 9, 9 4 3, 9 13 6, 9 14 8, 10 6 2, 10 7 4, 10 8 8,
10 10 7, 11 7 2, 11 12 7, 12 11 8, 13 2 8, 13 11 3, 14 2 9, 14 5 7, 14 10 8, 15 10 8, 15 11 9,
16 4 7, 16 7 8, 16 11 2, 17 2 3, 17 9 5, 18 2 9, 18 11 4, 19 5 3, 19 8 7, 20 7 8, 20 8 2, 20 15 8;
#-------------------------------------------------------------------------------------------
```

Example-1.5.run

```
reset;
option solver cplex1260;
model Example-1.5.mod;
model Example-1.5.dat;
solve;
```

```
print '***********************************************************';
print '            Solution Example 1.5:';
print '***********************************************************';
print ' Total cost of protection:', sum{i in 1..20} c[i]*x[i] + sum{j in 1..5} d[j]*u[j];
print '------------------------------------------------------------';
print ' Indices of the zones to be protected:',{i in 1..20 : x[i]=1}i;
print '------------------------------------------------------------';
print ' Number of protected species:', f;
print '------------------------------------------------------------';
print ' Indices of protected species:', {k in 1..15 : y[k]=1}k;
print '***********************************************************';
```

To solve the considered example, it is sufficient to type "model Example-1.5.run;" in the console ampl: **ampl: model Example-1.5.run;**
which starts the resolution using the CPLEX 12.6.0.0 solver and then displays the resulting solution:

```
CPLEX 12.6.0.0: optimal integer solution; objective 10
20 MIP simplex iterations
0 branch-and-bound nodes
*******************************************
            Solution Example 1.5:
*******************************************
Total cost of protection: 11
------------------------------------------------------------
Indices of the zones to be protected: 1 2 6 10 18
------------------------------------------------------------
Number of protected species: 10
------------------------------------------------------------
Indices of protected species: 1 2 3 6 7 8 9 10 11 14
*******************************************
```

program $P_{1.6}$ with the modified economic function, $\sum_{k\in\underline{S}} y_k - \varepsilon \sum_{i\in\underline{Z}} c_i x_i$, where ε is a sufficiently small constant.

Example 1.6. Let us take again the instance described in figure 1.1, set θ_k to 7 for every k of $\underline{S}$ and assume that we have a budget of 8 units. An optimal use of this budget, when the number of species protected by a reserve, R, is evaluated by $\text{Nb}_2(R)$, is provided by the resolution of $P_{1.6}$. It consists in protecting the 6 zones z_2, z_6, z_8, z_{10}, z_{16}, and z_{18}, which allows the protection of 10 species, all the species except s_3, s_5, s_{12}, s_{14}, and s_{15}.

1.3.3 Remarks on the Problems Addressed in Sections 1.3.1 and 1.3.2

In all the problems addressed in sections 1.3.1 and 1.3.2, it is possible to give a different importance to the protection of each species by replacing in the corresponding mathematical programs the economic function $\sum_{k\in\underline{S}} y_k$ with the economic function $\sum_{k\in\underline{S}} w_k y_k$ where w_k represents the weight assigned to the species s_k. These weights reflect the relative importance of the different species considered. It should also be noted that, for all these problems, a decision-maker may be interested in knowing their optimal solution for different values of the available budget, B. In this way, he/she can easily assess the marginal effect of an additional investment. This can be done by solving the corresponding mathematical programs with different values of B. It is also possible to look, almost equivalently, at the minimal budget needed to achieve a certain level of species protection. Let us consider, for example, the case where the number of species protected by a reserve, R, is assessed by $\text{Nb}_1(R)$. To know, in this case, the budget necessary to protect, at least, Ns species for all possible values of Ns, it is sufficient to solve program $\text{P}_{1.7}$ by varying Ns from 1 to m.

$$\text{P}_{1.7}: \begin{cases} \min \sum_{i\in\underline{Z}} c_i x_i \\ \text{s.t.} \left| \begin{array}{llll} y_k \le \sum_{i\in\underline{Z}_k} x_i \quad k\in\underline{S} & (1.7.1) & \mid \quad x_i\in\{0,1\} \quad i\in\underline{Z} & (1.7.3) \\ \sum_{k\in\underline{S}} y_k \ge \text{Ns} & (1.7.2) & \mid \quad y_k\in\{0,1\} \quad k\in\underline{S} & (1.7.4) \end{array} \right. \end{cases}$$

Constraint 1.7.2 states that the number of species protected by the reserve must be greater than or equal to Ns. It should be noted that the number of species actually protected may be greater than $\sum_{k\in\underline{S}} y_k$. It is in fact equal to the cardinal of the set $\{k : \sum_{i\in\underline{Z}_k} x_i \ge 1\}$. If we wish to solve the problem under consideration while maximizing the number of protected species, we can deduct from the economic function of $\text{P}_{1.7}$ the quantity $\varepsilon\sum_{k\in\underline{S}} y_k$ where ε is a sufficiently small constant.

Example 1.7. Let us again take the instance described in figure 1.1 assuming that the number of species protected by a reserve, R, is assessed by $\text{Nb}_1(R)$ and that $v_{ik} = 4$ for each pair (i,k). Table 1.1 gives the optimal solution of $\text{P}_{1.7}$ – after subtracting $\varepsilon\sum_{k\in\underline{S}} y_k$ to the economic function – for all possible values of Ns. Figure 1.2 shows the curve illustrating the minimal cost of a reserve as a function of the number of species to be protected.

1.4 Gradual Establishment of a Reserve Over Time to Protect a Maximal Number of Species of a Given Set, with a Time-dependent Budget Constraint

As previously $Z = \{z_1, z_2,\ldots, z_n\}$ designates the set of candidate zones but now the protection of the zones of Z is done gradually over a time horizon, T, composed of r periods (r years for example), $T_1, T_2,\ldots, T_r$, in order to spread the costs. However, all the protection decisions are taken at the beginning of the horizon considered. In addition, any zone protected from a certain period remains protected for all the subsequent periods in the time horizon considered. Let $\underline{T} = \{1, 2,\ldots, r\}$. The set of

TAB. 1.1 – Resolution of program $P_{1.7}$ for the instance described in figure 1.1. Presentation of the best strategy to adopt and its cost, taking into account the number of species to be protected.

Minimal number of species to be protected (Ns)	Set of zones to be protected, of minimal cost	Cost	Number of species that are actually protected	Protected species
1	z_6	1	4	s_6 s_9 s_{11} s_{14}
2	z_6	1	4	s_6 s_9 s_{11} s_{14}
3	z_6	1	4	s_6 s_9 s_{11} s_{14}
4	z_6	1	4	s_6 s_9 s_{11} s_{14}
5	z_2 z_6	2	5	s_1 s_6 s_9 s_{11} s_{14}
6	z_6 z_{10}	3	7	s_6 s_7 s_8 s_9 s_{10} s_{11} s_{14}
7	z_6 z_{10}	3	7	s_6 s_7 s_8 s_9 s_{10} s_{11} s_{14}
8	z_6 z_{10} z_{18}	4	8	s_2 s_6 s_7 s_8 s_9 s_{10} s_{11} s_{14}
9	z_6 z_8 z_{10} z_{18}	5	9	s_2 s_6 s_7 s_8 s_9 s_{10} s_{11} s_{13} s_{14}
10	z_2 z_6 z_8 z_{10} z_{18}	6	10	s_1 s_2 s_6 s_7 s_8 s_9 s_{10} s_{11} s_{13} s_{14}
11	z_2 z_6 z_8 z_{10} z_{16} z_{18}	8	11	s_1 s_2 s_4 s_6 s_7 s_8 s_9 s_{10} s_{11} s_{13} s_{14}
12	z_1 z_2 z_6 z_8 z_{10} z_{16} z_{18}	10	12	s_1 s_2 s_3 s_4 s_6 s_7 s_8 s_9 s_{10} s_{11} s_{13} s_{14}
13	z_1 z_2 z_4 z_6 z_8 z_{10} z_{16} z_{18}	12	13	s_1 s_2 s_3 s_4 s_6 s_7 s_8 s_9 s_{10} s_{11} s_{12} s_{13} s_{14}
14	z_1 z_2 z_4 z_6 z_8 z_{10} z_{14} z_{16}	15	14	s_1 s_2 s_3 s_4 s_5 s_6 s_7 s_8 s_9 s_{10} s_{11} s_{12} s_{13} s_{14}
15	z_1 z_2 z_4 z_6 z_8 z_{10} z_{14} z_{16} z_{20}	19	15	s_1 s_2 s_3 s_4 s_5 s_6 s_7 s_8 s_9 s_{10} s_{11} s_{12} s_{13} s_{14} s_{15}

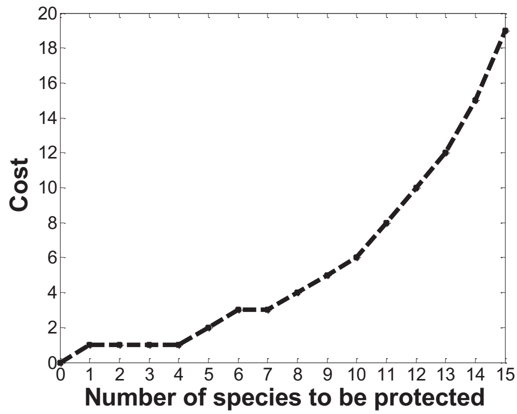

FIG. 1.2 – Curve associated with table 1.1: cost of the cheapest strategy according to the number of species to be protected.

species likely to be protected by a zone depends on the time at which that zone is protected. Indeed, the possible evolution of the environment between two periods may change the role of the different zones in protecting species. For example, a zone protected since the period T_t allows the protection of a certain set of species, but if this zone is protected only from the period $T_{t+\tau}$ it no longer allows the protection of all the species of this set. Typically, for each pair $(z_i,\ T_t)$ we know the set of species that are protected until the end of the time horizon if we protect z_i from the beginning of the period T_t. We denote by Z_{kt} the set of zones which, if they are protected from the beginning of the period T_t, protect species s_k. We denote by $\underline{Z}_{kt}$ the set of corresponding indices. The objective is to determine the zones to be protected from the beginning of each period and under a period-specific budgetary constraint so that a maximal number of species are protected at the end of the r periods. The cost of protecting the zones can vary over time. Thus the cost related to the decision to protect zone z_i at the beginning of the period T_t is denoted by c_{it}, $i \in \underline{Z}$, $t \in \underline{T}$, and this cost must be borne at the beginning of the period T_t. We know the available budget, B_t, at the beginning of the period T_t. In the simple model we consider, it is assumed that all the data are known at the beginning of the horizon T and do not change during this horizon. The problem can be formulated as a mathematical program. To do this, with each zone z_i and each period T_t is associated a Boolean variable, x_{it}, which, by convention, is equal to 1 if and only if we decide to protect zone z_i from the beginning of period T_t. As in the previous models, with each species s_k is associated a Boolean variable, y_k, which is equal to 1 if and only if species s_k is protected. This results in program $P_{1.8}$.

$$\mathrm{P}_{1.8}: \begin{cases} \max \sum_{k \in \underline{S}} y_k \\ \text{s.t.} \left| \begin{array}{llll} y_k \leq \sum_{t \in \underline{T}} \sum_{i \in \underline{Z}_{kt}} x_{it} & k \in \underline{S} & (1.8.1) & | \quad x_{it} \in \{0,1\} \quad i \in \underline{Z}, t \in \underline{T} \quad (1.8.4) \\ \sum_{t \in \underline{T}} x_{it} \leq 1 & i \in \underline{Z} & (1.8.2) & | \quad y_k \in \{0,1\} \quad k \in \underline{S} \quad (1.8.5) \\ \sum_{i \in \underline{Z}} c_{it} x_{it} \leq B_t & t \in \underline{T} & (1.8.3) & | \end{array} \right. \end{cases}$$

The economic function of $\mathrm{P}_{1.8}$ expresses the number of protected species. Because of constraints 1.8.1 and the fact that we are seeking to maximize the expression $\sum_{k \in \underline{S}} y_k$, variable y_k takes the value 1, at the optimum of $\mathrm{P}_{1.8}$, if and only if at least one of variables x_{it}, $t \in \underline{T}$, $i \in \underline{Z}_{kt}$, is equal to 1, in other words, if and only if there is at least one period T_t, at the beginning of which at least one zone of Z_{kt} is protected. Constraints 1.8.2 express that any zone can only be protected from a single period of the horizon. Constraints 1.8.3 correspond to period-specific budgetary constraints. They express that the budget allocated to the protection of the zones at the beginning of each period T_t should not exceed the available budget, B_t. In this model, the resources not used in the period T_t are lost. If this does not correspond to reality, constraints 1.8.3 can be replaced by the set of constraints $\mathrm{C}_{1.1}$. In this case, the resources available but not used in the period T_t can be used from period T_{t+1}.

$$\mathrm{C}_{1.1}: \begin{cases} \sum_{i \in \underline{Z}} c_{i1} x_{i1} + \delta_1 = B_1 & \\ \sum_{i \in \underline{Z}} c_{it} x_{it} + \delta_t = B_t + \delta_{t-1} & t \in \underline{T}, t \geq 2 \\ \delta_t \geq 0 & t \in \underline{T} \end{cases}$$

Variable δ_t, $t \in \underline{T}$, corresponds to the quantity of unused resources at the beginning of period T_t. The first constraint expresses, on the one hand, that the expenses at the beginning of period T_1 must not exceed the available budget, B_1, and, on the other hand, that variable δ_1 is equal to the amount of unused resources, *i.e.*, the quantity $B_1 - \sum_{i \in \underline{Z}} c_{i1} x_{i1}$. The following set of constraints expresses, on the one hand, that the expenses at the beginning of period T_t, $t \in \underline{T}, t \geq 2$, must not exceed the budget available at the beginning of this period, *i.e.*, $B_t + \delta_{t-1}$ and, on the other hand, that variable δ_t is equal to the amount of resources not used at the beginning of period T_t, *i.e.*, the quantity $B_t + \delta_{t-1} - \sum_{i \in \underline{Z}} c_{it} x_{it}$. In other words, variable δ_t corresponds to the resources not yet used up to period T_t, including period T_t, *i.e.*, the quantity $B_1 + B_2 + \cdots + B_t$ minus the quantity $\sum_{i \in \underline{Z}} c_{i1} x_{i1} + \sum_{i \in \underline{Z}} c_{i2} x_{i2} + \cdots + \sum_{i \in \underline{Z}} c_{it} x_{it}$.

1.5 Reserve Necessarily Including Certain Zones

All the problems discussed in this chapter consist in selecting an “optimal” set of zones to be protected. It may be that, for different reasons, some of the candidate zones must be selected.

1.5.1 Selection of a Reserve Taking into Account Already Protected Zones

It may be that in the problems studied in sections 1.2 and 1.3, some of the zones that could form the reserve are already protected zones. They have acquired this status in the past and still have it when the time comes to establish a new optimal reserve. To take this constraint into account, it is sufficient to solve the mathematical program corresponding to the problem under study by setting variables x_i to 1 for zone z_i whose protection is mandatory. One way of doing this is to add to this program constraint $x_i = 1$ for all the indices i concerned. Remember that in all the mathematical programs considered in these two sections, the Boolean variable x_i takes the value 1 if and only if zone z_i is selected to form the reserve.

Example 1.8. Let us look again at the instance described in figure 1.1 and consider the problem of determining a reserve, R, which respects a budgetary constraint and maximizes $\text{Nb}_1(R)$. Suppose, as in example 1.4, that $v_{ik} = 4$ for any couple (i, k) and that we have a budget of 8 units. If the protection of zone z_5 is mandatory, an optimal use of this budget consists in protecting the 4 zones z_2, z_5, z_6, and z_{10}, which allows the 8 species s_1, s_6, s_7, s_8, s_9, s_{10}, s_{11}, and s_{14} to be protected. If the protection of zone z_{11} is mandatory, an optimal use of this budget consists in protecting the 4 zones z_2, z_6, z_{10}, and z_{11}, which allows the 9 species s_1, s_6, s_7, s_8, s_9, s_{10}, s_{11}, s_{12}, and s_{14} to be protected. Remember that if there are no zones whose protection is mandatory, an optimal use of a budget of 8 units is to protect the 6 zones z_1, z_2, z_6, z_8, z_{10}, and z_{18}, which allows all species to be protected except s_4, s_5, s_{12}, and s_{15} (see example 1.4).

1.5.2 Gradual Establishment of a Reserve Over Time: Review of Decisions Taken at the Beginning of the Considered Horizon

Let us return to the constitution of a reserve discussed in section 1.4. A disadvantage of the considered model is that the decisions are made, definitively, at the beginning of the time horizon even if some zones are effectively protected only from a certain period of this horizon. We discuss below the possibility of revising these decisions over time, taking into account possible changes in projected costs, budgets and capacities of zones to protect species. Suppose that in the solution to the problem in section 1.4, the list of zones to be protected at each period is defined by $x_{it} = \tilde{x}_{it}$, $i \in \underline{Z}$, $t \in \underline{T}$ ($\tilde{x}_{it}$ is a constant which is 0 or 1). Let us also assume that we have arrived at the beginning of the period T_j and that the forecasts for the coming periods are reviewed, including the available budget. Thus Z_{kt} becomes $\hat{Z}_{kt},\ k \in \underline{S}, t \in \underline{T}, t \geq j$, B_t becomes $\hat{B}_t,\ t \in \underline{T}, t \geq j$, and c_{it} becomes $\hat{c}_{it},\ i \in \underline{Z} - \underline{R}_j$, $t \in \underline{T}, t \geq j$, where R_j is the reserve already constituted and $\underline{R}_j$ the set of corresponding indices. The decisions that had been taken at the beginning of the horizon considered for the periods T_j, T_{j+1},..., T_r can be abandoned and new optimal decisions can be sought, taking into account not only the new forecasts but also the reserve already constituted and the species that it allows to be protected. We are, therefore, in the case of establishing a reserve which must necessarily include certain zones. One could also consider abandoning some zones, but this is not considered here (see chapter 12, section 12.3.3.2). One way to formulate the problem is to slightly modify program $P_{1.8}$: Constraints 1.8.3 are now to be taken into account only for $t \geq j$ and constraints 1.9.4 which stipulate that some zones have already been selected must be added. It is also necessary to set $\hat{Z}_{kt} = Z_{kt},\ k \in \underline{S}, t \in \underline{T}, t \leq j-1$ to take into account the species already protected by R_j. This gives program $P_{1.9}$.

$$
P_{1.9}: \begin{cases} \max \sum_{k \in \underline{S}} y_k \\ \text{s.t.} \left| \begin{array}{lll} y_k \leq \sum_{t \in \underline{T}} \sum_{i \in \underline{\hat{Z}}_{kt}} x_{it} & k \in \underline{S} & (1.9.1) \\ \sum_{t \in \underline{T}} x_{it} \leq 1 & i \in \underline{Z} & (1.9.2) \\ \sum_{i \in \underline{Z}} \hat{c}_{it} x_{it} \leq \hat{B}_t & t \in \underline{T}, t \geq j & (1.9.3) \\ x_{it} = \tilde{x}_{it} & i \in \underline{Z}, t \in \underline{T}, t \leq j-1 & (1.9.4) \\ x_{it} \in \{0,1\} & i \in \underline{Z}, t \in \underline{T} & (1.9.5) \\ y_k \in \{0,1\} & k \in \underline{S} & (1.9.6) \end{array} \right. \end{cases}
$$

1.6 Case Where Several Conservation Actions are Conceivable in Each Zone

1.6.1 The Problem

This section considers a slightly different case from the ones studied in the previous sections insofar as several different protection actions are possible for each zone. Thus, for each candidate zone, a decision can be made to protect it or not to protect it, but if it is decided to protect it, several protection actions can be considered. We present below an example of reserve selection relevant to this issue and developed drawing on the references (Cattarino *et al.*, 2015; Salgado-Rojas *et al.*, 2020). In this example it is considered that a species present in a given zone is exposed to different threats and that, if the protection of that zone is decided, different actions can be taken to remove all or part of these threats. An optimal reserve is a reserve that maximizes an “overall ecological benefit” for the species considered within an available budget. This benefit takes into account both protected zones and actions taken in these zones to eliminate certain threats.

Let $S = \{s_1, s_2, \ldots, s_m\}$ be the set of species, animal or plant, in which we are interested and $Z = \{z_1, z_2, \ldots, z_n\}$ be the set of zones that we can decide whether or not to protect. S_i refers to the set of species present in zone z_i. In addition, there are a number of threats, $M = \{\mu_1, \mu_2, \ldots, \mu_g\}$, affecting these species. We denote by M_{ik} ($\subseteq M$) the set of threats affecting species s_k in zone z_i and M_i the set of threats to be considered in zone z_i. We have thus $M_i = \cup_{k:\, s_k \in S_i} M_{ik}$. The protection of zone z_i costs c_i and the elimination of threat μ_j in zone z_i costs d_{ij}. As we have said, the protection strategy has two levels. It is defined by the set of zones that it has decided to protect and, for each of these zones, by the set of threats that it has decided to eliminate. For a species s_k living in zone z_i of the reserve, we consider that the degree of protection of s_k in z_i is equal to the ratio between the number of eliminated threats weighing on s_k in z_i and the total number of threats weighing on s_k in z_i. The degree of protection of a species s_k present in a protected zone z_i where it is not threatened is equal to 1 for that zone. The degree of protection of a species s_k in an unprotected zone z_i is equal to 0. We denote by w_{ik} the square of the degree of protection of species s_k in zone z_i. A we have seen, the value of this variable results from the strategy adopted for zone z_i: not protecting it or protecting it and eliminating a number of threats. The problem is to determine the optimal strategy given the available budget. The value of a strategy is measured by the sum of the squares of the degrees of protection, w_{ik}, for all pairs $(z_i,\ s_k)$ where z_i is a candidate zone and s_k is a species present in that zone. We denote, respectively, by $\underline{S}$, $\underline{S}_i$, $\underline{Z}$, $\underline{M}$, $\underline{M}_i$, and $\underline{M}_{ik}$ the set of indices of the sets S, S_i, Z, M, M_i, and M_{ik}.

1.6.2 Mathematical Programming Formulation

We use the Boolean variables x_i, $i \in \underline{Z}$, which take the value 1 if and only if zone z_i is selected to be part of the reserve and the Boolean variables y_{ij}, $i \in \underline{Z}, j \in \underline{M}_i$, which take the value 1 if and only if we decide to eliminate the threat μ_j from zone z_i. The problem considered can be formulated as program $\mathrm{P}_{1.10}$.

$$
\mathrm{P}_{1.10}: \begin{cases}
\max \sum_{i \in \underline{Z}} \sum_{k \in S_i} w_{ik} \\
\text{s.t.} \left| \begin{array}{lll}
\sum_{i \in \underline{Z}} c_i x_i + \sum_{i \in \underline{Z},\, j \in \underline{M}_i} d_{ij} y_{ij} \le B & & (1.10.1) \\
w_{ik} \le \left(\sum_{j \in \underline{M}_{ik}} y_{ij} / |M_{ik}| \right)^2 & i \in \underline{Z}, k \in \underline{S}_i, |M_{ik}| > 0 & (1.10.2) \\
w_{ik} \le x_i & i \in \underline{Z}, k \in \underline{S}_i & (1.10.3) \\
x_i \in \{0,1\} & i \in \underline{Z} & (1.10.4) \\
y_{ij} \in \{0,1\} & i \in \underline{Z}, j \in \underline{M}_i & (1.10.5) \\
w_{ik} \in \mathbb{R} & i \in \underline{Z}, k \in \underline{S}_i & (1.10.6)
\end{array} \right.
\end{cases}
$$

Since variable w_{ik} represents the square of the degree of protection of species s_k in zone z_i, the economic function represents the sum, for all pairs $(z_i,\ s_k)$ where s_k is a species present in zone z_i, of the square of the degree of protection of species s_k in zone z_i. If zone z_i is not selected – $x_i = 0$ – then, due to constraints 1.10.3, $w_{ik} = 0$ for all the species living in this zone. If zone z_i is selected – $x_i = 1$ – then two cases are possible: (1) species s_k is not threatened in this zone – $|M_{ik}| = 0$ – and $w_{ik} = 1$ because of constraints 1.10.3 and the economic function to be maximized, (2) species s_k is threatened in this zone – $|M_{ik}| > 0$ – and because of constraints 1.10.2 and the economic function to be maximized $w_{ik} = \left(\sum_{j\in \underline{M}_{ik}} y_{ij}/|M_{ik}|\right)^2$. The economic function, therefore, expresses well the sum of the squares of the degrees of protection, w_{ik}, for all pairs (z_i, s_k) where z_i is a protected zone and s_k, a species present in this zone. Constraint 1.10.1 is the budget constraint and constraints 1.10.4 and 1.10.5 specify the Boolean nature of variables x_i and y_{ij}.

1.6.3 Example

Consider the instance described in figure 1.3 (20 zones and 15 species). The optimal protection strategies are given in table 1.2 when the available budget is 25 units.

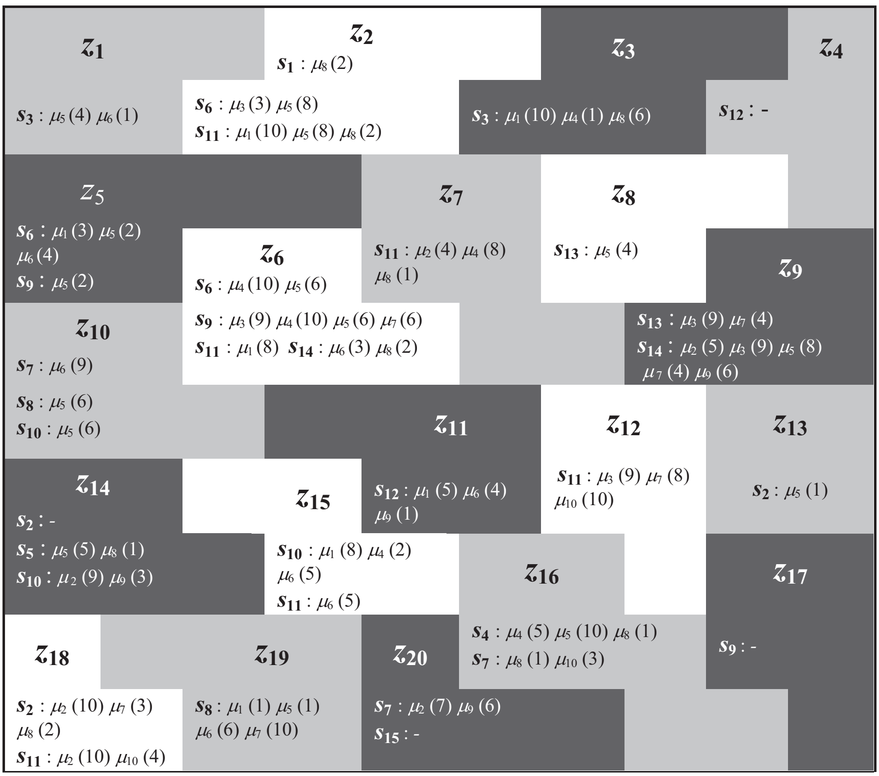

FIG. 1.3 – Twenty zones, z_1, z_2,…, z_{20}, are candidates for protection and fifteen species, s_1, s_2, …, s_{15}, living in these zones are concerned. For each zone, the species present and the threats associated, with their removal costs in brackets are indicated. The cost of protecting the white zones is equal to 1 unit, the cost of protecting the light grey zones is equal to 2 units and the cost of protecting the dark grey zones is equal to 4 units. For example, species s_7 and s_{15} are present in zone z_{20}, threats μ_2 and μ_9 affect species s_7 in this zone and there are no threats to species s_{15}. The cost of protecting this zone is equal to 4 units and the cost of removing threats μ_2 and μ_9 in this zone is equal to 7 and 6 units, respectively.

TAB. 1.2 – Optimal protection strategies for the instance described in figure 1.3 when the available budget is 25 units.

Protected zone	Protection cost of the zone	Species present in the zone	Threats associated to the couple (zone, species)	Threats removed	Total cost of removal threats in the zone	Square of the degree of protection of the couple (zone, species)
z_2	1	s_1	μ_8	μ_8	2	1
		s_6	μ_3 μ_5	–		0
		s_{11}	μ_1 μ_5 μ_8	μ_8		0.11
z_4	2	s_{12}	–	–	0	1
z_{10}	2	s_7	μ_6	–	6	0
		s_8	μ_5	μ_5		1
		s_{10}	μ_5	μ_5		1
z_{13}	2	s_2	μ_5	μ_5	1	1
z_{14}	4	s_2	–	–	1	1
		s_5	μ_5 μ_8	μ_8		0.25
		s_{10}	μ_2 μ_9	–		0
z_{17}	4	s_9	–	–	0	1
Total	15				10	7.36

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